Macroeconomic Dynamics (2024), **29**, e56, pp. 1–23
doi:10.1017/S1365100524000634

CAMBRIDGE UNIVERSITY PRESS

ARTICLE

Investments, credit guarantees, and government subsidies in a regime-switching framework*

Xiang Liu[1,2] and Zhaojun Yang[3]

[1]Yingyang School of Financial Technology, Zhejiang University of Finance and Economics, Hangzhou, China
[2]School of Finance, Zhejiang University of Finance and Economics, Hangzhou, China
[3]Department of Finance, Southern University of Science and Technology, Shenzhen, China
Corresponding author: Zhaojun Yang; Email: yangzj@sustech.edu.cn

Abstract

This paper examines how credit guarantees and government subsidies impact investment in a regime-switching model. We provide new explicit pricing formulas for a general standard asset. Almost all common corporate securities' prices can be easily derived by the explicit formulas though project cash flows are driven by both a Brownian motion and a two-state Markov chain. We provide a method about how governments should specify a proper tax subsidy standard for a given tax rate to motivate a firm to invest in a project in the way they wish. If the tax subsidy is sufficiently high (low), an overinvestment (underinvestment) occurs. The higher the tax rate, the more significant the overinvestment (underinvestment). We pin down the subsidy amount required for motivating a firm to invest immediately and fix the optimal capital structure with government subsidies.

Keywords: real options; regime-switching model; credit guarantees; government subsidies; capital structure

JEL classifications: D82; G12; G32

1. Introduction

Micro, small and medium-sized enterprises (MSMEs) are new drivers of the national economy of a country; actually, almost all large companies originate from a small business. Unfortunately, it is a long standing problem that most of MSMEs suffer from financing difficulties. In particular, small businesses have limited access to capital to start a project. Borrowing with credit guarantees is often the last resort. These problems are widely studied. However, an analysis of investment decisions with government subsidies and credit guarantees remains absent. This paper aims to fill this gap.

At the end of the 19th century, credit guarantee schemes first appeared in Europe. Credit guarantees are generally regarded as the most common and most effective program for government to support MSMEs (Cowling and Siepel (2013)). In a guarantee contract, governments play an important role (Beck et al. (2010), Honohan (2010)). For instance, it is necessary for governments to provide those MSMEs having significant positive externalities in overcoming global challenges (e.g., climate change, environmental deterioration, rising unemployment, increasing inequality, balancing economic needs with societal needs, etc.) with low guarantee fees and direct subsidies. Excessive fee cuts or subsidies are often far away from the primary objective and fail to realize the

*We thank the editor and an anonymous referee for their patience and helpful comments. The research reported in this paper was supported by the National Natural Science Foundation of China (Project Nos. 72031003 and 72432005). The major work of the paper was finished while Xiang Liu was a Ph.D student at the Southern University of Science and Technology.

effective use of government funds, while too less fee reduction or subsidies are unable to motivate MSMEs to invest in projects with valuable positive externalities but insufficient private profits.

To finance MSMEs with government subsidies, there are many different financial arrangements. Blanchard and Perotti (2002) present that positive government spending shocks have a positive effect on output, while taxes have the opposite one. Economides et al. (2022) study the implications of changes in the tax spending-public finance policy mix on private incentives and the macroeconomy. By contrast, we wonder how to make reasonable and effective use of government funds to subsidize enterprises with credit guarantees, and how much money the government should provide in a subsidy at least to make an entrepreneur invest immediately in a project having enough positive externalities. To the best of our knowledge, these problems incorporating credit guarantees are not considered in the literature. In particular, economic environment is experiencing more and more frequent random shocks, say the recurrence of COVID-19 pandemic. Therefore, in this paper, we address MSMEs' investments, credit guarantees and government subsidies in a regime-switching framework.

Currently, it is highly recognized that investors should integrate environmental, social and governance (ESG) factors into their investment plan instead of a pure economic profit or loss. Mayeres and Proost (1997) study the optimal externality tax problem and public investment rules. Montmartin and Herrera (2015) indicate positive spatial spillovers among private R&D investments and compare R&D subsidies and fiscal incentives. Doyle (2010) proposes that the government should stimulate investors' investment behavior through reasonable tax subsidies, so as to internalize information externalities. Actually, to motivate or discourage an investment, government intervention is getting more popular. For example, The CHIPS and Science Act just passed provides \$52.7 billion for American semiconductor research, development, manufacturing, and workforce development. This paper internalizes investment externalities and takes tax subsidies through both guarantee premium reduction and cash subsidy as an incentive instrument to motivate MSMEs to invest in a project that has positive externalities but possibly with a negative private value.

Our work is related to the long line of real options, which originated with the work of Myers (1977). Brennan and Schwartz (1985) examine natural resource investment through option pricing theory. According to Dixit et al. (1994), investment opportunities are regarded as options on real assets and the optimal investment policy is realized by maximizing the value of options. Real option theory is still attracting researchers' interest.

Concerning regime-switching models, Hamilton (1989) proposes regime shifts in explaining the cyclical features, which is now widely accepted. Guo et al. (2005) propose and solve a model of investment decisions with regime shift. Hackbarth et al. (2006) introduce regime shifts in the aggregate shock, and analyze the impact of macroeconomic conditions on dynamic capital structure and credit risk. Chen (2010) introduces the macroeconomic conditions into a consumption based asset pricing model. Hwu et al. (2021) develop an N-regime endogenous Markov-switching regression model.

Combined with option pricing and regime switching, Zhang and Guo (2004) present closed-form solution for perpetual American put option and study optimal stopping time problem in a regime-switching model. Luo et al. (2019) consider the real option with a continuous-time two-state Markov chain under double exponential jump-diffusion assumptions, and present an explicit expression with partial information. Luo and Yang (2017) consider real options and contingent convertibles with a regime-switching model. Luo and Yang (2019) address the growth option pricing method with equity default swaps in a regime-switching framework. However, neither of them considers credit guarantees taking government subsidies and investment externalities into account.

This paper is most closely related to Luo and Yang (2019). However, the distinctions are significant. First, we assume that both the drift and volatility of the regime-switching framework are determined by the state of economy regime, while the volatility in their model keeps constant. Clearly, the volatility is also an important determinant of the optimal investment policy. In other words, as the volatility changes over the business cycle, so does the value-maximizing investment policy. In a two-state regime-switching model, when one regime is assumed to dominate the other, it is generally recognized that the better regime has a higher drift. However, the diffusion may be ambiguous, see Guo et al. (2005) and Jeon and Nishihara (2015) among others. We thus make more general assumptions than usual: In our cash flow model, both the drift and diffusion depend on the economic state. That is, we extend the common geometric Brownian motion (GBM) cash flow model to a general Markovian regime-switching GBM. The generalization of the cash flow model makes the related pricing more challenging than before. Second, we provide a closed-form valuation for a general standard asset *à* la Leland (1994). Last and most importantly, we incorporate credit guarantee schemes, government subsidies and investment externalities into the regime-switching model.

The main contributions of this paper are summarized below. We develop a regime-switching model on a real investment with credit guarantees, government subsidies while investment externalities are taken into account. We derive the closed-form solutions for financial decisions and provide explicit pricing formulas for a general standard asset, which are never derived before. In particular, almost all common corporate securities' price can be easily derived by the explicit formulas even though the cash flow generated by the project is driven by both a Brownian motion and a two-state Markov chain. We present a detailed analysis of investment decisions with credit guarantees and government subsidies. We fix the amount of subsidies required for entrepreneurs to make an immediate investment and pin down the optimal capital structure under the government subsidy policy. We provide a method with respect to how governments should specify a proper tax subsidy coefficient for a given tax rate to motivate a firm to invest in a project in the way they wish. If the tax subsidy coefficient is greater (less) than 1, an overinvestment (underinvestment) occurs. The higher the tax rate, the more significant the overinvestment (underinvestment).

The structure of the paper is as follows: Section 2 describes the model setup and assumptions. Section 3 discusses the general corporate security pricing. Section 4 addresses the tax value and the fair credit guarantees with government subsidies. Section 5 provides the pricing and timing of the option to invest in a project. Section 6 presents numerical analysis. Section 7 concludes.

2. Model setup

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space endowed with the information flow $\{\mathcal{F}_t\}_{0\leq t<\infty}$ satisfying the usual conditions,[1] on which a standard Brownian Motion W and two-state continuous-time Markov chain L are defined. The Markov chain L is independent of the Brownian motion and right-continuous with values in the set $\{0, 1\}$, where 0 and 1 represent the recession and boom state of the economy respectively.

Consider that an entrepreneur or firm has an option to invest in a project with limited financial support from a government. The investment is irreversible but delayable with the sunk cost I. The earnings before interest and tax (EBIT) of the project is governed by the following Markovian regime-switching GBM:

$$\frac{dX_t}{X_t} = \mu\,(L_t)\,dt + \sigma\,(L_t)\,dW_t,\ X_0 = x, \tag{1}$$

where $\mu(L_t)$ is the risk-adjusted drift and $\sigma(L_t)$ is the state-dependent volatility. As usual, we assume $\mu(0) \leq \mu(1) < r$ (the risk-free interest rate), and $\sigma(1) > 0$ may bigger or smaller than $\sigma(0) > 0$. This is a classical real options problem with a new feature. A new considerable challenge arises from the fact that we are not sure whether we should invest earlier in boom than in recession if the volatility in boom is higher than that in recession. For the same reason, for a levered firm, we are not sure whether we should default later in boom than in recession if the volatility in boom is less than that in recession. To overcome the challenge, we use a guess-and-verify method.

Taking investment exteralities, e.g., ESG concerns, into account, we assume that governments will provide a subsidy (or economic punishment) to the investment depending on the firm's investment externalities. Specifically, the amount of the subsidy depends on investment externalities and the tax value generated by the investment. Naturally, a negative externality means a negative subsidy or a punishment for the investment. The higher the positive externality or the higher the tax value generated, the higher the subsidy.

We suppose that the entrepreneur has no money to exercise the investment and must borrow from a bank with a credit guarantee provided by an insurer. Thanks to the guarantee, the debt is risk-free and thus the bank lends money to the entrepreneur (borrower) at the lowest rate, i.e. the risk-free interest rate of the market. In return for the guarantee, the borrower pays the insurer the guarantee fee, which is a fraction of the money borrowed, with all the subsidy provided by the government for the investment. In this way, all the government subsidy is harvested by entrepreneurs through credit guarantees. The government exogenously determines the subsidy intensity with the tax rate and indirectly control the pricing and timing of the entrepreneur's option to invest in the project.

In accord with Goldstein et al. (2001), we simplify tax structure, including corporate tax and personal tax. We assume corporate effective tax rate $\tau_f \equiv 1-(1-\tau_c)(1-\tau_d)$, where τ_c denotes corporate profit tax rate, and τ_d denotes effective dividend tax rate. The bank interest tax rate or personal tax rate is denoted by τ_i.

3. Pricing corporate securities

In this section, we derive an analytical price of a general corporate security. We note that our model is time-homogeneous, and thus the investment and default thresholds are independent of time. Armed with the general pricing formula, we give the value of debt and equity according to their cash flow levels and boundary conditions.

3.1 A general pricing formula for a standard asset

In corporate finance, almost all the existing corporate securities can be priced by considering a special form of the standard asset we discussed here. By a standard asset we mean that its cash flow has a linear relationship with the firm's EBIT, i.e. $aX_t + K$ for some constant a and K when the EBIT falls in an economic state-dependent domain $\mathfrak{D}_{L_t}, L_t \in \{0,1\}$. The dynamics of the cash flow EBIT X is governed by the Markovian regime-switching GBM (1). We denote by $T_{\mathfrak{D}}$ the first time for the cash flow exiting the current domain, i.e. $T_{\mathfrak{D}} \equiv \inf\{t \geq 0 : X_t \notin \mathfrak{D}_{L_t}\}$, $L_t \in \{0,1\}$. At the stopping time $T_{\mathfrak{D}}$ the owner gets nothing but a claim whose value equals to $G(X_{T_{\mathfrak{D}}}; L_{T_{\mathfrak{D}}})$. For example, if the standard asset is debt, then $T_{\mathfrak{D}}$ would be the default time and $G(X_{T_{\mathfrak{D}}}; L_{T_{\mathfrak{D}}})$ represents the liquidation value. If there is no regime-switching, the pricing of the standard asset is well addressed by Tan and Yang (2017). Model (1) generalizes the models considered by Guo et al. (2005) and Luo and Yang (2017) *à* la Leland (1994). This generalization makes the pricing challenging.

Our model is time-homogenous and thus we assume the current time is zero without losss of generality. According to the risk-neutral pricing theory, the value of the standard asset is given by

$$Q_l(x) = \mathbb{E}\left[\int_0^{T_{\mathfrak{D}}} e^{-rt}(aX_t + K)dt + e^{-rT_{\mathfrak{D}}} G(X_{T_{\mathfrak{D}}}; L_{T_{\mathfrak{D}}}) \mid X_0 = x, L_0 = l\right].$$

We consider two cases: $\mathcal{D}_0 \subseteq \mathcal{D}_1$ and $\mathcal{D}_1 \subseteq \mathcal{D}_0$ respectively, which are naturally determined in advance to compute these prices. Following Guo et al. (2005) and Luo and Yang (2017), the rang of the EBIT can be partitioned into three regions if $\mathcal{D}_0 \subseteq \mathcal{D}_1$:[2] $\mathcal{D}_0$, $\mathcal{D}_1 - \mathcal{D}_0$ and $\overline{\mathcal{D}_1}$ (the complement of $\mathcal{D}_1$), which are called continuation region, transient region and stopping region respectively. In continuation region, the standard asset generates cash flow in both recession and boom states. In stopping region, or in transient region with the recession state, the claimant of the asset gets nothing but a lump-sum payoff, and by contrast, in transient region with the boom state, the asset continue to generate the cash flow. According to Ito's formula, the function $Q_0(x)$ and $Q_1(x)$ satisfy the following system of ordinary differential equations (ODEs):

$$\begin{cases} rQ_0(x) = ax + K + \mu_0 x Q_0'(x) + \frac{\sigma_0^2 x^2}{2} Q_0''(x) + \lambda_0 \left(Q_1(x) - Q_0(x)\right), \\ rQ_1(x) = ax + K + \mu_1 x Q_1'(x) + \frac{\sigma_1^2 x^2}{2} Q_1''(x) + \lambda_1 \left(Q_0(x) - Q_1(x)\right), \end{cases} \tag{2}$$

for $x \in \mathcal{D}_0$;

$$\begin{cases} Q_0(x) = G(x, 0), \\ rQ_1(x) = ax + K + \mu_1 x Q_1'(x) + \frac{\sigma_1^2 x^2}{2} Q_1''(x) + \lambda_1 \left(G_0(x, 0) - Q_1(x)\right), \end{cases} \tag{3}$$

for $x \in \mathcal{D}_1 - \mathcal{D}_0$; and

$$\begin{cases} Q_0(x) = G(x, 0), \\ Q_1(x) = G(x, 1), \end{cases}$$

for $x \in \overline{\mathcal{D}_1}$.

Define the following quadratic function of variable β:

$$h_l(\beta) = r + \lambda_l - \frac{1}{2}\sigma_l^2 \beta(\beta - 1) - \mu_l \beta, \; l \in \{0, 1\}. \tag{4}$$

In the following, we denote by β_1 and β_2 the two roots of the equation $h_1(\beta) = 0$, by β_3 and β_4 the two roots of the equation $h_0(\beta) = 0$. The equation $h_0(\gamma)h_1(\gamma) = \lambda_0\lambda_1$ has two negative roots denoted by γ_1 and γ_2 and two positive roots denoted by γ_3 and γ_4. We derive the explicit pricing for all standard assets as follows.

Proposition 3.1. *Suppose that the standard asset defined by its terminal payoff $G(x, l) = \phi_l x + \kappa_l$,[3] $l \in \{0, 1\}$, and its cash flow $ax + K$ for some constant a and K if the firm's cash flow does not exit the domain $\mathcal{D}_l$. Then if $\mathcal{D}_0 \subseteq \mathcal{D}_1$, the value $Q_l(x)$ of the standard asset has the following form:*

$$Q_0(x) = \begin{cases} \sum_{i=1}^4 A_i x^{\gamma_i} + aq_0 x + \frac{K}{r}, & \text{if } x \in \mathcal{D}_0, \\ G(x, 0), & \text{if } x \in \overline{\mathcal{D}_0}, \end{cases} \tag{5}$$

and

$$Q_1(x) = \begin{cases} \sum_{i=1}^4 B_i x^{\gamma_i} + aq_1 x + \frac{K}{r}, & \text{if } x \in \mathcal{D}_0, \\ C_1 x^{\beta_1} + C_2 x^{\beta_2} + \frac{a + \lambda_1 \phi_0}{r + \lambda_1 - \mu_1} x + \frac{K + \lambda_1 \kappa_0}{r + \lambda_1}, & \text{if } x \in \mathcal{D}_1 - \mathcal{D}_0, \\ G(x, 1), & \text{if } x \in \overline{\mathcal{D}_1}; \end{cases} \tag{6}$$

and if $\mathcal{D}_1 \subseteq \mathcal{D}_0$,

$$Q_1(x) = \begin{cases} \sum_{i=1}^4 \hat{A}_i x^{\gamma_i} + aq_1 x + \frac{K}{r}, & \text{if } x \in \mathcal{D}_1, \\ G(x, 1), & \text{if } x \in \overline{\mathcal{D}_1}, \end{cases} \tag{7}$$

and

$$Q_0(x) = \begin{cases} \sum_{i=1}^{4} \hat{B}_i x^{\gamma_i} + aq_0 x + \frac{K}{r}, & \text{if } x \in \mathcal{D}_1, \\ \hat{C}_1 x^{\beta_3} + \hat{C}_2 x^{\beta_4} + \frac{a+\lambda_0\phi_1}{r+\lambda_0-\mu_0} x + \frac{K+\lambda_0\kappa_1}{r+\lambda_0}, & \text{if } x \in \mathcal{D}_0 - \mathcal{D}_1, \\ G(x,0), & \text{if } x \in \overline{\mathcal{D}_0}, \end{cases} \tag{8}$$

where

$$q_l = \frac{r + \lambda_0 + \lambda_1 - \mu_0 - \mu_1 + \mu_l}{(r + \lambda_0 - \mu_0)\,(r + \lambda_1 - \mu_1) - \lambda_0\lambda_1},$$

and $B_i = \frac{h_0(\gamma_i)}{\lambda_0} A_i$, $\hat{B}_i = \frac{\lambda_0}{h_0(\gamma_i)} \hat{A}_i$.

Proof. We only consider the case $\mathcal{D}_0 \subseteq \mathcal{D}_1$ since the proof under the opposite case is similar. For the given standard asset defined by the cash flow $ax + K$ with domain D_l and terminal payoff $G(x, l)$, at the current economic state $l \in \{0, 1\}$, the standard asset value in continuation region $\mathcal{D}_0$ satisfying (2) consists of two components: the unlimited liability value of the perpetual asset $aq_l x + \frac{K}{r}$ and the value adjustment from the possible regime switching and terminal risk. In the transient region $\mathcal{D}_1 - \mathcal{D}_0$, the asset value also has two components: The first is the unlimited liability value of the perpetual asset in boom, satisfying (3) and $G(x, 0) = \phi_0 x + \kappa_0$, which is given by $\frac{a+\lambda_1\phi_0}{r+\lambda_1-\mu_1} x + \frac{K+\lambda_1\kappa_0}{r+\lambda_1}$; the second is the value adjustment due to the termination in boom and the boom-to-recession regime shift. All the coefficients in (5) and (6) are determined additionally by the continuousness of the value functions and the differentiability of value function $Q_1(\cdot)$ within the interior of domain $\mathcal{D}_1$, following Guo et al. (2005), Zhang and Guo (2004) and Luo et al. (2019). In the end, by using the guess-and-verify method, the conclusions are proved following the subsequent proof of Proposition 5.1. □

We point out that to the best of our knowledge, the conclusions of Proposition 3.1 are never derived before. In particular, almost all common corporate securities' price can be derived by the explicit formulas presented in Proposition 3.1, even if the cash flow generated by the firm is driven by both the Brownian motion and the two-state Markov chain. Accordingly, we extend Leland (1994) and Tan and Yang (2017)'s closed-form results to the Markovian regime-switching GBM cash flow model.

3.2 The pricing of corporate securities

To price corporate securities, we use a backward induction method and the prices can be derived from some standard asset prices we addressed in the last subsection.

First, after the debt default has taken place, the debtholders take over the firm and get nothing but a claim of which the value equals to $(1-\alpha)P_l(x) = (1-\alpha)(1-\tau_f)q_l x$. Here, α is the bankruptcy loss rate and $P_l(x) = (1-\tau_f)q_l x$ is the unlevered firm's value. This is because the unlevered firm has a claim whose cash flow equals to $(1-\tau_f)X_t$ forever, i.e. a special form of the above-mentioned standard asset with $a = (1-\tau_f)$, $K = 0$ and $\mathcal{D}_0 = \mathcal{D}_1 = (0, \infty)$.

Now we turn to the valuation of debt for given default thresholds. The default threshold in recession, denoted by x_0^b, and that in boom, denoted by x_1^b are endogenously determined by shareholders. We emphasize that a firm would default earlier in recession than in boom, but the opposition would also hold true, depending on model parameters. This is because a higher drift of the cash flow would induce a later default while a less volatility would lead to an earlier default. To be specific, we assume $x_1^b < x_0^b$, i.e. the firm's default happens earlier in recession than in boom, in the following derivations.

Debt is perpetual and debtholders receive cash flow $(1-\tau_i)c$ until default happens. At default, the debtholders get nothing but a liquidation value equaling to $(1-\alpha)(1-\tau_f)q_l x$ just derived. Therefore, to price the debt, we take it as a special standard asset with $a=0$, $K=(1-\tau_i)c$, $\mathcal{D}_0=(x_0^b,\infty)$, $\mathcal{D}_1=(x_1^b,\infty)$, $G(x,0)=(1-\alpha)(1-\tau_f)q_0 x$ and $G(x,1)=(1-\alpha)(1-\tau_f)q_1 x$. Accordingly, we conclude from Proposition 3.1 the following debt value:

$$D_0(x)=\begin{cases} A_1x^{\gamma_1}+A_2x^{\gamma_2}+\frac{(1-\tau_i)c}{r}, & \text{if } x>x_0^b,\\ (1-\alpha)(1-\tau_f)q_0x, & \text{if } x\le x_0^b, \end{cases} \tag{9}$$

and

$$D_1(x)=\begin{cases} B_1x^{\gamma_1}+B_2x^{\gamma_2}+\frac{(1-\tau_i)c}{r}, & \text{if } x>x_0^b,\\ C_1x^{\beta_1}+C_2x^{\beta_2}+\frac{\lambda_1(1-\alpha)(1-\tau_f)q_0x}{r+\lambda_1-\mu_1}+\frac{(1-\tau_i)c}{r+\lambda_1}, & \text{if } x_1^b<x\le x_0^b,\\ (1-\alpha)(1-\tau_f)q_1x, & \text{if } x\le x_1^b. \end{cases} \tag{10}$$

According to value-matching (continuousness) conditions, we have

$$D_0(x_0^b+)=D_0(x_0^b-),\ D_1(x_0^b+)=D_1(x_0^b-),\ D_1(x_1^b+)=D_1(x_1^b-).$$

Additionally, from the differentiability of the value functions within the interior of domain $\mathcal{D}_1$ we conclude $D'_1(x_0^b+)=D'_1(x_0^b-)$. Hence, we derive that the coefficient vector$(A_1,A_2,C_1,C_2)'$ is the solution of the system of linear equations: $\Gamma_1X=\Phi_1$, where

$$\Gamma_1=\begin{pmatrix} \left(x_0^b\right)^{\gamma_1} & \left(x_0^b\right)^{\gamma_2} & 0 & 0\\ \frac{h_0(\gamma_1)}{\lambda_0}\left(x_0^b\right)^{\gamma_1} & \frac{h_0(\gamma_2)}{\lambda_0}\left(x_0^b\right)^{\gamma_2} & -\left(x_0^b\right)^{\beta_1} & -\left(x_0^b\right)^{\beta_2}\\ \frac{\gamma_1h_0(\gamma_1)}{\lambda_0}\left(x_0^b\right)^{\gamma_1} & \frac{\gamma_2h_0(\gamma_2)}{\lambda_0}\left(x_0^b\right)^{\gamma_2} & -\beta_1\left(x_0^b\right)^{\beta_1} & -\beta_2\left(x_0^b\right)^{\beta_2}\\ 0 & 0 & \left(x_1^b\right)^{\beta_1} & \left(x_1^b\right)^{\beta_2} \end{pmatrix},$$

and

$$\Phi_1=\begin{pmatrix} (1-\alpha)(1-\tau_f)q_0x_0^b-\frac{(1-\tau_i)c}{r}\\ \frac{\lambda_1(1-\alpha)(1-\tau_f)q_0x_0^b}{r+\lambda_1-\mu_1}-\frac{\lambda_1(1-\tau_i)c}{r(r+\lambda_1)}\\ \frac{\lambda_1(1-\alpha)(1-\tau_f)q_0x_0^b}{r+\lambda_1-\mu_1}\\ (1-\alpha)(1-\tau_f)(q_1-\frac{\lambda_1q_0}{r+\lambda_1-\mu_1})x_1^b-\frac{(1-\tau_i)c}{r+\lambda_1} \end{pmatrix}.$$

In a similar way, thanks to Proposition 3.1, we get the following equity value:

$$E_0(x)=\begin{cases} A_3x^{\gamma_1}+A_4x^{\gamma_2}+(1-\tau_f)(q_0x-\frac{c}{r}), & \text{if } x>x_0^b,\\ 0, & \text{if } x\le x_0^b, \end{cases} \tag{11}$$

and

$$E_1(x)=\begin{cases} B_3x^{\gamma_1}+B_4x^{\gamma_2}+(1-\tau_f)(q_1x-\frac{c}{r}), & \text{if } x>x_0^b,\\ C_3x^{\beta_1}+C_4x^{\beta_2}+(1-\tau_f)(\frac{x}{r+\lambda_1-\mu_1}-\frac{c}{r+\lambda_1}), & \text{if } x_1^b<x\le x_0^b,\\ 0, & \text{if } x\le x_1^b. \end{cases} \tag{12}$$

According to the continuousness and differentiability conditions, the coefficient vector $(A_3, A_4, C_3, C_4)'$ satisfies the system of linear equations: $\Gamma_1 X = \Phi_2$, where

$$\Phi_2 = \begin{pmatrix} -\left(1-\tau_f\right)\left(q_0 x_0^b - \frac{c}{r}\right) \\ \left(1-\tau_f\right)\left(\left(\frac{1}{r+\lambda_1-\mu_1} - q_1\right)x_0^b + \frac{\lambda_1 c}{r(r+\lambda_1)}\right) \\ \left(1-\tau_f\right)\left(\frac{1}{r+\lambda_1-\mu_1} - q_1\right)x_0^b \\ -\left(1-\tau_f\right)\left(\frac{x_1^b}{r+\lambda_1-\mu_1} - \frac{c}{r+\lambda_1}\right) \end{pmatrix}.$$

As said before, the bankruptcy boundaries are determined endogenously by shareholders and thus according to smooth-pasting condition $E'_0(x_0^b+) = E'_0(x_0^b-)$ and $E'_1(x_1^b+) = E'_1(x_1^b-)$, the optimal default threshold (x_0^b, x_1^b) is a solution of the following system of equations:

$$\begin{cases} \gamma_1 A_3 \left(x_0^b\right)^{\gamma_1} + \gamma_2 A_4 \left(x_0^b\right)^{\gamma_2} + (1-\tau_f)q_0 x_0^b = 0, \\ \beta_1 C_3 \left(x_1^b\right)^{\beta_1} + \beta_2 C_4 \left(x_1^b\right)^{\beta_2} + \frac{(1-\tau_f)x_1^b}{r+\lambda_1-\mu_1} = 0. \end{cases} \tag{13}$$

The solution is not explicit but easy to get a numerical solution for it. Therefore, after the project is launched, the total firm value can be derived immediately by $V_l(x) = E_l(x) + D_l(x), l \in \{0, 1\}$.

4. Tax value and credit guarantees with government subsidies

In this section, we assume that an entrepreneur (firm) facing financial constraints invests in a project. S/he is unable to get a loan from a bank directly and has to resort to an insurer with a government subsidy, where the government will give a subsidy to the entrepreneur according to the social value (externality) brought from the investment. Specifically, the entrepreneur enters into an agreement with an insurer and a bank (lender) to obtain the required bank loan: When the entrepreneur defaults or a business goes bankrupt, the insurer promises to take all the loss. With the full guarantee, the lender is willing to provide bank loan at the risk-free rate. In return for the guarantee, the entrepreneur pays the insurer a guaranteed cost premium, while the government gives a kind of subsidies to the insurer also.

We introduce the value of tax first, which is the sole source of government subsidies. The government subsidizes the entrepreneur by providing her/him with a fraction of tax value brought from the investment according to the externality, a side effect or consequence of the investment. As a result, the guaranteed cost premium rate required is decreased since the guarantee market is supposed to be fully competitive, meaning that the insurer's net present value is zero.

First, the value of the taxes brought from the investment is given by

$$TX_l(x) = \mathbb{E}\left[\int_0^{T_{\mathfrak{D}}} e^{-rt}\left(\tau_f\left(X_t - c\right) + \tau_i c\right)dt + e^{-rT_{\mathfrak{D}}} G(X_{T_{\mathfrak{D}}}; L_{T_{\mathfrak{D}}}) \mid X_0 = x, L_0 = l\right],$$

where $G(X_{T_{\mathfrak{D}}}; L_{T_{\mathfrak{D}}})$ represents the value of the taxes after default. The taxes can be considered as a special standard asset discussed before; thus, for the given default thresholds assumed before, we conclude from Proposition 3.1 the following tax value:

$$TX_0(x) = \begin{cases} A_5 x^{\gamma_1} + A_6 x^{\gamma_2} + \tau_f\left(q_0 x - \frac{c}{r}\right) + \frac{\tau_i c}{r}, & \text{if } x > x_0^b, \\ \tau_f(1-\alpha)q_0 x, & \text{if } x \le x_0^b, \end{cases} \tag{14}$$

and

$$TX_1(x) = \begin{cases} B_5 x^{\gamma_1} + B_6 x^{\gamma_2} + \tau_f \left(q_1 x - \frac{c}{r}\right) + \frac{\tau_i c}{r}, & \text{if } x > x_0^b, \\ C_5 x^{\beta_1} + C_6 x^{\beta_2} + \frac{(1+\lambda_1(1-\alpha)q_0)\tau_f x}{r+\lambda_1-\mu_1} + \frac{\lambda_1 \tau_i c + (\tau_i - \tau_f) rc}{r(r+\lambda_1)}, & \text{if } x_1^b < x \leq x_0^b, \\ \tau_f (1-\alpha) q_1 x, & \text{if } x \leq x_1^b. \end{cases} \tag{15}$$

where $B_5 = \frac{h_0(\gamma_1)}{\lambda_0} A_5, B_6 = \frac{h_0(\gamma_2)}{\lambda_0} A_6$. The column vector $(A_5, A_6, C_5, C_6)'$ is the solution of the system of equations: $\Gamma_1 X = \Phi_3$, where

$$\Phi_3 = \begin{pmatrix} -\alpha \tau_f q_0 x_0^b + \frac{\tau_f c}{r} - \frac{\tau_i c}{r} \\ \left(\frac{\tau_f(1+\lambda_1(1-\alpha)q_0)}{r+\lambda_1-\mu_1} - \tau_f q_1\right) x_0^b + \frac{\lambda_1 \tau_f c}{r(r+\lambda_1)} \\ \left(\frac{\tau_f(1+\lambda_1(1-\alpha)q_0)}{r+\lambda_1-\mu_1} - \tau_f q_1\right) x_0^b \\ \left(\tau_f(1-\alpha)q_1 - \frac{\tau_f(1+\lambda_1(1-\alpha)q_0)}{r+\lambda_1-\mu_1}\right) x_1^b - \frac{\lambda_1 \tau_i c + (\tau_i - \tau_f) rc}{r(r+\lambda_1)} \end{pmatrix}.$$

If the coupon rate of debt (loan) is c, then the entrepreneur naturally gets the amount of the bank loan equaling $(1-\tau_i)c/r$. We assume that the bank loan is risk-free and accordingly, for some given investment threshold $x_l^e, l \in \{0, 1\}$, depending on the economy state, the value of the guarantee liability, denoted by $D_l^{gua}(x^e)$, satisfies the following equation:

$$D_l\left(x_l^e\right) + D_l^{gua}\left(x_l^e\right) = \frac{(1-\tau_i)c}{r}, \ l \in \{0, 1\}, \tag{16}$$

where $D_l\left(x_l^e\right)$ is given by (9) or (10). In return for the guarantee, the borrower (entrepreneur) pays the insurer a fixed guaranteed cost premium rate $g_l, l \in \{0, 1\}$ together with a government's subsidy depending the investment's externality. The amount of the subsidy is $S_l(x_l^e) \equiv \theta TX_l(x_l^e), \ l \in \{0, 1\}$, where θ is called tax subsidy coefficient depending on the investment externality intensity. Naturally, the higher the investment externality, the higher the government subsidy and thus the higher the parameter θ recommended. Therefore, for given investment thresholds x_0^e in recession and x_1^e in boom, we fix the guaranteed cost premium rate g_l as follows:

$$D_l^{gua}\left(x_l^e\right) = \frac{g_l(1-\tau_i)c}{r} + \theta TX_l\left(x_l^e\right), \ l \in \{0, 1\}. \tag{17}$$

It says that the government subsidy reduces the cash payment made by the entrepreneur, i.e. the guaranteed cost premium rate g_l is reduced due to the subsidy. Hence, qualified entrepreneurs benefit from the subsidy in advance.

We highlight that if the externality and tax value are large enough, the government subsidy may be higher than the value of the guarantee liability. In this case, the guaranteed cost premium rate may have a negative value, meaning that the government provides the entrepreneur with a waived premium and additionally a positive net cash subsidy for the investment, and the amount of the cash subsidy is equal to $-g_l(1-\tau_i)c/r$. Naturally, all the subsidies are harvested by the entrepreneur since both the lender (bank) and insurer get a net present value equal to zero.

5. The pricing and timing of the option to invest in a project and optimal leverage

This section considers the pricing and timing of the option to invest in a project with credit guarantees and government subsides. After that, we focus on the optimal capital structure or optimal leverage under the assumption that the government subsidy policy is not changed no matter how much money is borrowed. Finally, to measure the total subsides' effect in Section 6, we compute the present value of the future random tax revenues.

5.1 The pricing and timing of the option to invest in a project

Government subsides are worthwhile and necessary to motivate a firm to invest in a project that has sufficiently positive externalities when the project itself is not profitable. Government subsidies are generally a fraction (θ) of tax revenues, i.e. $0 \le \theta \le 1$. Actually, the tax subsidy coefficient θ can be negative ($\theta < 0$) for a negative externality project; it also can be greater than 1 ($\theta > 1$) for a sufficiently positive externality project.

For a given tax subsidy coefficient θ, we first fix the optimal investment threshold $x_l^e, l \in \{0, 1\}$ under the government subsidy specified in the last section.

Due to the time-homogenous model, we assume that the current time is zero without loss of generality as before. If the current cash flow level (EBIT)[4] is $X_0 = x$ and the economy state is $l \in \{0, 1\}$, the value of the investment option is given by

$$R_l(x) = \max_{\tau} \mathbb{E}\left[e^{-r\tau} \left(V_l\left(X_\tau\right) - I + S_l(X_\tau)\right) \mid X_0 = x, L_0 = l\right], \tag{18}$$

where τ represent the investment time, a stopping time with respect to $\{\mathcal{F}_t\}_{t \ge 0}$, and $S_l(X_\tau) = \theta T X_l(X_\tau)$ represents the government subsidy. This is because at the investment time, the entrepreneur's value is

$$E_l(X_\tau) - I + \frac{(1-\tau_i)c}{r}(1-g_l) = V_l(X_\tau) - I + S_l(X_\tau)$$

according to (16) and (17).

Similar to the preceding discussion on default threshold, we are not sure which of two investment thresholds x_0^e and x_1^e is higher. To be specific, we assume $x_1^e < x_0^e$ in the following derivations, i.e. the investment in boom is earlier than in recession, while the discussions under $x_1^e \ge x_0^e$ are similar and thus omitted. Hence, the domain $\mathcal{D}_0^e = (0, x_0^e)$ and $\mathcal{D}_1^e = (0, x_1^e)$ satisfy $\mathcal{D}_1^e \subseteq \mathcal{D}_0^e$. Following Guo et al. (2005), the regime-switching space $(0, \infty)$ of real investment can be partitioned into three regions: $\left(0, x_1^e\right)$, $\left[x_1^e, x_0^e\right)$ and $\left[x_0^e, \infty\right)$, which are called inaction region, transient region and action region, respectively. In inaction region, no investment occurs. Investment is triggered in transient region if the economy is in boom. In action region, investment is exercised immediately no matter what the economy state is.

We derive the following explicit solutions for the pricing and timing of the option to invest in the project with credit guarantees and government subsidies.

Proposition 5.1. *Suppose that the default occurs earlier in recession than in boom, i.e.* $x_0^b \ge x_1^b$.[5] *Then if* $x_1^e \le x_0^e$, *the value of the option to invest in the project defined by (1) with the guarantee and government subsidy defined by (17) is given by*

$$R_1(x) = \begin{cases} A_7 x^{\gamma_3} + A_8 x^{\gamma_4}, & \text{if } x < x_1^e, \\ V_1(x) - I + \theta T X_1(x), & \text{if } x \ge x_1^e, \end{cases} \tag{19}$$

and

$$R_0(x) = \begin{cases} B_7 x^{\gamma_3} + B_8 x^{\gamma_4}, & \text{if } x < x_1^e, \\ C_7 x^{\beta_3} + C_8 x^{\beta_4} + M_1 x^{\gamma_1} + M_2 x^{\gamma_2} + \frac{\lambda_0 (1-\tau_f+\theta\tau_f) q_1 x}{r+\lambda_0-\mu_0} & \\ + \frac{\lambda_0}{r+\lambda_0}\left(\frac{(1-\theta)(\tau_f-\tau_i)c}{r} - I\right), & \text{if } x_1^e \le x < x_0^e, \\ V_0(x) - I + \theta T X_0(x), & \text{if } x \ge x_0^e, \end{cases} \tag{20}$$

where $B_7 \equiv \frac{\lambda_0}{h_0(\gamma_3)} A_7, B_8 \equiv \frac{\lambda_0}{h_0(\gamma_4)} A_8, M_1 \equiv \frac{\lambda_0(B_1+B_3+\theta B_5)}{r+\lambda_0-\mu_0\gamma_1-\sigma_0^2\gamma_1(\gamma_1-1)/2},\ M_2 \equiv \frac{\lambda_0(B_2+B_4+\theta B_6)}{r+\lambda_0-\mu_0\gamma_2-\sigma_0^2\gamma_2(\gamma_2-1)/2}$; *and if* $x_0^e < x_1^e$, *we have*

$$R_0(x) = \begin{cases} \hat{A}_7 x^{\gamma_3} + \hat{A}_8 x^{\gamma_4}, & \text{if } x < x_0^e, \\ V_0(x) - I + \theta T X_0(x), & \text{if } x \geq x_0^e, \end{cases} \tag{21}$$

and

$$R_1(x) = \begin{cases} \hat{B}_7 x^{\gamma_3} + \hat{B}_8 x^{\gamma_4}, & \text{if } x < x_0^e, \\ \hat{C}_7 x^{\beta_1} + \hat{C}_8 x^{\beta_2} + \hat{M}_1 x^{\gamma_1} + \hat{M}_2 x^{\gamma_2} + \frac{\lambda_1(1-\tau_f+\theta\tau_f)q_0 x}{r+\lambda_1-\mu_1} & \\ + \frac{\lambda_1}{r+\lambda_1}(\frac{(1-\theta)(\tau_f-\tau_i)c}{r} - I), & \text{if } x_0^e \leq x < x_1^e, \\ V_1(x) - I + \theta T X_1(x), & \text{if } x \geq x_1^e, \end{cases} \tag{22}$$

where $\hat{B}_7 \equiv \frac{h_0(\gamma_3)}{\lambda_0} \hat{A}_7, \hat{B}_8 \equiv \frac{h_0(\gamma_4)}{\lambda_0} \hat{A}_8, \hat{M}_1 \equiv \frac{\lambda_1(A_1+A_3+\theta A_5)}{r+\lambda_1-\mu_1\gamma_1-\sigma_1^2\gamma_1(\gamma_1-1)/2},\ \hat{M}_2 \equiv \frac{\lambda_1(A_2+A_4+\theta A_6)}{r+\lambda_1-\mu_1\gamma_2-\sigma_1^2\gamma_2(\gamma_2-1)/2}$, *and function* $h_l(\cdot)$, $l \in \{0, 1\}$, *is defined by (4).*

Taking $x_1^e \leq x_0^e$[6] *in particular, we conclude from the value-matching condition and smooth-pasting condition that the column vector* $(A_7, A_8, C_7, C_8)'$ *is the solution of the system of linear equations:* $\Gamma_2 X = \Phi_4$, *where*

$$\Gamma_2 = \begin{pmatrix} (x_1^e)^{\gamma_3} & (x_1^e)^{\gamma_4} & 0 & 0 \\ \frac{\lambda_0}{h_0(\gamma_3)} (x_1^e)^{\gamma_3} & \frac{\lambda_0}{h_0(\gamma_4)} (x_1^e)^{\gamma_4} & -(x_1^e)^{\beta_3} & -(x_1^e)^{\beta_4} \\ \frac{\gamma_3\lambda_0}{h_0(\gamma_3)} (x_1^e)^{\gamma_3} & \frac{\gamma_4\lambda_0}{h_0(\gamma_4)} (x_1^e)^{\gamma_4} & -\beta_3 (x_1^e)^{\beta_3} & -\beta_4 (x_1^e)^{\beta_4} \\ 0 & 0 & (x_0^e)^{\beta_3} & (x_0^e)^{\beta_4} \end{pmatrix},$$

and

$$\Phi_4 = \begin{pmatrix} (B_1+B_3+\theta B_5)(x_1^e)^{\gamma_1} + (B_2+B_4+\theta B_6)(x_1^e)^{\gamma_2} + (1-\tau_f+\theta\tau_f) q_1 x_1^e \\ +(1-\theta)\frac{(\tau_f-\tau_i)c}{r} - I \\ M_1 (x_1^e)^{\gamma_1} + M_2 (x_1^e)^{\gamma_2} + \frac{\lambda_0(1-\tau_f+\theta\tau_f)q_1 x_1^e}{r+\lambda_0-\mu_0} + \frac{\lambda_0}{r+\lambda_0}(\frac{(1-\theta)(\tau_f-\tau_i)c}{r} - I) \\ \gamma_1 M_1 (x_1^e)^{\gamma_1} + \gamma_2 M_2 (x_1^e)^{\gamma_2} + \frac{\lambda_0(1-\tau_f+\theta\tau_f)q_1 x_1^e}{r+\lambda_0-\mu_0} \\ (A_1+A_3+\theta A_5-M_1)(x_0^e)^{\gamma_1} + (A_2+A_4+\theta A_6-M_2)(x_0^e)^{\gamma_2} \\ +(1-\tau_f+\theta\tau_f)(q_0 - \frac{\lambda_0 q_1}{r+\lambda_0-\mu_0})x_0^e + \frac{r}{r+\lambda_0}(\frac{(1-\theta)(\tau_f-\tau_i)c}{r} - I) \end{pmatrix}.$$

The optimal investment thresholds x_0^e and x_1^e, which are endogenously determined by the entrepreneur, satisfy the following system of equations:

$$\begin{cases} \gamma_3 A_7 \left(x_1^e\right)^{\gamma_3} + \gamma_4 A_8 \left(x_1^e\right)^{\gamma_4} = \gamma_1 \left(B_1 + B_3 + \theta B_5\right) \left(x_1^e\right)^{\gamma_1} + \gamma_2 \left(B_2 + B_4 + \theta B_6\right) \left(x_1^e\right)^{\gamma_2} \\ + \left(1 - \tau_f + \theta \tau_f\right) q_1 x_1^e, \\ \beta_3 C_7 \left(x_0^e\right)^{\beta_3} + \beta_4 C_8 \left(x_0^e\right)^{\beta_4} = \gamma_1 \left(A_1 + A_3 + \theta A_5 - M_1\right)\left(x_0^e\right)^{\gamma_1} + \gamma_2 \left(A_2 + A_4 + \theta A_6 - M_2\right)\left(x_0^e\right)^{\gamma_2} \\ +(1 - \tau_f + \theta \tau_f)(q_0 - \frac{\lambda_0 q_1}{r+\lambda_0-\mu_0})x_0^e. \end{cases} \tag{23}$$

Proof. To be specific, we focus on the case of $x_0^b \geq x_1^b$ and $x_1^e \leq x_0^e$ in the following since the proofs for other cases are almost the same. The proof is similar to that of the proposition 3.1. We take the investment option as a special standard asset defined in Subsection 3.1. The standard asset has no regular cash flow, and its terminal payoff $G(x, l) = V_l(x) - I + \theta TX_l(x)$, where $V_l(x) = E_l(x) + D_l(x), l \in \{0, 1\}$, with $D_l(x)$ being given by (9) and (10) and $E_l(x)$ being given by (11) and (12), and $TX_l(x)$ is given by (14) and (15). We stress that the conclusions of Proposition 3.1 do not apply directly because its linear terminal payoff assumption does not hold here.

First, we conclude from the Bellman's principle (without control) that the value $R_l(x)$ defined by (18) of the option to invest satisfies the following ODE:

$$\begin{cases} rR_1(x) = \mu_1 x R_1'(x) + \frac{\sigma_1^2 x^2}{2} R_1''(x) + \lambda_1 \left(R_0(x) - R_1(x)\right), \\ rR_0(x) = \mu_0 x R_0'(x) + \frac{\sigma_0^2 x^2}{2} R_0''(x) + \lambda_0 \left(R_1(x) - R_0(x)\right), \end{cases} \tag{24}$$

for $x \in \left(0, x_1^e\right)$;

$$\begin{cases} R_1(x) = V_1(x) - I + \theta TX_1(x), \\ rR_0(x) = \mu_0 x R_0'(x) + \frac{\sigma_0^2 x^2}{2} R_0''(x) + \lambda_0 \left(V_1(x) - I + \theta TX_1(x) - R_0(x)\right), \end{cases} \tag{25}$$

for $x \in \left[x_1^e, x_0^e\right)$; and

$$\begin{cases} R_1(x) = V_1(x) - I + \theta TX_1(x), \\ R_0(x) = V_0(x) - I + \theta TX_0(x), \end{cases} \tag{26}$$

for $x \in \left[x_0^e, \infty\right)$. After that, we use the guess-and-verify method to solve (24)–(26). Specifically, motivated by (7) and (8) in Proposition 3.1, we guess that (24) (in the continuation region) has the following general solution:

$$R_1(x) = A_7 x^{\gamma_3} + A_8 x^{\gamma_4} + \sum_{i=1}^{2} \hat{A}_i x^{\gamma_i} \text{ and } R_0(x) = B_7 x^{\gamma_3} + B_8 x^{\gamma_4} + \sum_{i=1}^{2} \hat{B}_i x^{\gamma_i}.$$

Noting that $\lim_{x\to 0} Q_1(x) = \lim_{x\to 0} Q_0(x) = 0$ (the option is virtually worthless if the EBIT approaches zero), we get $\hat{A}_1 = \hat{A}_2 = \hat{B}_1 = \hat{B}_2 = 0$ since γ_1 and γ_2 are negative. Substituting the general solution into (24) leads to that γ_3 and γ_4 are a root of the equation $h_0(\gamma)h_1(\gamma) = \lambda_0\lambda_1$. Similarly, if $x \in \left[x_1^e, x_0^e\right)$ (transient region), we guess that $R_0(x) = C_7 x^{\beta_3} + C_8 x^{\beta_4} + M_1 x^{\gamma_1} + M_2 x^{\gamma_2} + \zeta_1 x + \zeta_2$. Substituting it into (25) and using the continuity and differentiability of the value functions, we get (19) and (20) after tedious derivations. Last, we use the well-known smooth-pasting conditions to derive the optimal investment thresholds x_0^e and x_1^e satisfying (23). □

Proposition 5.1 explains that in the continuation region ($x < x_1^e$), the real option value is subject to the possible adjustments arsing from the regime-switching in addition to the common investment option value. In the transient region ($x_1^e \leq x < x_0^e$) with a recession state, the real option value is the common value of waiting plus the increased value due to the possible economy switching into boom when the option is instantly exercised. These conclusions look complicated but fortunately are closed-form. Numerical tests are provided in Section 6.

5.2 Optimal leverage

For a given government subsidy policy, the entrepreneur would sufficiently make use of the policy benefit to maximize the value of the option to invest. Naturally, we wonder what the optimal capital structure or optimal leverage should be if the amount of money borrowed is unrestrained thanks to the guarantee and subsides. We now answer this question.

Clearly, this question comes down to finding optimal coupon rate, denoted by c_l^*, of debt at the investment threshold x_l^e, $l \in \{0, 1\}$. To do so, we emphasize that the prices of corporate securities derived before are a function of the coupon rate c of debt, which is exogenously given but must satisfy $(1 - \tau_i)c * (1 - g)/r \geq I$, i.e. the loan after paying the guarantee cost must cover the sunk cost I. To highlight their dependence on the coupon rate c, we denote the above-derived option value by $R_l(x, c)$ instead of $R_l(x)$ when the current EBIT is equal to x. In the same way, the above-derived debt value at the investment threshold (time) x_l^e is denoted by $D_l(x_l^e, c)$ instead of $D_l(x_l^e)$.

Therefore, the optimal coupon rate c_l^* at the investment threshold x_l^e solves the following optimization problem:

$$c_l^* \equiv \arg\sup_c R_l(x_l^e, c), \; l \in \{0, 1\}. \tag{27}$$

And then, we get the following optimal leverage at the investment threshold x_l^e:

$$\mathcal{L}^*(x_l^e) = \frac{D_l(x_l^e, c_l^*)}{V_l(x_l^e, c_l^*)}, \; l \in \{0, 1\}. \tag{28}$$

5.3 The pricing of the tax revenues

To compute the present value of the future random tax revenues, we note that the tax revenues are like a real option but with the known investment threshold $x_l^e, l \in \{0, 1\}$. Therefore, similar to Proposition 5.1, we provide the following results without a proof.

The tax value prior to investment is given by

$$T_l(x) \equiv \mathbb{E}\left[e^{-r(\tau - t)}(1 - \theta)TX_l\left(X_\tau\right) \mid X_0 = x, L_0 = l\right],$$

where τ represents the investment time given by (18).

Suppose that the default occurs earlier in recession than in boom, i.e. $x_0^b \geq x_1^b$, and that the investment occurs later in recession than in boom, i.e. $x_1^e \leq x_0^e$. Similar to the real option pricing method, the pre-investment tax value is given by

$$T_1(x) = \begin{cases} A_9 x^{\gamma_3} + A_{10} x^{\gamma_4}, & \text{if } x < x_1^e, \\ (1 - \theta)TX_1(x), & \text{if } x \geq x_1^e, \end{cases} \tag{29}$$

and

$$T_0(x) = \begin{cases} B_9 x^{\gamma_3} + B_{10} x^{\gamma_4}, & \text{if } x < x_1^e, \\ C_9 x^{\beta_3} + C_{10} x^{\beta_4} + M_3 x^{\gamma_1} + M_4 x^{\gamma_2} + \frac{(1-\theta)\lambda_0 \tau_f q_1 x}{r+\lambda_0-\mu_0} & \\ -\frac{(1-\theta)\lambda_0(\tau_f-\tau_i)c}{(r+\lambda_0)r}, & \text{if } x_1^e \le x < x_0^e, \\ (1-\theta)TX_0(x), & \text{if } x \ge x_0^e, \end{cases} \tag{30}$$

where $B_9 \equiv \frac{\lambda_0}{h_0(\gamma_3)} A_9, B_{10} \equiv \frac{\lambda_0}{h_0(\gamma_4)} A_{10}, M_3 \equiv \frac{(1-\theta)\lambda_0 B_5}{r+\lambda_0-\mu_0\gamma_1-\sigma_0^2\gamma_1(\gamma_1-1)/2}, M_4 \equiv \frac{(1-\theta)\lambda_0 B_6}{r+\lambda_0-\mu_0\gamma_2-\sigma_0^2\gamma_2(\gamma_2-1)/2}$. From the value-matching condition, we conclude that the column vector $(A_9, A_{10}, C_9, C_{10})'$ is the solution of the system of linear equations $\Gamma_2 X = \Phi_5$, where

$$\Phi_5 = \begin{pmatrix} (1-\theta)(B_5\left(x_1^e\right)^{\gamma_1} + B_6\left(x_1^e\right)^{\gamma_2} + \tau_f\left(q_1 x_1^e - \frac{c}{r}\right) + \frac{\tau_i c}{r}) \\ M_3\left(x_1^e\right)^{\gamma_1} + M_4\left(x_1^e\right)^{\gamma_2} + \frac{(1-\theta)\lambda_0\tau_f q_1 x_1^e}{r+\lambda_0-\mu_0} - \frac{(1-\theta)\lambda_0(\tau_f-\tau_i)c}{(r+\lambda_0)r} \\ \gamma_1 M_3\left(x_1^e\right)^{\gamma_1} + \gamma_2 M_4\left(x_1^e\right)^{\gamma_2} + \frac{(1-\theta)\lambda_0\tau_f q_1 x_1^e}{r+\lambda_0-\mu_0} \\ ((1-\theta)A_5 - M_3)\left(x_0^e\right)^{\gamma_1} + ((1-\theta)A_6 - M_4)\left(x_0^e\right)^{\gamma_2} \\ +(q_0 - \frac{\lambda_0 q_1}{r+\lambda_0-\mu_0})(1-\theta)\tau_f x_0^e - \frac{(1-\theta)(\tau_f-\tau_i)c}{r+\lambda_0} \end{pmatrix}.$$

In short, we conclude from (13) the optimal default threshold x_l^b for a given coupon rate c, from (23) the optimal investment threshold x_l^e for some given coupon rate c and default threshold x_l^b, and from (27) the optimal coupon rate c_l^* of debt for some given investment threshold x_l^e and default threshold x_l^b. Solving them simultaneously, we derive the optimal default threshold x_l^b, optimal capital structure c_l^*, and optima investment threshold x_l^e. Once decision variables $\{x_l^b, x_l^e, c_l^*\}$, $l \in \{0, 1\}$, are determined, the values of all other contingent claims including the firm's leverage can be derived accordingly. More specifically, we produce the following algorithm:

6. Numerical results and analysis

Baseline parameter values. To make a relevant comparative static analysis, we select exogenous variables and parameters as typical as possible. The baseline model parameter values are specified as follows unless otherwise stated: the risk-free interest rate $r = 0.08$, close to the historical average Treasury rate, following Huang and Huang (2012); low drift rate $\mu_0 = 0.01$ and high drift rate $\mu_1 = 0.04$ following Luo and Yang (2017); the jump intensity from recession to boom $\lambda_0 = 0.15$ and that from boom to recession $\lambda_1 = 0.1$; the volatility $\sigma_0 = 0.55$ in recession and $\sigma_1 = 0.5$ in boom; the corporate effective tax rate $\tau_f = 0.15$, interest tax $\tau_i = 0.05$ and the bankruptcy loss rate $\alpha = 0.25$; the project investment cost $I = 50$, tax subsidy coefficient $\theta = 0.4$, coupon rate of debt $c = 6$ and current cash flow level $x = 5$.

6.1 The optimal investment threshold and the guarantee cost

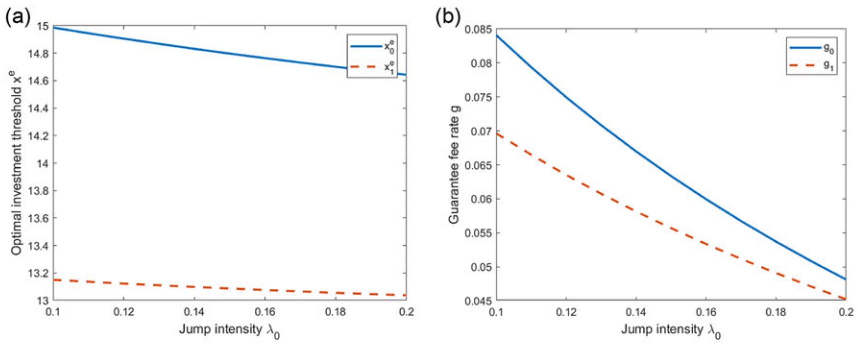
We first consider how the jump intensity from recession to boom impacts on the investment time and guarantee cost. Intuitively, the higher the intensity, the more profitable the project and thus the earlier the investment and due to the less the default risk, the less the guarantee cost. As we expected, Figure 1 tells the same story. Specifically, Figure 1(a) shows that the optimal

Algorithm 1. The algorithm for the pricing and timing of the real option

Input: $r, \mu_0, \mu_1, \sigma_0, \sigma_1, \lambda_0, \lambda_1, \tau_i, \tau_f, \alpha, I, \theta, \delta, c$

Output: $x_l^b, x_l^e, c_l^*, \mathcal{L}_l^*, R_l^*(\delta)$

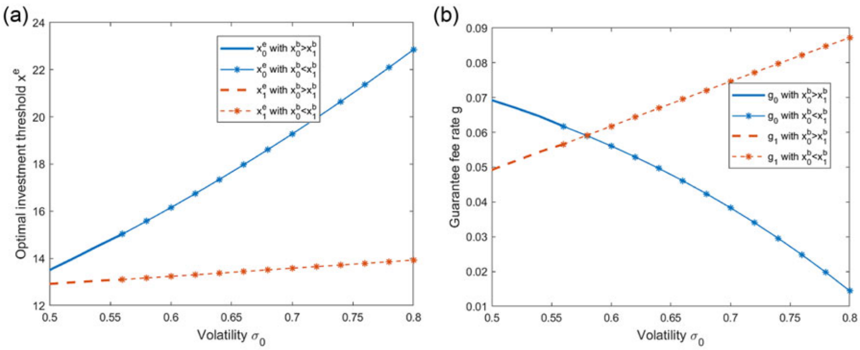
For given two-state regime-switching model, compute roots of equations:
$\beta_1, \beta_2 \Leftarrow h_1(\beta) = 0;\ \beta_3, \beta_4 \Leftarrow h_0(\beta) = 0,$
$\gamma_1, \gamma_2, \gamma_3, \gamma_4 \Leftarrow h_0(\gamma)h_1(\gamma) = \lambda_0\lambda_1,$
while regime-switching framework is set **do**
 // After investment and financing,
 while guess that $x_0^b \geq x_1^b$ **do**
  $D_0(x), D_1(x) \Leftarrow (9)\&(10),$
  $E_0(x), E_1(x) \Leftarrow (11)\&(12),$
  $TX_0(x), TX_1(x) \Leftarrow (14)\&(15),$
  $x_0^b, x_1^b \Leftarrow (13),$
  Verify, **if** conflict with $x_0^b \geq x_1^b$ **then**
   $D_l(x), E_l(x), TX_l(x), x_l^b \Leftarrow (7)\&(8),$
  end
 end
 // During financing with government subsidies, $S_l(x) = \theta TX_l(x),$
 $g_0, g_1 \Leftarrow (16)\&(17),$
 // Before investment and financing,
 while guess that $x_0^e \geq x_1^e$ **do**
  $R_0(x), R_1(x) \Leftarrow (19)\&(20);\ x_0^e, x_1^e \Leftarrow (23),$
  $T_0(x), T_1(x) \Leftarrow (29)\&(30),$
  Verify, **if** conflict with $x_0^e \geq x_1^e$ **then**
   $R_l(x), x_l^e, T_l(x) \Leftarrow (21)\&(22),$
  end
 end
 $c_0^*, c_1^*, \mathcal{L}_0^*, \mathcal{L}_1^*, R_0^*(x), R_1^*(x) \Leftarrow (27)\&(28),$
end



Investment threshold vs. jump intensity

Guarantee fee rate vs. jump intensity

Figure 1. The effect of jump intensity from recession to boom on (a) optimal investment threshold and (b) fair fair guarantee fee rate.



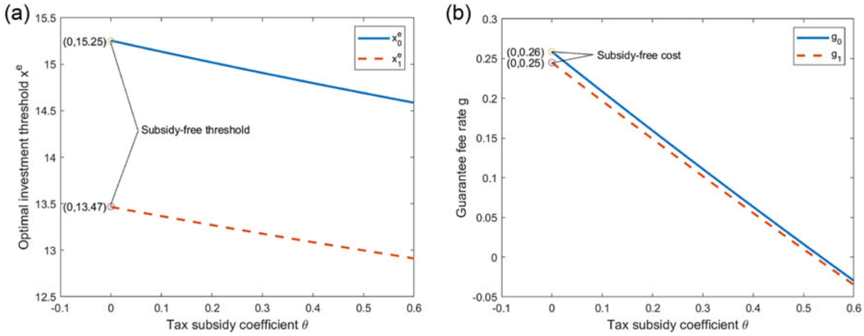
Investment threshold vs. volatility

Guarantee fee rate vs. volatility

Figure 2. The effect of volatility in recession on (a) optimal investment threshold and (b) fair guarantee fee rate.

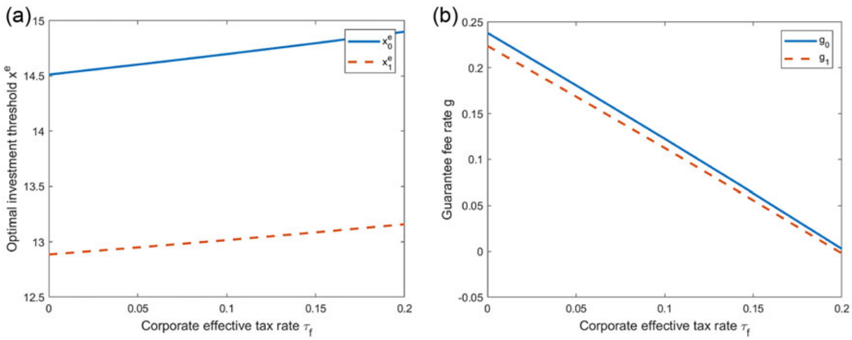
investment threshold decreases with the jump intensity from recession to boom. In addition, the investment is much earlier in boom than in recession. Figure 1(b) demonstrates that the increase of jump intensity reduces the guarantee cost and the guarantee cost in boom is smaller than that in recession.

Figure 2(a) explains that as volatility σ_0 in recession increases, the investment is postponed both in recession and in boom. This is in agreement with the well-known assertion that a higher volatility increases the value of waiting and delays investment. Moreover, an increase in volatility changes the bankruptcy trigger, postponing the default in recession, such that if volatility σ_0 surpasses a threshold, the default in recession happens later than in boom as shown in the figure. In addition, as we expected, the investment in boom occurs much earlier than in recession. Figure 2(b) shows that the guarantee fee rate in boom increases with the volatility, while that in recession decreases with the volatility. The former is intuitive since a high volatility has a negative impact on the value of debt, increases the guarantee liability, and naturally induces a high guarantee fee rate in return for the guarantee. However, the latter is counter-intuitive. It comes to light that a higher volatility induces higher investment threshold and government subsidies, which thus leads to a less default risk and a less guarantee liability. The final result is determined by which of the two opposite factors dominates the other. For the same reason, Figure 2(b) displays that when the volatility is high, the guarantee cost in boom is strangely higher than that in recession.



Investment threshold vs. tax subsidy   Guarantee fee rate vs. tax subsidy

Figure 3. The effect of the tax subsidy coefficient θ on (a) optimal investment threshold and (b) fair guarantee fee rate.



Investment threshold vs. tax rate   Guarantee fee rate vs. tax rate

Figure 4. The effect of corporate effective tax rate τ_f on optimal investment threshold and fair guarantee fee rate.

Figure 3(a) presents the impact of government subsidies through guarantee fee reduction or cash subsidy. It says that the higher the tax subsidy coefficient, the less the optimal investment threshold. This is because the greater the tax subsidy coefficient, equivalently the less the investment cost. Figure 3(b) plots the guarantee fee rate against the tax subsidy coefficient. It states that the increase in the tax subsidy coefficient reduces the guarantee fee rate as we expected. In particular, the guarantee fee rate would be negative, meaning that not only a waiver of the guarantee premium but also a cash subsidy of $-g_l(1-\tau_i)c/r$ is provided by the government. This seems unreasonable at first sight; but it would be really worthwhile for a project having a sufficiently positive externality but lacking in the private value. Figure 3(b) further says that the guarantee fee rate in boom is less than that in recession. Intuitively, this is because the default risk in boom is lower than that in recession.

Figure 4 illustrates the impact of corporate effective tax rates on investment thresholds and guarantee costs. Figure 4(a) says that an increase in the corporate effective tax rate leads to a steady rise in the optimal investment threshold. This is because the higher the tax rate, the less the firm gets from the investment; thus, the investment is postponed. In addition, economic boom accelerates investment relative to economic recession; this is clear. Figure 4(b) demonstrates that an increase in the tax rate reduces the guarantee cost. The reason behind this is that the higher the tax rate, the higher the investment threshold (i.e. the lower the default risk) and the higher the tax subsidy; both of them lead to a less fair guarantee cost. Last, as we expected, Figure 4(b) states that the guarantee cost is lower in boom than in recession for an arbitrary given tax rate.

Table 1. Optimal coupon rate c_l^* vs. the jump intensity from recession to boom

	Recession			Boom		
Jump intensity	c_0^*	L_0^*	R_0	c_1^*	L_1^*	R_1
λ_0=0.10	5.15	20.34%	46.82	5.35	21.74%	54.29
λ_0=0.12	5.25	20.43%	48.57	5.45	21.88%	55.59
λ_0=0.15	5.35	20.46%	50.84	5.55	21.96%	57.30
λ_0=0.20	5.55	20.71%	53.95	5.70	22.12%	59.63

Table 2. Optimal coupon rate c_l^* vs. the volatility in recession

Volatility	Recession			Boom		
Volatility	c_0^*	L_0^*	R_0	c_1^*	L_1^*	R_1
σ_0=0.50	5.20	22.31%	49.55	5.40	22.03%	56.93
σ_0=0.52	5.25	21.52%	50.07	5.45	21.97%	57.08
σ_0=0.55	5.35	20.46%	50.84	5.55	21.96%	57.30
σ_0=0.60	5.90	19.88%	52.13	5.95	22.68%	57.69

6.2 *The optimal capital structure and option value*

More often than not, we do not consider the optimal capital structure problem since the amount of money borrowed is determined by the funding gap for investing in a project. On account of that the government provides subsidies depending on the level of debt in our model, we address the optimal structure and fix the optimal coupon level to maximize the project value. Obviously, due to that the government subsidy is a fraction of the total tax revenue generated by the project, the benefit of issuing debt is decreased. In particular, if tax subsidy coefficient $\theta = 1$, then the merit of tax shields from debt disappears and thus the entrepreneur should not borrow if s/he has enough money to invest in the project. In this subsection, as assumed before, we suppose that the entrepreneur must borrow to start the project and s/he can borrow as much as s/he wants. We discuss how much s/he should borrow to maximize the project value and how the optimal coupon rate is related with the jump intensity λ_0, volatility σ_0 and externality of the project.

Table 1 reports the effect of the jump intensity from recession to boom on the optimal capital structure (i.e. optimal coupon rate c_l^* or optimal leverage $\mathcal{L}_l^*$, $l \in \{0, 1\}$) and the real option value. It says that an increased jump intensity increases the level of debt and the leverage both in boom and in recession. The former (the increased debt level) results from the fact that the higher the intensity, the less the default risk, and thus the more the optimal level of debt issued. The latter (the increased leverage) happens because of the increased debt level. However, the leverage may decrease with the jump intensity since the higher the jump intensity, the less the investment threshold, as predicted by Figure 1(a); thus, a lower leverage would be conversely preferred to prevent the firm from default. Moreover, as expected, the optimal level of debt and optimal leverage in boom are higher than that in recession; the higher the jump intensity, the higher the real option value.

Table 2 displays the effect of the volatility on the optimal capital structure and the real option value. It states that the optimal debt coupon both in recession and in boom increases with the volatility σ_0. This happens because the higher the volatility, the higher the investment threshold, i.e. the safer the debt, and thus, the higher the optimal debt coupon. By contrast, the change of the leverage is ambiguous since both the debt value and the firm value increases with the volatility. Moreover, the optimal debt coupon in boom is higher than that in recession. The reason behind is that the debt is exposed to less default risk in boom than in recession, which implies that increasing

Table 3. Optimal coupon rate c_i^* vs. tax subsidy coefficient

	Recession			Boom		
Subsidy coefficient	c_0^*	L_0^*	R_0	c_1^*	L_1^*	R_1
θ=0.1	9.10	30.45%	48.19	9.25	32.29%	54.32
θ=0.2	7.85	27.38%	49.05	8.00	29.06%	55.29
θ=0.3	6.60	24.06%	49.93	6.75	25.57%	56.28
θ=0.4	5.35	20.46%	50.84	5.55	21.96%	57.30

Table 4. Optimal coupon rate c_i^* vs. corporate effective tax rate

	Recession			Boom		
Tax rate	c_0^*	L_0^*	R_0	c_1^*	L_1^*	R_1
τ_f=0.15	5.35	20.46%	50.84	5.55	21.96%	57.30
τ_f=0.18	7.45	27.16%	49.84	7.60	28.82%	56.19
τ_f=0.20	8.80	31.19%	49.22	8.95	33.06%	55.48
τ_f=0.22	10.10	34.90%	48.61	10.20	36.82%	54.81

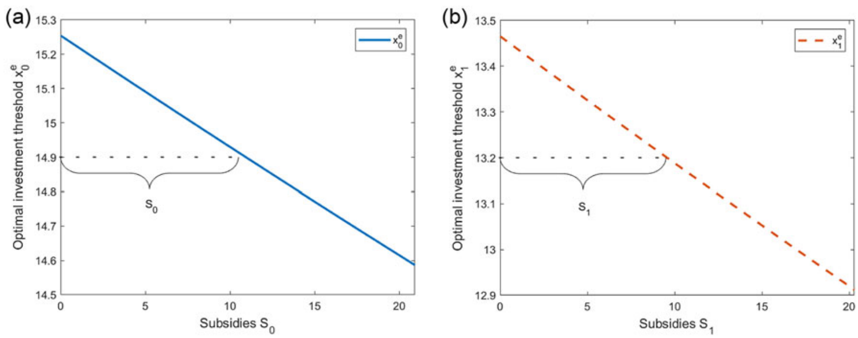
debt scale in boom is profitable relative to the debt scale in recession. Last, the table predicts that a higher volatility increases the value of the option. This is a well-known result.

Table 3 shows the relationship between government subsidies and optimal capital structure. Government subsidies depend on tax revenues; thus, government subsidies would make the tax shields effect insignificant. Therefore, the optimal coupon rate and optimal leverage decrease with the tax subsidy coefficient as argued in the first paragraph of the subsection and predicted by the table. The more the government subsidizes, the less the profit from tax shields and thus the less the optimal level of debt. The table displays further that the increase of the investment externalities induces a higher value of the investment option. This is because the higher the tax subsidy coefficient, the more the subsidies and all the subsidies are harvested by the entrepreneur.

Table 4 illustrates the effect of the corporate effective tax rate on the optimal coupon rate and the value of the real option. It displays that the optimal coupon rate and the leverage increase with the corporate effective tax rate. This is because the higher the tax rate, the tax shield effect becomes more valuable, leading to an increase in both the optimal coupon rate and the leverage. Noting that the tax subsidy coefficient $\theta = 0.4$, we conclude that the value of the real option decreases with the tax rate as shown in the table. Finally, Table 4 explains that the optimal coupon rate, optimal leverage, and the value of the real option in recession are less than their corresponding items in boom. Clearly, these are what we expect.

6.3 The subsidy required for an immediate investment

All real investment projects can be divided into three types according to their profitability: One is unprofitable (the project value is negative) and thus should be given up forever; The second type is profitable from the point of view of investors and should be therefore started sometimes; The third type has a positive total social value but it is not profitable for private investors. For the third type of projects governments should provide subsides. For example, when a country is faced with a major industrial or strategic demand and requires a special project to be started immediately. The government should use government funds to support the entrepreneur with the help of credit guarantees and government subsidies. We wonder how much subsides should be provided at least to let the entrepreneur start such a project immediately. Naturally, the larger



Investment threshold vs. tax subsidies | Guarantee fee rate vs. tax subsidies

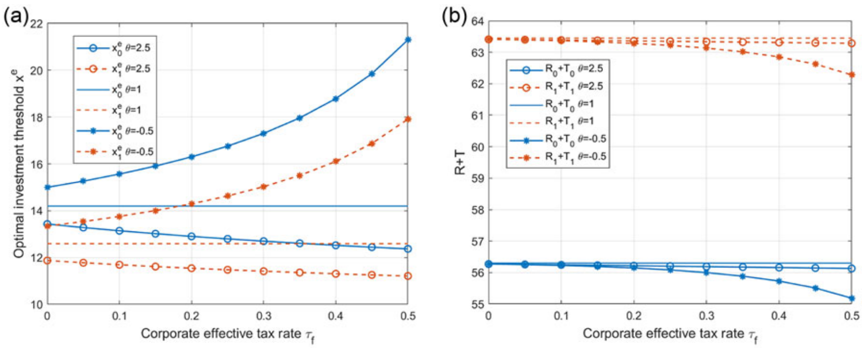
Figure 5. The figure demonstrates the government subsidies required for motivating an immediate investment under the two different economy state and with different cash flow levels.

the government subsidy, the lower the optimal investment threshold. In particular, without government subsidies, it would not lead to an immediate investment if the current cash flow level is too low. More specifically, we can consider the investment threshold x_l^e as a decreasing function of the amount S_l of government subsidies, denoted by $x_l^e(S_l)$, if Keeping all the other things the same. Clearly, to motivate the entrepreneur to invest instantly, the required minimum subsidy S_l^* is given by $S_l^* \equiv \inf\{S_l | x_l^e(S_l) \le x\},\ l \in \{0, 1\}$, where x is the current cash flow level. Figure 5 shows that the amount S_0 or S_1 of government subsidies at least is required for motivating the entrepreneur to conduct the immediate investment. Figure 5(a) shows the government subsidies S_0 required to take an immediate investment in recession. If the firm's cash flow is 14.9, the entrepreneur does not exercise the investment unless s/he gets the amount of government subsidies being $S_0 \approx 11$. The higher the current level of cash flow, the less the government subsidy required for the immediate investment. In a similar way, Figure 5(b) presents the corresponding results in boom. Last, if the externality of the investment is negative, e.g., an investment inducing environmental deterioration, the tax subsidy coefficient θ can be less than zero, meaning that the investment is punished in addition to getting no subsidies from a government.

6.4 The effect of tax rates on the pricing and timing of an alternative real option

Strictly speaking, an investment in a project not only creates the project value but also creates the value of tax revenues in addition to the value derived from its externalities. For an obvious reason, we take the sum of the created project value and the value of the created tax revenues, i.e. $R_l(x) + T_l(x)$, as the value of the investment option, while the value derived from its externalities is not easy to measure.[7] We call such investment option an alternative real option. We stress that its investment threshold is still determined by the entrepreneur as before; thus, only if $\theta = 1$, the investment threshold chosen by the entrepreneur is also optimal for the alternative real option.

Figure 6(a) says that if the tax subsidy coefficient θ is greater (less) than 1, the entrepreneur would overinvest (underinvest). Figure 6(b) states that both the overinvestment and underinvestment induce a value loss of the alternative real option. The higher the tax rate, the more significant the effect. All these results are easy to follow thanks to our previous analysis. Naturally, if a project has sufficiently positive externalities, we should let $\theta > 1$ to motivate the firm to accelerate investment, even if it would lead to the value loss of the alternative real option. For this reason, the tax subsidy coefficient (θ) is a powerful policy instrument to motivate an entrepreneur to invest in a positive externality project.



Investment threshold vs. tax rate

The real option value vs. tax rate

Figure 6. The effect of the corporate effective tax rate on the pricing and timing of the alternative real option.

7. Conclusion

Micro, small and medium-sized enterprises (MSMEs) are the new force of national economic development and have a strong external spillover effect. The externalities would be positive and negative; thus, we can not evaluate an investment in a project just according to the cash flow generated by it. These considerations are quite in agreement with the viewpoint that corporations and investors should take ESG concerns into account in making a financial decision. ESG concerns have been recently attracting more and more interests of academic researchers and government regulations. To stimulate or discourage an investment, government intervention is necessary and feasible.

MSMEs often experience financing constraints to start a project or operate a business. The financing difficulties are closely related to the external economic environment, which usually changes cyclically. This phenomenon is getting more significant with the spread of global pandemic coronavirus disease 2019 (COVID-19).

Given the above considerations, we develop a regime-switching model on real investment with credit guarantees and government subsides depending on investment externalities. We provide explicit pricing formulas for a general standard asset in a Markovian regime-switching GBM model *à* la Leland (1994). To the best of our knowledge, these formulas are never derived before. Almost all common corporate securities' price can be easily derived by the explicit formulas even though the project cash flow is driven by both a Brownian motion and a two-state Markov chain. Taking investment externalities into account, we present numerical analysis on the pricing and timing of the option to invest in a project with credit guarantees and government subsidies. We provide a method about how governments should specify a proper tax subsidy coefficient for a given tax rate to motivate a firm to invest in a project in the way they wish. If the tax subsidy coefficient is greater (less) than 1, an overinvestment (underinvestment) occurs. The higher the tax rate, the more significant the overinvestment (underinvestment). The subsidy required for an immediate investment depends not only on the current cash flow level but also on the state of the economy regime. The investment threshold and the fair guarantee fee rate decrease with the jump intensity from recession to boom and the amount of government subsidies. The higher the volatility in recession, the later the investment no matter what current economy state is.

These findings are helpful to address how to effectively motivate or discourage a firm to invest in a project according to investment externalities, of which the fundamental idea is quite in agreement with the recent ESG concerns. This paper is therefore of significance to the effective use of government funds to support MSMEs.

Notes

1 The information is perfect, and the information flow $\{\mathcal{F}_t\}_{t\geq 0}$ is generated by processes W and L and learned by participants.
2 Similarly, if $\mathcal{D}_1 \subseteq \mathcal{D}_0$, the corresponding assertions hold true also.
3 The linear terminal payoff assumption is not necessary to derive an explicit solution. However, the assumption holds for many common corporate securities. For example, the liquidation value of a common corporate security has such form in general.
4 The EBIT is not generated if the investment has not taken place but in many cases, we can infer the EBIT level from an existing market as widely assumed in the real options literature.
5 If $x_0^b < x_1^b$, the discussions are similar and thus omitted as we did before.
6 If $x_1^e > x_0^e$, the derivations are almost the same.
7 Actually, in our model, the externalities are exogenously given and reflected by the tax subsidy coefficient θ. The more positive the externalities, the higher the value of θ.

References

Beck, T., L.F. Klapper and J.C. Mendoza. (2010). The typology of partial credit guarantee funds around the world. *Journal of Financial Stability* 6(1), 10–25.
Blanchard, O. and R. Perotti. (2002). An empirical characterization of the dynamic effects of changes in government spending and taxes on output. *The Quarterly Journal of Economics* 117(4), 1329–1368.
Brennan, M.J. and E.S. Schwartz. (1985). Evaluating natural resource investments. *Journal of Business* 58(2), 135–157.
Chen, H. (2010). Macroeconomic conditions and the puzzles of credit spreads and capital structure. *The Journal of Finance* 65(6), 2171–2212.
Cowling, M. and J. Siepel. (2013). Public intervention in UK small firm credit markets: Value-for-money or waste of scarce resources? *Technovation* 33(8), 265–275.
Dixit, R. K., A. K. Dixit and R. S. Pindyck. (1994). *Investment Under Uncertainty*, Princeton, NJ: Princeton University Press.
Doyle, M. (2010). Informational externalities, strategic delay, and optimal investment subsidies. *Canadian Journal of Economics/Revue canadienne d'économique* 43(3), 941–966.
Economides, G., H. Park, A. Philippopoulos and S. Sakkas. (2022). On the mix of government expenditure and tax revenues. *Macroeconomic Dynamics* 26(1), 1–48.
Goldstein, R., N.J. Ju and H. Leland. (2001). An EBIT-based model of dynamic capital structure. *Journal of Business* 74(4), 483–512.
Guo, X., J. Miao and E. Morellec. (2005). Irreversible investment with regime shifts. *Journal of Economic Theory* 122(1), 37–59.
Hackbarth, D., J. Miao and E. Morellec. (2006). Capital structure, credit risk, and macroeconomic conditions. *Journal of Financial Economics* 82(3), 519–550.
Hamilton, J.D. (1989). A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica* 57(2), 357–384.
Honohan, P. (2010). Partial credit guarantees: Principles and practice. *Journal of Financial Stability* 6(1), 1–9.
Huang, J.-Z. and M. Huang. (2012). How much of the corporate-treasury yield spread is due to credit risk? *The Review of Asset Pricing Studies* 2(2), 153–202.
Hwu, S.-T., C.-J. Kim and J. Piger. (2021). An n-state endogenous markov-switching model with applications in macroeconomics and finance. *Macroeconomic Dynamics* 25(8), 1937–1965.
Jeon, H. and M. Nishihara. (2015). The effects of business cycle and debt maturity on a firm's investment and default decisions. *International Review of Economics & Finance* 38, 326–351.
Leland, H.E. (1994). Corporate debt value, bond covenants, and optimal capital structure. *The Journal of Finance* 49(4), 1213–1252.
Luo, P.F. and Z.J. Yang. (2019). Growth option and debt maturity with equity default swaps in a regime-switching framework. *Macroeconomic Dynamics* 23(6), 2250–2268.
Luo, P., J. Xiong, J. Yang and Z. Yang. (2019). Real options under a double exponential jump-diffusion model with regime switching and partial information. *Quantitative Finance* 19(6), 1061–1073.
Luo, P. and Z. Yang. (2017). Real options and contingent convertibles with regime switching. *Journal of Economic Dynamics and Control* 75, 122–135.
Mayeres, I. and S. Proost. (1997). Optimal tax and public investment rules for congestion type of externalities. *Scandinavian Journal of Economics* 99(2), 261–279.
Montmartin, B. and M. Herrera. (2015). Internal and external effects of r&d subsidies and fiscal incentives: Empirical evidence using spatial dynamic panel models. *Research Policy* 44(5), 1065–1079.
Myers, S.C. (1977). Determinants of corporate borrowing. *Journal of Financial Economics* 5(2), 147–175.

Tan, Y. and Z. Yang. (2017). Growth option, contingent capital and agency conflicts. *International Review of Economics and Finance* 51, 354–369.

Zhang, Q. and X. Guo. (2004). Closed-form solutions for perpetual american put options with regime switching. *SIAM Journal on Applied Mathematics* 64(6), 2034–2049.

Cite this article: Liu X and Yang Z (2024). "Investments, credit guarantees, and government subsidies in a regime-switching framework." *Macroeconomic Dynamics* **29**(e56), 1–23. https://doi.org/10.1017/S1365100524000634