where $\Delta^2 = \Sigma\{T_{xy,xy}\sin^2 xy + 2T_{xy,xz}\cos xy\cos xz - 2T_{xy,zu}\cos xy\cos zu - 2T_{xy}\cos xy\}$

wherein $T_{xy,xy} = (AB' - A'B)^2$

$T_{xy,xz} = (AB' - A'B)(AC' - A'C)$

$T_{xy,zu} = (AC' - A'C)(BD' - B'D) + (AD' - A'D)(BC' - B'C)$

$T_{xy} = (AC' - A'C)(BC' - B'C) + (AD' - A'D)(BD' - B'D)$ (8).

Now $f_1 = D\pi/\Delta_1$ and $f_2 = D'\pi/\Delta_2$

whence, on reduction, we find $f^2\Delta^2 = Q^2\pi^2$.

Substituting in (7), and remembering that $\sin xu = \pi/p$, we obtain finally

$$P = \frac{AD' - A'D}{\Delta}\sin xu.F \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

Δ having the form given in (8).

Kötters synthetic geometry of algebraic curves—Part II., involutions of the second and higher order.

[See Index.]

Amsler's Planimeter.

By Professor Steggall.

There are many proofs of the principle of this planimeter, but all that are accessible to me seem a little beyond the grasp of many students who use the instrument. It seems worth while, therefore, to notice the following proof, which, to the best of my knowledge, is new.

Let A (fig. 7) be a pivot, round which the pivoted rods AB, BC rotate, and let AB′, B′C′ be a consecutive position of the rods, when the point C has traced out the arc CC′ of the curve whose area is required; and let us first suppose that this curve does not include the point A. The element of area ABCC′B′ consists of three parts, namely :—

(1) The triangle ABB′.

(2) The parallelogram BCC″B′, where B′C″ is equal and parallel to BC.

(3) The triangle C′B′C″.

Now, since the planimeter in its circuit returns to its original position, the areas (1) have a sum zero; so have the areas (3); and the area of the closed figure is, therefore, equal to the sum of all such parallelograms as BCC″B′; and this sum equals their common side BC, multiplied by their total heights.

Next, let P be any fixed point on BC, then its (elementary) motion at right angles to BC consists of the line P′M + MN, which is perpendicular to BC and B′C′; the sum of these elements is the registration by a wheel at P, free to revolve on BC as an axis; but the sum of the elements P′M vanishes (as is the case with the areas (1) and (3)) from its twofold description, and, therefore, the registration of the wheel is equal to the sum of all the heights of the parallelograms BCC′B″. Hence the area of the closed curve is equal to BC × registration of wheel, where "registration of wheel" means number of revolutions × circumference.

If the curve enclose the point A, as the instrument is not constructed to allow BC to cross over AB, we must note that our curve involves, besides the parallelograms, the areas of the circles whose radii are AB, BC; and the registration only gives us the heights of the parallelograms, together with the circumference of the circle of radius BP. Thus we must add to our result obtained by the usual reading, the expression $\pi AB^2 + \pi BC^2 - 2\pi BP.BC$. If a circle of known area is described about A, this quantity can be at once found for any particular instrument.

Fifth Meeting, March 8th, 1889.

George A. Gibson, Esq., M.A., President, in the Chair.

An Elementary Discussion of the Closeness of the Approximation in Stirling's Theorem.

By Prof. Chrystal.

[The substance of this paper will appear in the second volume of Professor Chrystal's Algebra.]