

ARTICLE

Multiple Routes to Progress in Model-Based Economics

Till Grüne-Yanoff¹  and Caterina Marchionni^{2,3} 

¹Department of Philosophy, Royal Institute of Technology, Stockholm, Sweden, ²Philosophy, Faculty of Social Science, University of Helsinki, Helsinki, Finland and ³Department of Philosophy, University of Milan, Milan, Italy
Email: Caterina.marchionni@unimi.it

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Abstract

This article examines whether characteristic modeling practices in economics lead to progress, that is, improved explanations and predictions. We question the widespread assumption that progress in modeling occurs solely through successive vertical refinements toward a single “best” model. Instead, we propose that progress can occur through multiple pathways. Specifically, we identify five distinct horizontal paths to progress: increased isolation of causal factors, differentiation of targets and purposes, derivation of robust theorems, multiplication of inconsistent perspectival models, and exploration of novel possibilities. We argue that these pathways, when properly constrained, increase explanatory and predictive power and therefore lead to scientific progress.

1. Introduction

Does economics make progress? More specifically, do modeling practices in economics lead to better explanations and predictions of economic phenomena? This question, in different guises, has long concerned economists, methodologists, and philosophers of economics (e.g., Alexandrova and Northcott 2009; Backhouse 1997; Bohem et al. 2002; Boumans and Herfeld 2022; Hutchison 1978; Peruzzi and Cevolani 2022; Stigler 1983). Skepticism about economic progress often stems from the observation that economists rarely, if ever, discard their models even when faced with contrary empirical evidence. Rather than replacing older, inferior models with better ones, economics is characterized by a persistent plurality of models. Critics highlight that the models economists work with frequently fail to deliver accurate predictions (Rosenberg 1992), that economists rarely subject their models to rigorous empirical testing (Alexandrova and Northcott 2009), and that they often disregard falsifications when they occur (Blaug 1992). In response, some recommend substantial reforms, ranging from subjecting

The authors contributed equally to the article.

models to more stringent empirical tests (Blaug 1992) to reducing investments in model building while loosening economists' commitment to a unified theory (Northcott 2022).

Some of these concerns seem reasonable to us. However, we worry that the skepticism about progress in economics often relies on a narrow conception of theoretical progress, one that assumes that progress is simply a matter of getting closer to the truth. This notion of progress aligns with historical scientific developments, such as the shift from the Ptolemaic to the Copernican theory of the solar system. However, it does not seem to fit disciplines like economics, where much of the theoretical activity proceeds via idealized models and notions of truth and falsity do not straightforwardly apply (Alexandrova and Northcott 2009). Even if we redefine progress as getting closer to "the best model of a target," this still assumes that progress happens only through continuous refinements, with inferior models discarded along the way.¹

We argue, however, that in model-based sciences like economics, progress can occur through multiple pathways. Drawing on economist Dani Rodrik (2015), we claim that economics often advances "horizontally" rather than "vertically." As Rodrik puts it, "Knowledge accumulates in economics not *vertically*, with better models, replacing worse ones, but *horizontally*, with newer models explaining aspects of social outcomes that were unaddressed earlier" (emphasis added). This perspective is echoed by economists Gilboa et al. (2014, 2022), who argue that models in economics should be understood as theoretical cases, which cannot be refuted by empirical observation, and that progress is made through the formulation of new cases. Such views are buttressed by claims that empirical adequacy is one of many criteria for model development, evaluation, and selection—others include theoretical tractability, simplicity, or diversity—so poor empirical performance is often not seen as a sufficient reason to discard a model.

Furthermore, economists often construct new models by means of varying their syntactic structures, even before identifying a possible target for their model. A clear example of this developmental trajectory can be found in game theory. Mary Morgan (2012, 368–9) observes that "game theory has traditionally grown by a theorizing activity that fills in the empty cells in a taxonomy" and "by attempts to characterize particular economic situations . . . as game situations, and thus to type them or place them within a particular category of game." Robert Sugden (2011) has voiced a similar view, arguing that economists often create models "in search of observations."

These latter accounts indicate that the critics of economics (such as, among others, Blaug and Rosenberg) may be *descriptively* correct: Modeling in economics does not seem to follow the vertical conception of progress, where models are continuously refined to better represent a target, with failed models discarded along the way and expansion occurring only because of the need to model different targets. However, many practitioners seem to disagree with the critics' normative conclusions: They seem to think that their modeling practices are not thereby *normatively* problematic. We argue that the latter attitude makes sense if we view the horizontal proliferation

¹ Economists largely talk about "(theoretical) models," reserving the term *theory*, mainly for historical reasons, to name collections of models or construction manuals for models, for example, "game theory" or "oligopoly theory." In the face of these uses, we do not think that a meaningful distinction can be drawn between theories and models. In the rest of the article, except when mentioning those specific names, we refer to models both in horizontal and vertical contexts.

of models as an alternative route toward better explanations and more accurate predictions.² How such expansion is supposed to constitute or contribute to progress, however, remains unclear.

In this article, we explore the conditions under which horizontal model proliferation leads to cognitive progress. We focus on economics, as that is where our expertise lies. But we believe that our analysis has wider application: other disciplines too use multiple highly idealized models of the same targets in parallel (e.g., Levins 1966; Weisberg 2012; Batterman and Rice 2014). For these disciplines, the same questions apply as to economics: Can they also be regarded as making progress horizontally? For economics, we identify five horizontal routes and the conditions under which they contribute to progress: increased isolation of causal factors, differentiation of targets and purposes, derivation of robust theorems, multiplication of inconsistent perspectival models, and exploration of novel possibilities. Where applicable, we also indicate similar arguments for other disciplines.

Our aim is not to deny the relevance of vertical refinement, which we take for granted (for recent discussions, see Boumans and Herfeld 2022; Peruzzi and Cevolani 2022). Nor do we wish to offer an unqualified defense of economic modeling practices. To the contrary, the five horizontal routes we present each comes with each specific success conditions. It is clear to us that these success conditions can sometimes be satisfied in economics but often are not (we cannot vouch for this claim with respect to other disciplines). Our analysis therefore as much provides a novel tool for criticizing modeling practices in economics as it offers potential justifications. To legitimately claim that model proliferation contributes to horizontal progress in any of these five ways demands showing that this proliferation indeed satisfies the respective success conditions. If it does not, then model proliferation might indeed be as pointless as the critics claim it is.

2. Scientific progress

Progress is an axiological concept: It is goal directed (Niiniluoto 2024; Mäki 2002). A move from stage A to stage B counts as progress when stage B is better than stage A according to some specific criterion. Science aims for different kinds of progress, including empirical, methodological, technological, conceptual, and so on. Our focus here is on *cognitive progress*, understood as progress toward better explanations and more accurate predictions of the phenomena within the field's domain. In other words, progress from stage A to stage B occurs if there is an overall increase in predictive accuracy or explanatory power. This could be for a given target or because more targets are

² Note that we do not claim that the critical authors explicitly deny this claim. Rather, we believe this is a case of oversight. By default, not by explicit arguments, these authors assume that progress consists only in the replacement of models that have been shown to be inadequate—Some argue that models should be tested and improved (e.g., through deidealization) when falsified (Blaug 1992); others argue that model deidealization does not work and that therefore economics needs to be fundamentally reformed if it is to progress (Alexandrova and Northcott 2009; Northcott 2022). Neither considers the possibility of other routes to progress and hence conclude from a failure of vertical progress a failure of progress altogether. But in this case, *tertium datur*—and we aim to elaborate on this alternative in this article.

explained or predicted. Our focus is on cognitive progress because much of model-based activity in science and in economics is in fact driven by cognitive goals. This is not to say that cognitive goals are the only goals but rather to say that they are generally shared across the field.

To evaluate whether cognitive progress has occurred, different philosophical accounts—such as the epistemic, noetic, truthlikeness, and problem-solving accounts—rely on different criteria (Niiniluoto 2024). For example, on an epistemic account, scientific progress amounts to the accumulation of knowledge: Stage B is better than stage A if in B there is more knowledge than there is in A (Bird 2007). On a problem-solving account, progress occurs when, other things being equal, the number and significance of problems that can be solved in B are higher than they are in A (Kuhn 1962; Laudan 1978), and so on. Although these accounts differ, they are generally applicable to the achievement of cognitive goals like explanation and prediction. For example, more knowledge leads to better explanations and more accurate predictions, while an increased capacity to explain and predict amounts to solving more problems. For this reason, we set aside the finer distinctions between these accounts and define scientific progress in terms of achieving better explanations and predictions. Hence progress occurs when, other things being equal, either more targets are successfully predicted or explained in B than in A or the same target is better predicted or explained in B than in A. Determining when genuine progress has occurred thus requires criteria for what constitutes a good, and thus better explanation or prediction. Empirical accuracy is one such criterion, but not the only one. For example, explanatory power can be assessed on multiple dimensions (see Marchionni 2013; Ylikoski and Kuorikoski 2010). Similarly, adding more variables to a model may increase its complexity and decrease its fit with the data (Ruiz and Schulz 2023), and predictive success itself can be assessed on multiple criteria (Syrjänen, forthcoming). Different pathways may improve explanatory or predictive power on different dimensions.

Progress can be examined at different levels: across an entire field, within a specific subfield, or in individual modeling episodes. Although our goal is to assess whether specific modeling practices in economics are conducive to progress, our units of analysis will be families of models aimed at either the explanation or the prediction of a particular class of phenomena. Although we consider our cases to be representative of the way economic modeling practices proceed, we do not wish to claim that our analysis allows us to establish whether economics, as a whole, is a progressive science.

Last, we should ask how cognitive progress is achieved. In other words, one thing is to define what progress is—for us, better explanations and more accurate predictions; another is to say *how* progress is achieved. We argue that there are multiple pathways to progress. Stegmüller (1976) calls this *progress branching* (cf. Niiniluoto 2024). Therefore, unlike what Rodrik's terminology may suggest, we view progress as consisting not of two distinct types—horizontal and vertical—but rather of a single kind (improved explanatory and/or predictive power), which can be realized through either vertical development or horizontal expansion. For brevity, we refer to progress made vertically as vertical progress and to progress made horizontally as horizontal progress. Vertical progress occurs when a model is

improved leading to better explanations or predictions of a target system. In contrast, horizontal progress involves providing either better explanations or better predictions of a single target through multiple models or by extending explanations and predictions to additional targets using multiple models. We should keep in mind, however, that whether progress has actually occurred depends on whether there has been a genuine increase in explanatory or predictive power. Thus both vertical development and horizontal proliferation constitute progress only when they lead to an actual improvement in explanatory or predictive power.

To further illustrate the distinction between vertical development and horizontal proliferation, consider a set of models $\mathbf{M} = \{M_1, M_2, \dots, M_n\}$ used to represent targets within a domain $\mathbf{D} = \{T_1, T_2, \dots, T_k\}$. The representational use of these models involves mapping $\mathbf{M}$ onto $\mathbf{D}$, meaning that many members of $\mathbf{M}$ may represent one member of $\mathbf{D}$ and that some members of $\mathbf{D}$ may not be represented at all. Achieving progress through vertical development involves replacing or removing some of the members of $\mathbf{M}$. Model de-idealization is an example: when a model is replaced with a less idealized one which more accurately represents the target and hence allows for better explanations or predictions.³ Achieving progress through horizontal expansion, in contrast, involves adding new models to the set $\mathbf{M}$. Of course, the models that are added, replaced, or removed need to satisfy some conditions for the addition, replacement, or removal to count as progress. Some conditions pertain to the relationship between the new model and existing ones, whereas others concern the relationship between the model and its target. Importantly, long-term progress requires that our explanations and predictions become more empirically accurate—an improvement that occurs primarily through vertical refinement. Without such accuracy gains, model proliferation alone does not ensure genuine cognitive advancement. Hence achieving overall cognitive progress requires both introducing new models and refining existing ones.

3. Horizontal paths to progress

We identify five ways in which model proliferation can improve explanatory and predictive power. Drawing from the literature on model-based economics, these examples illustrate how, under specific conditions, horizontal model expansion contributes to scientific progress. The first three pathways assume that the models are *not* incompatible with one another. These models idealize their targets in different ways but do not necessarily contradict one other. In such cases, progress occurs by (1) capturing additional features of the same target, (2) expanding the range of models to represent more targets for different purposes, or (3) improving the reliability of explanations and predictions by establishing robustness theorems. In this sense, these pathways can be seen as extensions or adaptations of vertical progress to the case of multiple idealized models. In the remaining two pathways, the models are either inconsistent or incommensurable, yet the goal is not to determine which model provides the best explanation or prediction, and this is why we don't expect that progress will involve discarding purportedly inferior models. Instead, the

³ See Knuuttila and Morgan (2019) and Peruzzi and Cevolani (2022) for accounts of deidealization.

multiplication and coexistence of models serve an epistemic function, contributing to explanation and/or prediction in ways that go beyond selecting the best model. In this sense, the issue here is a version of the problem of using multiple conflicting idealized models to explain the same target (e.g., Morrison 2011; Rice 2019), now examined in the context of scientific progress. In other words, if we accept that multiple conflicting idealized models can be used to explain the same target, how can additional models, despite being inconsistent with the first, lead to better explanations of it?

3.1 Isolation of causal factors

Progress can occur through the multiplication of models that represent the same target but isolate distinct contributing factors or causal mechanisms. By *isolation*, we mean the process of developing models that focus on specific, independent causal factors, while other factors are held constant or left out by means of theoretical idealizations and omissions (Mäki 1992). The need for multiple models of the same target arises from the complexity of the systems under investigation, on one hand, and the idealized and partial nature of models, on the other (e.g., Mäki 1992, Mitchell 2002). For example, the plurality of models of patterns of residential segregation is well known in the philosophical literature. Different models can be thought of as isolating distinct potential factors responsible for segregation: Schelling's checkerboard model focuses on the causal contribution of mild-discriminatory preferences, while other models investigate the role of institutional discrimination, economic inequalities, and more.⁴ Together, these isolating models come to constitute what Ylikoski and Aydinonat (2014) aptly call a *menu of available explanations*. Adding new, relevant models to this menu increases our ability to identify the correct explanation for a wider range of real-world instances of segregation, thereby contributing to progress.

The explanations composing the menu (and hence the models representing them) are not mutually exclusive. There is no need to choose a single model for a given target. Instead, the causal factors they isolate combine in different ways to account for the target. Sometimes they combine additively as contributing causes; in other cases, only a subset may be relevant. In yet other cases, they overdetermine the outcome or the operation of one preempts the operation of the others. For example, the causal contribution of mild-discriminatory preferences represented in Schelling's model is in reality often preempted by factors like socioeconomic disparities and racism. This may account for why Schelling's model seems to perform poorly when tested against real-world data.

However, the model need not be discarded. First, the existence of mild-discriminatory preferences contributes to explaining why top-down efforts to create integrated neighborhoods frequently fail. Second, any attempt to explain residential segregation must now consider and possibly rule out the contribution of mild-discriminatory preferences vis-à-vis other possible causes of segregation (Ylikoski and Aydinonat 2014).

This kind of model proliferation should occur only if, and to the extent to which, different causal factors or mechanisms are in fact responsible for the phenomenon.

⁴ Isolating models are of course prominent in many other sciences. The classical-mechanics model of the planetary system, for example, describes the position of a planet as a function of time and disregards all its other properties (Cartwright 1989, chapter 5).

Furthermore, each isolating model should be developed vertically; that is, an isolating model can perform its explanatory function only if it is a sufficiently accurate representation of the contributing causal factors it aims to theoretically isolate.⁵ Expanding the set of models by adding models that do not represent an actual aspect of a target phenomenon will not contribute to progress via this route. For example, if it were found that mild-discriminatory preferences, as posited by Schelling, never played a role in explaining residential segregation, the inclusion of the Schelling model to the set of models of residential segregation could not be said to contribute to progress through this route.⁶

3.2 Differentiation of targets and purposes

Modelers often begin by investigating a target with a single model, only to discover that the assumed target is actually an agglomeration of heterogeneous items. Similarly, modelers often build a general-purpose model of a target, only to later realize that it only represents the target well enough for some purposes but not others. In these cases, new models are developed to represent each of these differentiated targets or to represent the same target for different purposes without, however, discarding the models they started with. This proliferation contributes to progress when it enables more fine-grained representations of a discipline's target domain or provides adequate representations of a target for more fine-grained purposes.

Oligopoly theory serves to illustrate this kind of horizontal development. Cournot proposed a first model in 1838, which was largely ignored until after his death, only to be mentioned by Walras as a forerunner and then uncharitably criticized by Bertrand in 1883. Bertrand's criticism led him to formulate an alternative model (Friedman 2000), which he saw as a replacement of Cournot's (i.e., a kind of vertical development). Later economists tended to hold that neither model is superior to the other. The accuracy of the predictions of each model will vary from context to context, depending on the actual situation of the companies represented. Roughly put, the Cournot model is preferable in situations where firms must make production decisions in advance and are unable to adjust their quantity levels to quantities produced by their rivals. The Bertrand model, in contrast, is preferable in situations where output quantities are sufficiently flexible and firms are capable of meeting any market demand that arises at the price level that they set (Quin and Stuart 1997). Thus, rather than replacing one model with the other, economists retain both as tools to be used for different targets and purposes.

The toolbox was further enriched by the Stackelberg leadership model in 1934, which, like the Cournot model, represents competition based on quantity. Yet the Stackelberg model introduces a dynamic feature: The "leader" first chooses the quantity it produces, and the "follower" observes the leader's decision. This leads to a substantial difference in the rational decision of the competitors, compared to the Cournot model. Over time, more models were developed, none of which replaced the Cournot, Bertrand, or Stackelberg model. Instead, they were seen as further

⁵ What it means for a model to be a sufficiently accurate representation of a target is a debated question in the philosophy of modeling. We don't take a stand here, except to say that standards of accuracy also depend on the purpose to which a model is put.

⁶ Mild-discriminatory preferences may still explain some hitherto unobserved cases. In such cases, its addition may contribute to progress as in the differentiation case that we discuss next.

differentiations of cases and purposes, allowing economists to select the most appropriate model for the case at hand (Horton Tremblay and Tremblay 2019).

This kind of model proliferation reflects the fragmented nature of the economics' subject matter, as well as the multiple purposes it serves. Economic institutions vary across history and cultures. There is no reason to believe that a single model should be capable of covering this variety. Instead, to capture these different phenomena with sufficient accuracy for a given purpose, economists develop and maintain many different representational vehicles. The variability of economic phenomena, both historically and geographically, implies that models that have proven to be of limited use in particular applications may become relevant in a different context or at a later time (Rodrik 2015, 5).

Furthermore, economics has multiple purposes. On a macro-level, economics responds to numerous demands, including explanatory, predictive, normative, and policy-guiding ones. This breadth is rarely demanded of other sciences. For example, no physicist is expected to build bridges. Each of these macro-demands separates into different micro-purposes, for example, when two models of segregation can both be explanatory and serve different purposes (e.g., answer different explanation-seeking questions). Responding to such a variety of demands gives further justification to model proliferation.

By collecting and retaining models that at some point have proven useful, economists increase their toolbox of representational vehicles for an ontically diverse world. In such a world, a large model set increases the chances of finding the right model for a new purpose. However, these considerations also set limits to their justification: Once the fragmented nature of a discipline's subject domain is covered with sufficient accuracy, and the different purposes are adequately addressed, further model proliferation ceases to be necessary.⁷ This kind of horizontal expansion thus can contribute to scientific progress, yet only as far as the given ontic and functional diversity of a discipline's domain demands.

This "target and purpose differentiation" account shows that model proliferation can contribute to progress. However, it is not obvious how different this is from vertical development. It could be argued that in this case, it is still possible to aim for the best model *given* a target-purpose dyad and that it is finding that model that counts as progress. If so, rather than an alternative route to progress, this should be considered as a more refined conception of vertical development in a discipline with an ontically fragmented domain and multiple purposes (in line with general philosophy of science arguments by, e.g., Giere 2010; Parker 2020). Maybe. For us, however, the important point is not to claim that there is an important qualitative difference between horizontal expansion and vertical development. As long as it is clear that no model is entirely discarded—because it may prove useful in the future or for a purpose not yet conceived—horizontal development plays an independent role in making progress.

⁷ Rodrik (2015) argues that economists are uncertain about the targets of their models changing in the future—hence there is never an end to such proliferation. We consider this argument to have little practical or methodological consequence. Although such uncertainty might certainly raise the expectation of future proliferation, it does not—without knowledge of what changes to expect—justify model proliferation in the present.

3.3 Derivation of robust theorems

The proliferation of models for the purpose of discovering robust theorems may also contribute to progress. Robust theorems are discovered through what Kuorikoski, Lehtinen, and Marchionni (2010) call *derivational robustness analysis*. The purpose of derivational robustness analysis is to identify a theorem that remains robust (and hence holds) across variations of those modeling assumptions introduced for tractability purposes. The models involved in this process share a common core that is hypothesized to represent the set of causes or causal mechanisms believed to be responsible for the phenomenon. If across such variations in tractability assumptions the result remains the same, confidence in the validity of the robust theorem, which comprises the common core and result, increases. For robustness analysis to work, however, several conditions need to be satisfied. First, the common core should be a sufficiently accurate representation of the specific causal factors that are believed to be central to the phenomenon under study. Second, there should be enough variation in tractability assumptions. Third, the varied assumptions should be *independent* of one another. How to best cash out this independence is highly contested, however (see, e.g., Harris 2021). For our purposes, what matters is to recognize that the independence requirement places a significant constraint on whether derivational robustness analysis delivers a genuinely robust theorem.

Kuorikoski, Lehtinen, and Marchionni (2010) work out the example of geographical economics. They claim that its development was characterized by the construction of several models that differed only in one or two assumptions while sharing a core set of assumptions, those laid out in Krugman (1991), thought to represent an important mechanism for the spatial agglomeration and dispersion of economic activity. At least part of the proliferation of these relatively similar models could be rationalized as an instance of derivational robustness analysis. Insofar as an increase in confidence in the robust theorem contributes to better explanations and predictions, robustness-driven proliferation qualifies as an instance of horizontal expansion favoring progress.⁸ For this to occur, there ought to be genuine independence between the different assumptions that are varied across derivations. For example, McCann (2005) argues that the functional forms attributed to transportation costs in several of the models of geographical economics all share the same bias. Lisciandra (2017) argues that this is so for the models discussed by Kuorikoski, Lehtinen, and Marchionni (2010). Whether this is the case or not, this debate clearly illustrates that establishing the relevant independence of assumptions is far from straightforward. Independence, however, is a precondition for derivational robustness analysis to yield epistemic benefits. Therefore, for horizontal expansion to contribute to progress through this route, the following conditions must be satisfied: a set of models that share a common core, which is a sufficiently accurate representation of a set of causal factors or a causal mechanism; that all derive the same result; and that have enough variation of genuinely independent assumptions.

⁸ The derivation of robustness theorems is an important issue also in biology, as Weisberg (2013, chapter 9) argues at the hand of the Lotka–Volterra model.

3.4 Multiplication of inconsistent models

In the two previous cases, the models at hand were different yet complementary. They isolated distinct causes, targeted different aspects, or served different purposes. But could progress also occur through the proliferation of models that represent the same target, explain or predict it equally well, yet ascribe incompatible features to it? When faced with inconsistent models, progress is often thought to occur when scientists establish grounds for deciding which model to retain while discarding the others. As models are refined and improved, one may prove to explain the data better or account for a broader range of phenomena. Alternatively, a new model may emerge that supersedes previous models, resolving their inconsistencies rather than simply incorporating them as special cases. All these strategies align with the vertical route to progress.

In philosophy of science, the issue of justifying the simultaneous existence of models that give inconsistent descriptions of the same target has been discussed as the *problem of inconsistent models* (Morrison 2011; see also Rice 2019). The main challenge is how to square the use of inconsistent models with the requirement that explanations be *factive*—that is, grounded in truth. For example, Morrison (2011) characterizes two (or more) models as inconsistent with each other if (i) one describes the target in ways that contradicts the assumptions of the other, (ii) these descriptions/assumptions are “fundamental” in the sense that they concern the target structure responsible for determining specific types of behavior, and (3) neither model emerges as having greater predictive power with fewer internal difficulties and contradictions with experimental findings. Although justifying a plurality of inconsistent models is a slightly different endeavor from explaining whether and how their multiplication contributes to progress, the two issues are closely related: If there are epistemic reasons to accept the coexistence of multiple inconsistent models of a given target, then in some cases, the proliferation of such models may contribute to progress. The jury is still out on how to assess such situations. Morrison herself was rather skeptical.⁹ While we are not committing to a perspectivalist account of models or to its success in justifying the problem of inconsistency models, we believe that if it can be made to work, perspectivalism would open up another route to horizontal progress.

According to perspectivalist accounts of models, a target may exhibit certain features from one perspective and entirely different features from another (Giere 2019). For instance, an object may move along a well-defined trajectory at any given time from the classical-mechanics perspective, while not having a well-defined position at every moment from the quantum mechanics perspective (Frigg 2022, 445). Whether this reliance on perspectives offers a convincing solution to the problem of inconsistent models remains debated: Some argue that it collapses into antirealism or into uncontroversial claim that the models are not inconsistent after

⁹ Morrison (2011, 347) describes the problems of inconsistent models as follows: Each “provides some ‘insight’ . . . however, none offers more than partial ‘truths’ and each is in conflict with claims made by the others. While each model has its particular successes and together are sometimes taken as complementary insofar as each contributes to an overall explanation of much of the available experimental data, many basic questions are left unanswered precisely due to the lack of a comprehensive account.”

all, as they highlight different aspects of the same target (Frigg 2022; Morrison 2011). We do not aim to resolve the debate here. Instead, for the sake of the argument, let us assume that some inconsistent models are perspectival in such a way that their respective truth cannot be adjudicated outside a given perspective. In this framework, the proliferation of models can lead to improvements in explanatory or predictive power if a newly introduced perspectival model reveals features of the phenomenon that would remain hidden within extant perspectives. Moreover, the interaction among these distinct perspectives may itself be epistemically productive: It pushes researchers to refine their models in ways that would not occur if only a single perspective were available (Chang 2012; Massimi 2022).

To identify possible cases of perspectival models in economics, we can proceed in two ways. First, we can begin by observing that two or more models of the same system are inconsistent and hypothesize that their inconsistency is due to the perspectives they endorse. Second, we can note that multiple theoretical approaches coexist in economics and hypothesize that these approaches reflect different perspectives and hence, when applied to the same target, they are likely to produce inconsistent models.

Let us start with the first approach. As an example, consider the case in which the same decision situation is described as either a decision under risk or a strategic decision. A decision under risk attributes a probability to states of the world, which determine the outcome of choices; many accounts demand that the decision situation be described so that these probabilities are independent of the choices. A strategic perspective, in contrast, describes the outcomes of a decision situation as the result of many agents' choices, where each agent is looking at the decision situation in similar ways. From this perspective, it does not make sense to rely on unconditional probability assignments in one's decisions, as the choice of one agent depends on what they consider rational for the others to choose, and vice versa (Bermúdez 2015).¹⁰

Strategic and risky decision descriptions are inconsistent in the sense that some of their core assumptions contradict each other and that they yield different predictions and normative recommendations. Despite this inconsistency, it might be beneficial to develop and maintain both perspectives simultaneously, even when investigating a single situation. First, it is often epistemically very difficult to determine which perspective agents have adopted, as these concern psychological factors that can change quickly and imperceptibly. Second, there may not be a fact of the matter about which perspective is correct, even for a single situation. An agent might assume a strategic perspective, only to draw a conclusion about the probability of an outcome, then immediately consider a strategic response to that conclusion, thus toggling between these perspectives until a choice is made.

Even when investigating a single decision situation, expanding the model set to include both a risky and a strategic perspective may promote progress. However, at least two conditions must be met for such an expansion to be progressive. First, the

¹⁰ Morrison (2011), in her discussion of inconsistent models, draws on examples from nuclear physics, showing that this is not a problem of economics alone.

modeler must have no evidence indicating with substantially higher probability that one of the perspectives is the correct one. For example, if there is substantial evidence that agents in a particular situation are taking the risky perspective, then the only model that promises accurate predictions or valid normative recommendations is the risky one, and the strategic model should be discarded. Second, there must not be an acceptable model that synthesizes both perspectives, where acceptability includes desiderata like simplicity, tractability, and cognitive salience. If a model could be developed that accurately and intelligibly represented how agents toggle between the risky and the strategic frames, then that model would be preferable to both the risky and the strategic perspectives as the better predictor of the agents' choices. Many situations meet these conditions, making the horizontal expansion of inconsistent models a possible path to progress in economic modeling.

The second way of identifying inconsistent models starts by noting that in economics, rival schools of thought exist—including neoclassical, Marxian, Keynesian, Austrian, and feminist. We can think of each school of thought as a scientific perspective, understood as the set of field-specific practices including a body of knowledge, theoretical, empirical and technological resources, and a set of methodological and epistemic principles for the justification of knowledge claims (Massimi 2022). (cf. Lari 2021) As an illustration, neoclassical economics and feminist economics models of the same target might be inconsistent because they rely on inconsistent assumptions about the nature of unpaid domestic labor and child-rearing, that is, on whether they represent the economic activities that are essential to the “provisioning of human life” (Nelson, cited in Lari 2024). The existence of inconsistent assumptions provides a *prima facie* reason to consider it a case of perspectival modeling, even if establishing that they are indeed inconsistent would require examining specific modeling instances. Improving explanations and predictions will thus require the proliferation of perspectives, at least as far as these are scientifically legitimate (e.g., their practices of knowledge production are reliable and justificatory) and as long as none of them is superior to the others. Moreover, if at least part of the epistemic benefits arises from the productive engagement between rival perspectives, then more interaction than is now the case should characterize the relation between different schools of thought in economics (cf. Chang 2015).

3.5 Exploration of possibilities

Under certain conditions, which we discuss shortly, the proliferation of models for the purpose of exploring possibilities can also contribute to progress. As an illustration, consider Slutsky, 1927, (1937) article “The Summation of Random Causes as a Source of Cyclic Processes” (originally published in a Russian journal as Slutsky), where he showed that it was possible for apparently cyclic patterns to emerge from random shocks to the economy if the latter were modeled using a stable stochastic difference equation with certain technical properties. Slutsky constructed several models for this purpose. The most prominent of these consisted of a 10-item moving summation of a random number series, expressed in the equation

$e_k : y(t) = \sum_{k=1}^{10} e_k$. Using this model, he showed that the summation of random causes generated wavelike phenomena.

Furthermore, by superimposing the graph of the model-generated “random” cycle onto the graph of the index of English business cycles for 1855–77, Slutsky demonstrated that these wavelike fluctuations could imitate cycles exhibiting approximate regularity and thus imitate observed actual business cycles. The widely recognized conclusion from Slutsky’s model was that observed business cycles could possibly be explained as the summation of purely random causes and did not necessarily require explanation by an underlying periodic mechanism (Barnett 2006). This opened a new approach to business cycle modeling by hypothesizing that the interaction of chance events could generate periodicity where none existed before.¹¹

What is important for our argument is that Slutsky used the model to demonstrate only that it was *possible* for regular business cycles to have random causes, not that they actually did. His model was intended to elucidate a possibility, not to represent real business cycles. For this reason, it cannot be seen as a replacement of earlier models of the business cycle; rather, it is an expansion of the set of such models, which now includes those that elucidate mere possibilities. Slutsky’s contribution thus expands rather than refines existing business cycle models. Nevertheless, his model has been universally recognized as an important contribution to the analysis of economic fluctuations and is frequently described as “groundbreaking” (Barnett 2006). Arguably, it improves actual periodic explanations by identifying the conditions under which this explanation would not apply, thus expanding the range of answers to relevant counterfactual questions (Verreault-Julien and Grüne-Yanoff 2025). Therefore it should be considered an exemplar of horizontal expansion contributing to economic progress.

Slutsky’s modeling result had such a groundbreaking effect not simply because it elucidated some possibility but also because the possibility satisfied important epistemic conditions. First, the possibility it elucidated was of great epistemic relevance. At the time, existing models of business cycles assumed that cycles were *necessarily* generated by some underlying periodic mechanism, such as sunspots or delays in replacement of basic investments. It was against this implicit necessity claim that the possibility Slutsky elucidated gained epistemic relevance: If random causes are a genuine possibility, then this refutes the necessity claim about periodic mechanisms (for discussion of such argument patterns in a different context, see Grüne-Yanoff 2009). Second, Slutsky’s model convinced readers that he elucidated an *economic* possibility, not just a logical or statistical one. He did so by linking the processes involving moving summation to actual developments in the economy, specifically linking rainfall over many days to crop yields:

It seems probable that an especially prominent role is played in nature by the process of *moving summation* with weights of one kind or another.... For

¹¹ Such possibility exploration has also been documented in other disciplines. Weisberg (2013), for example, discusses how the modeling of unnatural biochemical systems—specifically xeno nucleic acids (XNAs), which are materially different from but functionally similar to DNA and RNA—supports the hypothesis that it is possible to have materially different realization bases for the essential functions of living systems, in particular the ability to carry genetic information.

example, let causes ... x_{i-2} , x_{i-1} , x_i , ... produce the consequences ... y_{i-2} , y_{i-1} , y_i , ... where the magnitude of each consequence is determined by the influence, not of one, but of a number of the preceding causes, as for instance, the size of a crop is determined, not by one day's rainfall, but by many. (Slutzky, 1927, 1937, 108)

By illustrating the modeled process with real-world examples, Slutsky showed that processes like that were occurring in nature. Consequently, the results of these processes, although modeled mathematically, could be seen as genuine economic possibilities, not just statistical artifacts.

These are demanding conditions. Economic modelers cannot expect that any horizontal expansion into novel possibilities will lead to better explanations and predictions. To the contrary, the proliferation of possibility claims in some disciplines is often seen as a mere apology of unsuccessful modeling attempts, precisely because they do not meet these epistemic conditions. If horizontal expansion through new possibility-elucidating models contributes to progress, it does so only under qualified conditions.

The Slutsky model sought to argue for an objective possibility: It concerned the question whether an actual phenomenon could possibly be produced by certain causes. But some models may elucidate epistemic, rather than objective, possibilities. Epistemic possibilities concern whether a certain claim might be true in light of current evidence or knowledge—that is, whether current knowledge either allows or rules out the truth of such a claim (for more details on this distinction and its relevance for science, see Sjölin Wirling and Grüne-Yanoff 2024).

For example, consider the well-known underdetermination problem: The basic idea is that scientific success criteria (including but not restricted to empirical adequacy) typically do not suffice to select a single model, leaving many candidate models for a given purpose. This problem is further complicated by the fact that many possible models remain unconceived at any given time. The history of science is full of examples when earlier scientists failed to conceive theoretical possibilities that later displaced existing theories (Stanford 2009). To address this problem, scientists not only collect existing models that are equally supported by the evidence but actively seek to construct possible alternatives, expecting that these alternatives will be just as well supported by the evidence as our best current models. Model ensembles, representing uncertainty about which model is the most adequate representation of a target for a specific purpose, can be found in climate science, for example (Betz 2009). These ensembles represent not objective possibilities, as Slutsky's model did, but rather epistemic possibilities, reflecting the uncertainty about which model is the most accurate given the current state of knowledge. An expansion of such an ensemble might not seem to contribute to progress, but rather the opposite: Expanding the set indicates an increase of uncertainty. Although knowledge of such uncertainty can be useful, it does not directly help with either explanation or prediction and therefore should not be considered as contributing to progress.

4. Conclusions

Horizontal model proliferation is a fact in economics, as well as in many other scientific disciplines. Instead of identifying a current model as an inaccurate

representation of a certain target or inadequate for a given purpose, and hence replacing it with a better one, economists often construct another model of the same target or for the same purpose and then use it alongside the previous one without committing explicitly to the superiority of either. Practicing economists and economic methodologists have already noted this feature of economics (Rodrik 2015; Gilboa et al. 2022; see also Aydinonat 2018 and references therein). However, describing these practices fact does not provide a justification for them. To the contrary. Critics of economics have seen them as highly problematic. The only justifications that Rodrik (2015) and Gilboa et al. (2022) offered, as we discussed in section 3.2, concerned the differentiation of targets and purposes, thus leading in effect to an extension of the scope of economics. In this article, we both developed this particular justification further and advanced four novel justifications for horizontal progress: Proliferation of models might support the isolation of causes, it might clarify and distinguish multiple perspectives on a phenomenon, it might help explore novel possibilities, and it might help with derivational robustness analysis.

We marked each of these as a potential justification, conditional on specific success conditions. This is the second contribution of our article. The success conditions we identify can be and have been satisfied in economics, as some of the cases we have presented show. We also pointed to discussions of such potential justifications in other disciplines, but we do not want to judge whether the success conditions there have or even can be satisfied. In general, although a universal skepticism against horizontal progress seems indefensible, there is no guarantee that the success conditions will always or even often be satisfied and hence that the corresponding modeling practices will be justified in economics or elsewhere. Instead, our identification of these success conditions can be a renewed basis for criticism of such practices: Besides not contributing to vertical progress, they might now also be diagnosed to fail to contribute to horizontal progress. Our arguments should therefore not be mistaken for a blanket endorsement of current modeling practices in economics. Instead, by differentiating the multiple pathways through which progress can occur and by unpacking the conditions required for their success, we provide a novel framework for critically engaging with contemporary modeling practices in economics.

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