

BOOK REVIEW

H.H. PANJER, G.E. WILLMOT (1992): *Insurance risk models*. Society of Actuaries, Schaumburg IL 60713-2226, USA, 442 pages, US\$ 35.00 (overseas: + 50%).

The main topic of the present book is evaluation of the distribution of the aggregate amount of claims incurred in an insurance portfolio during a specified period. The main emphasis is on recursive methods, and the authors present numerous models that allow such methods.

The book consists of five main parts. Part I (Chapters 1-4) gives the statistical preliminaries. Part II discusses some basic risk models, using the traditional distinction between individual (Chapter 5) and collective (Chapter 6) models. In Part III some more advanced claim frequency models within the collective set-up are presented. Chapter 7 concentrates on compound claim frequency models whereas Chapter 8 is devoted to mixed claim frequency models. The surprisingly close connection between mixed and compound models is discussed. In Part IV the authors discuss fitting, estimating, and approximating. Whereas in the earlier parts of the book the underlying models have been assumed to be known, Chapter 9 discusses how to choose an appropriate model and estimate its parameters. The chapter concentrates on claim frequency models; fitting of severity distributions has been treated by HOGG and KLUGMAN (1984). Chapter 10 is devoted to asymptotic results on the tail of the aggregate claims distribution. Finally Part V discusses ruin theory within the frame-work of compound Poisson processes, using the mathematical tools introduced in earlier chapters.

The book contains a lot of numerical examples. At the end of most of the chapters there are exercises. Also at the end of each chapter there are references to related literature.

The kernel of the book is the extensive treatment of recursive methods for evaluation of aggregate claims distributions, and models allowing such methods. This is an area where the authors have given several valuable contributions during the latest fifteen years. At present no other text-book gives such an extensive treatment of this area. It must have been a difficult task to write such a book at the present stage as so many new results steadily appear. In the preface the authors write that the manuscript has been in progress since 1984. The theory on recursive methods has developed tremendously since then. Thus it is not surprising that some recent important contributions have not been discussed in the book; the reviewer would in particular like to mention DE PRIL'S (1989) extension to the general individual model of methods that he had earlier deduced for the life model, SCHRÖTER'S (1990) extension of the (a, b) class, and DE PRIL and DHAENE'S (1992) reduction to the half of the upper bound for the approximation error displayed in formula (6.5.11). It is

interesting to note that in Numerical illustration 6.5.2, the approximation error is very close to DE PRIL and DHAENE'S (1992) upper bound.

The book contains much interesting material. However, the presentation could have been better. Very little motivation is given. A reader who already knows the area, will find the book interesting as he would already be motivated, but the reviewer is more uncertain about how suitable the book would be as a text-book for students. The main topic of the book is evaluation of aggregate claims distributions, but very little is said about why it is interesting to study such distributions. In the introduction to the last chapter the authors say that the purpose of this and *subsequent chapters* is to explore some applications. This could indicate that the authors had actually intended to include more chapters on applications. However, this remark is made on p. 357, and many readers could then have already been lost. At the end of the chapters on basic risk models there are interesting sections on calculation of pure reinsurance premiums. However, the reviewer finds that the theory of these sections should have been indicated much earlier to show the reader that the theory actually has practical applications.

The lack of motivation is very clear in Part I on statistical preliminaries. There are different ways of presenting such a preliminary part. It could be made rather condensed, presuming that most of the readers already know the theory, and that the other readers could supplement with other texts. In that case the need for motivation is not that essential. However, in the present book, Part I is 125 pp., and the reader ought to be given some ideas of why he should read such an extensive text. In this direction much could be achieved with simple means. Why not in Section 2.9 on compound distributions mention the insurance risk model where it is assumed that the individual claim amounts are mutually independent and identically distributed and independent of the number of claims? Why not in Section 2.8 on mixed distributions mention the typical situation in experience rating where individual differences between policies are modeled by unknown random risk parameters?

In a large majority of the proofs and deductions in the book, the authors utilise Laplace transforms and probability generating functions. The reviewer will not deny that such transforms can often be useful tools, in particular to prove asymptotic results. However, one should not overdo using them. Let us assume that you are in Paris as a tourist. You have just been to the Notre Dame on the island Île de la Cité in the river Seine, and now you want to see the Sacré Cœur on the top of the hill Montmartre. It is very convenient to go there by the métro. So you enter the underground train at Cité, leave it at Montmartre, and after of a couple of minutes walk you stand in front of the Sacré Cœur. You entered the train on a flat island down in the river, and to your great amazement you discover that you are now on the top of a tall, steep hill! But how did you get there? Your métro ride did not give you any clue; you only noticed a couple of not too interesting platforms of stations that you passed. However, you got no clues that the landscape had completely changed. The reviewer finds that with most of the deductions based on transforms in the present book you are in the same situation. You get quickly from one place to

another, but you do not see much on the way. You have proved a result, so you know that the result holds, but you have not learned anything else. You have entered the métro at the station of your assumptions and left it at the station of your result, but you have not seen the gradual change of your probabilistic landscape on the way. Sometimes the authors do not even bother to leave the stations; sometimes they state both assumptions and results in terms of transforms; you start under ground at one station and are left under ground at another station. As a rule of a thumb, the reviewer believes that if you can prove a result both directly and with transforms with about equally long proofs, then the direct proof is preferable as you will have a better feeling of what is going on. Even a longer direct proof may be preferable to a shorter proof based on transforms.

The extensive use of transforms is closely related to the lack of motivation mentioned above. Relations are sometimes stated in terms of transforms instead of distributions and moments. In Section 3.6 on both processes there is a good example. In formula (3.6.12) the intensity function of the process is without proof expressed as a ratio between derivatives of the Laplace transform of the mixing distribution. Very few students would intuitively feel that such a result has to hold. On the other hand, one easily accepts that the claim intensity is the conditional mean of the random Poisson parameter given the number of claims that have already occurred. However, such an expression for the intensity function is not presented in the book.

Unfortunately there are a lot of careless formulations in the book, e.g. “The Pascal distribution can be seen to be a compound binomial random variable ...”; “... the variate of interest is $Y = X_1/X_2$ where X_1 and X_2 are known independent distributions”; “... it is assumed that X_1, X_2, \dots are mutually independent and identical random variables” (presumably identically distributed). At first glance it may seem pedantic to point out such slips of the pen as the intelligent reader would have no problem understanding what is meant. However, for a teacher struggling to get his students to formulate themselves decently, it is unfortunate when such formulations appear in their text-book. Likewise the reviewer dislikes expressions like $Y = X|X > 0$. Such a formulation may give the inexperienced reader the impression that the random variable Y is a function of the random variable X .

The methods presented in the book are illustrated with numerical examples, and various approximations are compared with the exact methods. Unfortunately these comparisons are as a rule performed on the probability density and/or the cumulative distribution, not on other quantities like stop loss premiums. Again a question of why one is actually interested in calculating the aggregate claims distribution. Are we interested in the distribution itself or some functional of it like the stop loss transform? Unfortunately the book does not give a clear answer.

Both for the recursive methods and for the asymptotic results it is assumed that the underlying model and its parameters are completely specified. Model fitting is discussed in Chapter 9. To be able to fit a model, one should preferably have a substantial amount of data. However, in practice it is also

often desirable to say something about the tail or fractile of the aggregate claims distribution in situations when very little data are available, perhaps only estimates of a couple of moments. In such cases more traditional approximation methods like e.g. the Normal Power approximation still seem to be relevant, in particular in solvency control systems. This could have been pointed out in the book. In the present reviewer's opinion it would be too simple to believe that because of modern computers and new exact methods, one could forget the simple approximations.

In spite of the deficiencies discussed above, the present reviewer would strongly recommend the book to people interested in the theory of aggregate claims distributions. However, as a text-book for a lecture course, it demands an effort from the lecturer to motivate and clarify. The reviewer looks forward to a second edition of the book for two reasons. Firstly, in the area of aggregate claims distributions so much is happening that it is likely that it will soon be desirable to update it. Secondly, the presentation could be much improved with simple means.

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