Massive Stars Near and Far
Proceedings IAU Symposium No. 361, 2024
J. Mackey, J. S. Vink, & N. St-Louis, eds.
doi:10.1017/S1743921322002320

Entrainment in 3D hydrodynamics simulations of neon burning

F. Rizzuti[1], R. Hirschi[1,2], C. Georgy[3], W. D. Arnett[4], C. Meakin[5] and A. StJ. Murphy[6]

[1]Astrophysics Group, Lennard-Jones Laboratories, Keele University, Keele ST5 5BG, UK
email: f.rizzuti@keele.ac.uk

[2]Kavli IPMU (WPI), University of Tokyo, 5-1-5 Kashiwanoha, Kashiwa 277-8583, Japan

[3]Geneva Observatory, Geneva University, CH-1290 Sauverny, Switzerland

[4]Steward Observatory, University of Arizona, 933 N. Cherry Avenue, Tucson AZ 85721, USA

[5]Pasadena Consulting Group, 1075 N Mar Vista Ave, Pasadena, CA 91104 USA

[6]School of Physics and Astronomy, University of Edinburgh, Edinburgh EH9 3FD, UK

Abstract. Our knowledge of massive star evolution is limited by uncertainties linked with multi-dimensional processes taking place in stellar interiors. Important examples are convective boundary mixing (CBM) and entrainment, which are implemented in 1D stellar evolution models assuming simplified prescriptions. 3D hydrodynamics models can improve these prescriptions by studying realistic multi-D processes for a short timerange (minutes or hours). In these proceedings, we present results coming from a new set of high-resolution hydrodynamics simulations of a neon-burning shell in a massive star, and discuss how the entrainment law can be calibrated from 3D models and then used to improve 1D stellar evolution prescriptions.

Keywords. convection - hydrodynamics - turbulence - stars: evolution - stars: interiors - stars: massive

1. Introduction

Stellar evolution models are one-dimensional (1D) tools employed to simulate the entire lifetime of a star. Their efficiency is gained by making simplifying assumptions, spherical symmetry being the main one. Calibrating the prescriptions often requires comparison of results from specific simulations to observational data. As an example, the mixing length theory (MLT, Böhm-Vitense 1958) is used in 1D models to predict the extension of convective regions in the star.

Multi-dimensional hydrodynamics models provide useful insight in this sense, having several advantages compared to 1D stellar models. In particular, removing the assumption of spherical symmetry, it is possible to model fluid instabilities and reproduce complex processes, e.g. convection, rotation and magnetic activity, without the simplifying assumptions made in 1D models. On the other hand, multi-D models have a high computing cost, much larger than for 1D models, so they are bound to reproduce only short time scales and localized spatial extents. Therefore, an advancement in understanding stellar evolution can be achieved by combining the strengths of 1D and multi-D models, using the latter for constraining prescriptions, and subsequently including them in the former.

In these proceedings, we present a new set of hydrodynamics models simulating convection in a neon-burning shell of a 15 $M_\odot$ star. The simulations have been run with the `PROMPI` code, successfully used in the past for studying in a similar way other convective regions of massive stars (see Meakin & Arnett 2007; Viallet et al. 2013; Cristini et al. 2019). In particular, we focus here on the evolution of the convective zone in the neon shell, and the modelling of the convective boundary mixing (CBM) employing the entrainment law.

2. Methodology

Initial conditions were taken from a 1D stellar evolution simulation of a 15 $M_\odot$ star, obtained with the `GENEC` code (Eggenberger et al. 2008). Assumptions of this 1D model include solar metallicity, Schwarzschild criterion for convective boundaries, and penetrative overshoot for hydrogen- and helium-burning cores. For our neon-shell model, we mapped the 1D radial profiles of fundamental variables (e.g. density, temperature, abundances) from the second neon-burning shell onto a cubic box of side 0.64×10^8 cm. The numerical resolution chosen for the model is a grid of 512^3 cells, i.e. each cell is a cube of 1.25×10^5 cm side. All simulations have been run starting from this initial setup.

In our simulations, convection is driven using an explicit nuclear network for neon burning, including the isotopes ^{4}He, ^{16}O, ^{20}Ne, ^{24}Mg and ^{28}Si (the ^{4}He abundance is assumed to be at nuclear equilibrium in this late burning stage). Having all the same burning network, the simulations differ for the so-called "boosting factor", a constant factor that multiplies the nuclear energy generation rates. The factor has been introduced in order to boost convection and reduce the computing cost, but also to study the dependence of key quantities on the driving energy. For this reason, we produced a set of four simulations with boosting factors 1, 10, 100, 1000 times the nominal 1D case (see Rizzuti et al. 2022), in a similar way to what has been done in the work of Cristini et al. (2019) for the carbon shell. The boosting factor has a strong impact on the computing power required to run the simulations. In particular, with a fixed computing budget we are able to simulate a longer time range for models with a larger boosting factor.

3. Results

In Figure 1 we display a vertical cross-section across the domain of the nominal-luminosity simulation, with the velocity magnitude in colour scale. The convective zone is easily recognizable as the central region where the flow has higher speed and forms plumes and eddies. Above and below the convective zone, the stable region is dominated by low-speed internal gravity waves.

In order to study the evolution of this convective zone in our simulations, we need first to identify the convective boundaries. We define them by means of the chemical composition of the layers, using the fact that the convective zone is always homogeneously mixed. In this way, we define the convective boundaries as the location where the horizontally-averaged mean atomic mass is equal to the mid-point between the value in the convective and in the stable zone. With this definition, we are able to follow the time evolution of both the upper and lower convective boundary locations, as shown in Figure 2 for the four simulations with different boosting factors.

At the beginning of the simulations an intermediate phase is present, that we call "initial transient", before convection is fully developed with eddies and plumes. The length of this transient depends on the boosting factor, since a stronger energy generation produces faster convection. After the initial transient the simulation is in a "quasi-steady" state, and it is possible to analyse the results. Figure 2 clearly shows the growth in time of the convective zone, both upwards and downwards. This is due to a phenomenon

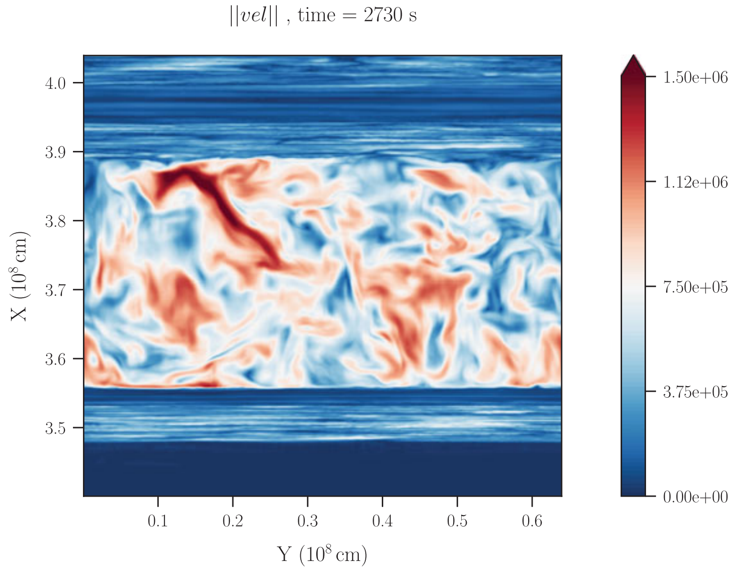

Figure 1. Vertical cross section of the velocity magnitude (values in colour scale, cm s^{-1}) taken towards the end of the nominal-luminosity simulation, at 2730 seconds. The high-velocity eddies identify the convective zone.

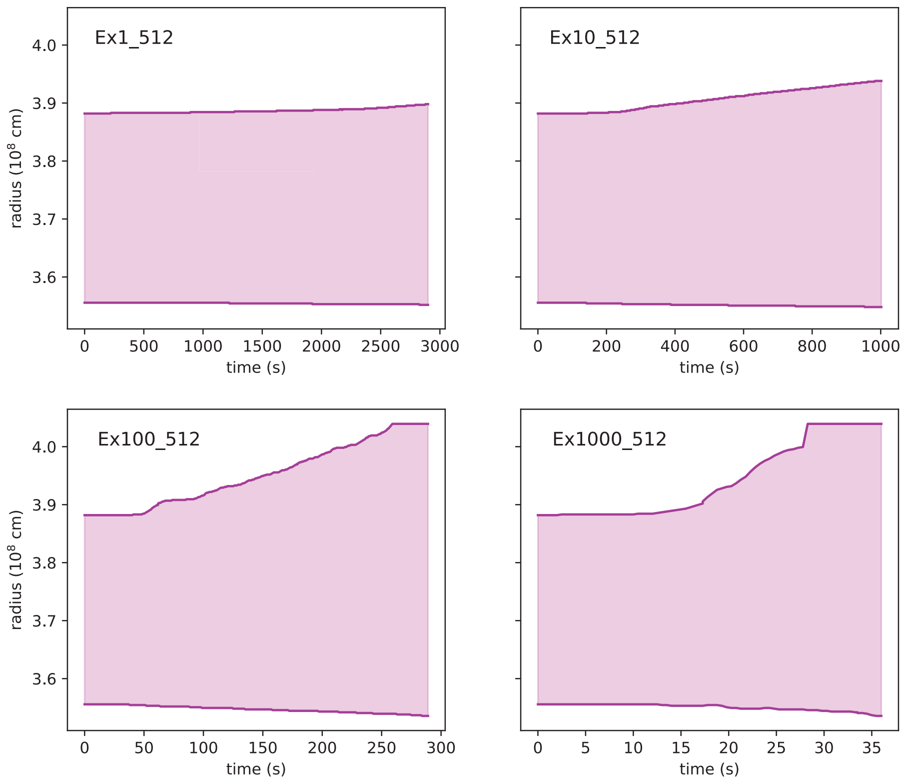

Figure 2. Time evolution of the upper and lower boundaries enclosing the convective zone (in magenta), for the four simulations with boosting factors 1, 10, 100 and 1000. After an initial transient, whose length depends on the boosting, the convective zones grow due to entrainment. It is noticeable how strongly the boosting affects the timescale for evolution.

https://doi.org/10.1017/S1743921322002320 Published online by Cambridge University Press

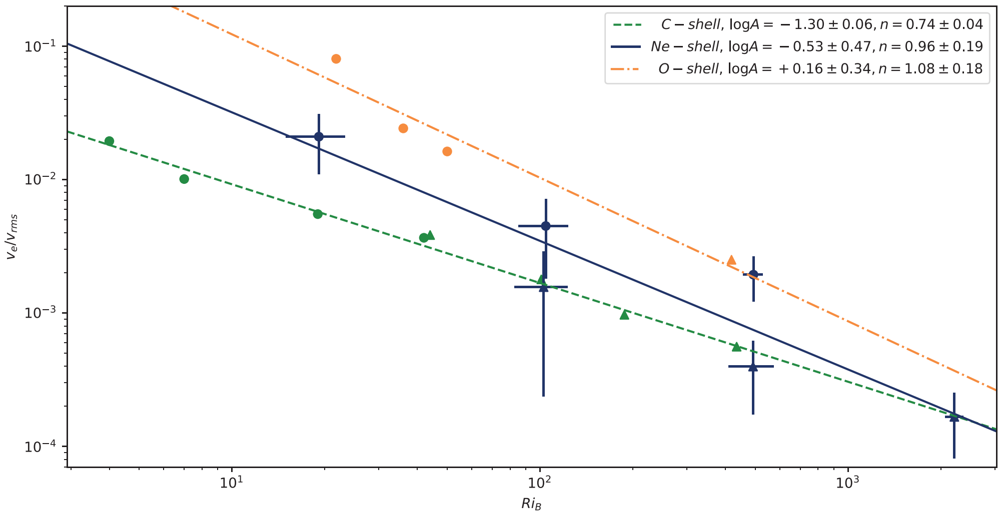

Figure 3. Entrainment rate versus bulk Richardson number in log scale and respective linear regressions: Ne-shell from this study (blue, solid), C-shell from Cristini et al. (2019) (green, dashed), O-shell from Meakin & Arnett (2007) (orange, dot-dashed). Triangles are measurements for the lower convective boundary, circles for the upper boundary. Error bars (only for Ne-shell) are standard deviations. Parameter estimates for the entrainment law $v_e/v_{rms} = A\ Ri_B^{-n}$ are listed in the legend. See Rizzuti et al. (2022).

called "turbulent entrainment", well-known in the literature (see Maeder 1976; Zahn 1991). It results from the shear mixing that takes place near the convective boundaries, where material is continuously entrained from the stable region into the convective zone, resulting in more fluid becoming convective.

Entrainment can be parametrized with a simple law, assuming a dependence of the entrainment rate (i.e. entrainment velocity over convective velocity) on the bulk Richardson number, Ri_B (a measure of the convective boundary stiffness): $E = v_e/v_{rms} = A\ Ri_B^{-n}$, coming from geophysics (Fernando 1991) and applied to stellar interiors by Meakin & Arnett (2007). For our simulations, the entrainment velocities have been obtained from the time derivatives of Figure 2, noticing that a larger boosting factor produces stronger entrainment, and that the entrainment velocity is smaller for the lower boundary, given its larger stiffness. The bulk Richardson number has been computed for each boundary by integrating the Brunt-Väisälä frequency as described in Cristini et al. (2019).

We finally show in Figure 3 the measurements of entrainement rate versus bulk Richardson number for our three simulations with boosting factors 1, 10 and 100 (in blue, solid). We decided to exclude boosting factor 1000 from the analysis, since we found that the extreme fluid velocities in the simulation disrupt the boundaries, losing the information about CBM. With these data points, we are able to estimate the free parameters of the entrainment law given above. We display in Figure 3 the line of best fit in log scale for our neon-shell simulations alongside other two `PROMPI` simulations for the carbon (Cristini et al. 2019) and oxygen (Meakin & Arnett 2007) shells of massive stars. The error bars for neon shell come from our statistical analysis of the simulations. The estimates of the entrainment law parameters A and n are given in the legend for each study.

From the parameter estimates, we can conclude that entrainment takes place in a similar way in multiple burning stages of massive stars. This is also compatible with other hydrodynamics simulations of convective zones in stellar interiors (e.g. Mocák et al.

2009; Gilet et al. 2013; Horst et al. 2021). The best fit for our new neon-shell simulations provides $n = 0.96 \pm 0.19$, which is compatible with the value of 1 expected from geophysics (Fernando 1991). Estimates for A are generally more uncertain, taking into account the large dispersion of measurements and that the fitting has been done in log scale. Our results for the neon shell indicate that values lie between 0.1 and 1.0. The small errors in the estimated parameters may suggest that uncertainties have been underestimated.

4. Conclusions

We present in these proceedings a new set of 3D hydrodynamics simulations of a neon-burning shell in a 15 $M_\odot$ star, obtained with the `PROMPI` code. We studied in detail the development and evolution of the convective zone, as well as the mixing which takes place near the convective boundaries. Importantly, we analyzed the entrainment of stable material into the convective zone and parametrized it with a simple law.

We found that significant entrainment is present in our simulations when using the exact same conditions taken from a state-of-the-art 1D stellar model, particularly in the case where the energy generation was not modified (nominal-luminosity case). Estimating the free parameters for the entrainment law in our simulations, we found that results for neon shell are in agreement with previous hydrodynamics studies of other burning phases in massive stars, confirming the occurrence and importance of CBM for the stellar evolution.

There is still a discrepancy between predictions for entrainment in hydrodynamics models versus stellar evolution models. This is well represented by the disagreement on the A parameter for the entrainment law. We refer here to the studies conducted by Staritsin (2013) and Scott et al. (2021), the only ones so far in the literature to have tested the entrainment law in 1D stellar models.

Synergy and convergence between one- and multi-dimensional models are crucial steps towards a better understanding of the complex phenomena in the stellar interiors. On the one hand, hydrodynamics models rely on the initial conditions assumed from 1D models, and they show disagreement on the strength of CBM assumed for 1D stellar evolution. On the other hand, adopting the entrainment law in 1D, especially for late convective phases, will provide a realistic way of modelling CBM, as well as produce more accurate initial conditions for starting new 3D simulations.

Acknowledgements

RH acknowledges support from the World Premier International Research Centre Initiative (WPI Initiative), MEXT, Japan and the IReNA AccelNet Network of Networks (National Science Foundation, Grant No. OISE-1927130). CG has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation program (Grant No. 833925). WDA acknowledges support from the Theoretical Astrophysics Program (TAP) at the University of Arizona and Steward Observatory. CG, RH, and CM acknowledge ISSI, Bern, for its support in organising collaboration. This article is based upon work from the ChETEC COST Action (CA16117) and the European Union's Horizon 2020 research and innovation programme (ChETEC-INFRA, Grant No. 101008324). The authors acknowledge PRACE for awarding access to the resource MareNostrum 4 at Barcelona Supercomputing Center, Spain, and the STFC DiRAC HPC Facility at Durham University, UK (Grants ST/P002293/1, ST/R002371/1, ST/R000832/1, ST/K00042X/1, ST/H008519/1, ST/ K00087X/1, ST/K003267/1). The University of Edinburgh is a charitable body, registered in Scotland, with Registration No. SC005336.

References

Böhm-Vitense E., 1958, Zeitschrift für Astrophysik, 46, 108
Cristini A., Hirschi R., Meakin C., Arnett D., Georgy C., Walkington I., 2019, MNRAS, 484, 4645
Eggenberger P., Meynet G., Maeder A., Hirschi R., Charbonnel C., Talon S., Ekström S., 2008, AP&SS, 316, 43
Fernando H. J. S., 1991, Annual Review of Fluid Mechanics, 23, 455
Gilet C., Almgren A. S., Bell J. B., Nonaka A., Woosley S. E., Zingale M., 2013, ApJ, 773, 137
Horst L., Hirschi R., Edelmann P. V. F., Andrássy R., Röpke F. K., 2021, A&A, 653, A55
Maeder A., 1976, A&A, 47, 389
Meakin C. A., Arnett D., 2007, ApJ, 667, 448
Mocák M., Müller E., Weiss A., Kifonidis K., 2009, A&A, 501, 659
Rizzuti F., Hirschi R., Georgy C., Arnett W. D., Meakin C., StJ. Murphy A., 2022, arXiv e-prints, p. arXiv:2207.03223
Scott L. J. A., Hirschi R., Georgy C., Arnett W. D., Meakin C., Kaiser E. A., Ekström S., Yusof N., 2021, MNRAS, 503, 4208
Staritsin E. I., 2013, Astronomy Reports, 57, 380
Viallet M., Meakin C., Arnett D., Mocák M., 2013, ApJ, 769, 1
Zahn J. P., 1991, A&A, 252, 179

Discussion

Question: Do your simulations eventually reach the sub-surface convection? How far from the surface do they get?

Answer: The burning shell is deep so it doesn't reach the surface. Simulating convection below the surface would be a different environment. Our simulations are not including the surface.

Question: I was very excited to see that you also see changes at lower boundary as well as the upper boundary, because in 1D stellar models we just use overshooting and not undershooting. I'd like to know if we should try to include undershooting in our models to get closer to 3D results.

Answer: Yes, this is something we see happening in 3D, so it should be included. We are expecting it from the fluid motions. Entrainment is much slower at the lower boundary, but for completeness it also should be included.

Follow-up question: So why is this? Why do we not see the same effect at the upper and lower boundary?

Follow-up answer: It depends on the properties of the boundary. The bulk Richardson number is a measurement of the stiffness of boundary. The lower boundary is much stiffer, it has a higher bulk Richardson number, so we have much different properties.

https://doi.org/10.1017/S1743921322002320 Published online by Cambridge University Press