



ARTICLE

# When Diversity in Voting Trumps Ability

Jonas Karge 

Technische Universität Dresden, Dresden, Germany

Email: [jonas.karge@tu-dresden.de](mailto:jonas.karge@tu-dresden.de)

(Received 28 October 2024; revised 28 February 2025; accepted 21 April 2025)

## Abstract

Would you prioritize assembling a group of more competent individuals or a more diverse group? According to the well-known Diversity-Trumps-Ability Theorem (DTA), when two groups of similar size are compared, the more diverse group generally outperforms the more competent group. Despite considerable criticism regarding the DTA's mathematical rigor, it has sparked substantial interdisciplinary discussion. While most research on the DTA focuses on its implications within specific fields or on empirical simulations to test its validity, we aim to establish precise conditions under which diversity indeed surpasses ability in a particular decision-making context: voting. To this end, we integrate the DTA into a voting model inspired by the *Condorcet Jury Theorem* (CJT) and model *diversity* based on the classic dependency model in the CJT literature, namely the influence of an *opinion leader*.

**Keywords:** diversity-trumps-ability; correlated voting; condorcet jury theorem; opinion leader; epistemic democracy

## 1 Introduction

Imagine you are faced with a challenging decision and you have the option of assembling a large group of people whose goal is to identify the best possible choice with a high degree of accuracy. Would you prefer to assemble a group of more competent individuals or one that is more diverse? According to the well-known Diversity-Trumps-Ability Theorem (DTA), when comparing two groups of similar size, the more diverse group will generally outperform the more competent group.

Since its publication nearly two decades ago (Hong and Page 2004), the DTA has received considerable attention, though it has also faced substantial criticism regarding its mathematical rigor (Thompson 2014). Nevertheless, the theorem has sparked widespread discussion in a number of disciplines, including the social sciences, humanities, and computer science.

Most notably, Hélène Landemore (2012) employs the DTA in defense of democracy. Interpreting the theorem to suggest that a diverse group of less competent individuals can generally outperform a group of highly skilled experts, she argues that democratic processes are likely to produce superior decisions compared to those made by a small group of experts (Christiano and Bajaj 2024). In doing so, Landemore aligns the DTA

with similar arguments within the realm of epistemic democracy, which studies and defends democracy based on its ability to effectively track the truth (Sakai 2020). This is akin to the Condorcet Jury Theorem (CJT) (Genta 2024), which, under fairly restrictive assumptions, posits that as the number of voters increases, the probability of the group identifying the correct option converges toward 1.

Discussions of DTA and diversity in decision-making typically focus on three different epistemic frameworks: search problems, deliberation, and voting (Niesen et al. 2024). In this paper, we focus specifically on voting, understood as an election mechanism involving a set of voters, a set of options, and the aggregation of the voters' selected choices (Niesen et al. 2024).

While most works discussing the DTA tend to either (i) explore its implications within their respective fields, (ii) highlight its formal limitations, or (iii) perform experimental simulations to validate or challenge its claims, our contribution focuses on offering mathematically rigorous bounds that specify the conditions under which diversity truly exceeds ability.

More specifically, we first embed the DTA problem in a voting model that operates within the framework of the CJT. This serves three purposes: (1) it forms a bridge between two of the main arguments in the field of epistemic democracy, (2) it allows to clearly define *diversity* in a voting setting, associating the increasing influence of a so-called opinion leader on the electorate with a decrease in diversity, (3) it provides probabilistic guarantees for the identification of the correct option, which depend on this notion of diversity.

Second, within this model we derive a precise threshold for when diversity truly exceeds ability, inspired by techniques recently introduced by Karge (2024), where similar bounds are derived for a scenario involving the aggregation of imprecise probabilistic beliefs.

Finally, we devote a section to positioning our results within the philosophical framework of *epistemic democracy*. In this section, we (1) revisit the long-standing relationship between epistemic democracy and the CJT, (2) provide an overview of key generalizations of the CJT, and (3) examine its connection to the DTA. In each case, we highlight how our results contribute to the ongoing discussion.

## 2 Preliminaries

In this section, we introduce the notation and formal foundations of this paper. To ensure accessibility, we present the preliminaries to our framework in natural language wherever possible, focusing only on the parameters essential to the main discussion. For a formal treatment of the underlying framework, we refer the reader to section 7.1 in the appendix.

We begin by introducing the CJT, a central result from the epistemic view of voting. This is done in three parts: First, we present the original CJT result, and second, we discuss a recently proposed generalization that simultaneously weakens all of the original assumptions. Finally, we define *diversity* and *ability* within this framework.

*The CJT.* As a fundamental theorem in voting theory, the CJT provides probabilistic guarantees for determining the presumed correct alternative among a set of alternatives under certain conditions. Originally, the CJT assumed that agents are equally competent (*homogeneity*), that they are more likely to vote for the correct alternative than for a competitor (*reliability*), that they do not influence each other or be influenced by an external factor in the voting process (*independence*), and that they choose exactly one

(completeness) out of two alternatives (dichotomy) in majority voting. Thus the classic CJT (Condorcet 1785) states the following.

**Theorem 1. (Condorcet 1785).** *For odd-numbered, homogeneous groups of independent and reliable agents in a dichotomic voting setting, the probability that majority voting identifies the correct alternative*

- (1) *increases monotonically with the number of agents and*
- (2) *converges to 1 as the number of agents goes to infinity.*

Since the real world cannot guarantee these ideal conditions, it is essential in CJT research to seek generalizations that maintain the asymptotic part under weakened assumptions, while the monotonic increase fails once heterogeneously competent agents are allowed (Owen et al. 1989).

Recently, a novel generalization of CJT has been proposed that simultaneously weakens all of the assumptions underlying the original CJT (Karge et al. 2024). In this generalization, agents are allowed to vote for an arbitrary finite number of alternatives, while allowing for heterogeneous competence levels and a degree of correlation among voters modeled by an *opinion leader* (OL), the classic dependency model in the CJT literature (Boland et al. 1989).

The opinion leader (OL) serves as an abstract external influence in the voting process, without actively participating in it. Instead, the OL approves or disapproves presented alternatives based on her own competence  $\hat{p}$ , which represents the probability of approving the correct alternative. Subsequently, her choice influences the votes of the agents: each agent votes according to the preference of the OL rather than her own “inner voice” with a certain probability  $\pi$ . In the standard OL model, this probability  $\pi$  is identical for all agents, i.e., it is uniformly distributed.

## 2.1 Voting probabilistic framework

We now introduce the underlying voting and probabilistic framework.

Let  $\mathcal{W} = \{\omega_1, \dots, \omega_m\}$  denote a finite set of  $m$  alternatives. For example, in a political context, each  $\omega_j$  could represent a policy that can be voted on. Moreover, we define a finite set  $\mathcal{N} = \{a_1, \dots, a_n\}$  consisting of  $n$  agents. The underlying voting method in our model is *approval voting*, where each agent  $a_i$  can vote for any number of alternatives from the set  $\mathcal{W}$ . In a given election, the alternative that receives the most votes wins the approval vote.

**Example 1.** Suppose we have  $m = 3$  alternatives, i.e.,  $\mathcal{W} = \{\omega_1, \omega_2, \omega_3\}$  and that we have  $n = 2$  agents, i.e.,  $\mathcal{N} = \{a_1, a_2\}$ . Suppose  $a_1$  votes for the alternatives  $\omega_1, \omega_2$ , and  $a_2$  votes only for  $\omega_1$ . Then  $\omega_1$  has a strictly higher score than either of the competing alternatives and wins the approval vote.

Within the set  $\mathcal{W}$ , we assume that exactly one alternative is correct, denoted as  $\omega_*$ . Given this fixed alternative, the voting scenario unfolds as a random process that determines both the opinion leader’s (OL) approved choices and the agents’ votes. First, the OL endorses a subset of alternatives without directly participating in the voting process. The probability that the OL includes the correct alternative in this subset is given by  $\hat{p}$ , representing her competency. Second, each agent independently selects a subset of alternatives before casting their vote, with the probability of correctly identifying  $\omega_*$  determined by the parameter  $p_i^{\omega_*}$ . Finally, a fraction of the agents follows

the OL's endorsements, meaning their final votes align with the alternatives approved by the OL. The likelihood of an agent adhering to the OL's choices is captured by the parameter  $\pi$ , which is assumed to be uniform across all agents. Once the subset of OL-following agents is established, the final voting occurs: each agent either votes according to their private selection or adopts the OL's approved alternatives.

**Example 2.** We provide some example values for each parameter: Suppose that the OL has a rather high probability of approving the correct alternative  $\hat{p} = 0.6$  and that agents are relatively inclined to follow the OL  $\pi = 0.4$ . Suppose also that  $a_1$  has a high probability of voting for the first two alternatives when uninfluenced, but never votes for the third:  $p_1^{\omega_1} = 0.8, p_1^{\omega_2} = 0.75, p_1^{\omega_3} = 0$ .

To derive the generalized CJT result, two additional assumptions are required. The first, known as  $\Delta p$ -group reliability, is defined in terms of the average probability that an agent privately approves the correct alternative, denoted as  $\bar{p}^{\omega_*}$ , and the corresponding probability for any incorrect alternative, denoted as  $\bar{p}^{\omega_{\dagger}}$ . The  $\Delta p$ -group reliability assumption states that  $\bar{p}^{\omega_*}$  must exceed  $\bar{p}^{\omega_{\dagger}}$  by at least a margin of  $\Delta p$  for any incorrect alternative.

The second assumption, referred to as *private agent approval independence*, requires that each agent's private selection of an alternative occurs independently, ensuring that agents do not influence one another in this process. However, this does not imply independence in the final election outcome; agents' choices may still be correlated through the influence of the opinion leader.

Based on these assumptions, a bound on the worst-case minimum probability of success,  $P_{\min}$ , for identifying the correct alternative through approval voting can be established (Karge et al. 2024). Throughout this paper, we interpret an increase in  $\Delta p$  as a rise in average competence, while a higher  $\pi$  reflects greater correlation among agents.

**Theorem 2.** Consider an approval voting setting with  $m > 1$  alternatives, satisfying private agent approval independence and  $\Delta p$ -group reliability for some  $\Delta p \in (0, 1]$ , influenced by an opinion leader with  $\pi \in \left[0, \frac{\Delta p}{\Delta p + 1}\right)$  and  $\hat{p} \in [0, 1]$ . Then it is guaranteed that the success probability of the approval voting process is at least

$$1 - (m - 1)(\hat{p}e^{-\frac{1}{2}n\Delta p^2(1-\pi)^2} + (1 - \hat{p})e^{-\frac{1}{2}n(\Delta p(1-\pi)-\pi)^2}).$$

Observe that, in order to guarantee that the minimal success probability converges to one as the number of agents goes to infinity, a bound on the probability of following the OL is required, i.e.,  $\pi \in \left[0, \frac{\Delta p}{\Delta p + 1}\right)$ . This theorem will play a crucial role in determining the threshold for when DTA later in the paper. To see how a combination of different parameters yields a minimum probability of success, called  $P_{\min}$ , consider the following example:

**Example 3.** Consider the following parameters: Suppose we have a relatively large number of alternatives,  $m = 20$ , and a set of agents who, on average, are 40% more likely to vote for the correct alternative than for any other,  $\Delta p = 0.4$ . Suppose also that we have a very competent OL who approves the correct alternative with a probability of 80%, i.e.,  $\hat{p} = 0.8$ . Finally, suppose we have 100 agents, where each agent has a 10% chance of following the OL:  $\pi = 0.1$ , and  $n = 100$ . This gives a minimum probability of success for the correct alternative to win the approval vote of  $P_{\min} = 0.85$ .

Naturally, the minimum probability of success can be adjusted by modifying these parameters. Intuitively,  $P_{\min}$  increases with a larger number of agents, as this amplifies the *wisdom of the crowds* effect, and with higher competency levels among both the agents and the opinion leader. Additionally, reducing the number of alternatives improves the likelihood of identifying the correct one, as fewer options generally make the correct choice easier to discern. Finally, the success probability increases when the likelihood of agents following the opinion leader decreases. This is because our bound represents a worst-case scenario where, in the worst case, the opinion leader selects only incorrect alternatives, causing the electorate to be misled toward these wrong choices.

Having defined the underlying formal framework, we now define the two concepts central to this work, *diversity* and *ability*.

When it comes to diversity, there are a variety of ways to find an appropriate model. Within voting, a common approach is to model diversity as a lack of positive correlation that affects all voters in the same way, as in the OL model (Niesen et al. 2024). *Ability*, on the other hand, is more straightforward. In a truth-tracking voting setting, we let *ability* refer to the probability of an agent identifying the correct alternative, i.e., with the agent's uninfluenced competence.

**Definition 1. (Diverse Groups).** Consider two groups of equal size, i.e., both groups compromise the same number of agents. Let  $\pi_1$  and  $\pi_2$  denote the uniformly distributed probabilities of following the OL for group 1 and group 2, respectively. We say that group 1 is more diverse than group 2 if  $\pi_1 < \pi_2$ , i.e., if the first group has a lower positive correlation than the second.

In the subsequent step, a group of agents with higher levels of competence or *expertise* is defined.

**Definition 2. (Competent Groups).** Given two groups of equal size. Let  $\Delta p_1$  and  $\Delta p_2$  denote the margins by which each group is, on average, more likely to vote for the correct alternative than a competitor. We say that group 1 is more competent than group 2 if  $\Delta p_1 > \Delta p_2$ .

With these definitions in hand, we proceed to derive a threshold for when diversity exceeds ability or competence.

### 3 A diversity threshold

In this section, our goal is to delineate an exact threshold for when diversity truly exceeds ability within the Condorcet voting model.

Recall the original problem statement: You want to assemble a group of experts that is either more diverse or more competent. Suppose your group already consists of a few agents, and you want to increase the diversity, i.e., reduce the uniformly distributed probability,  $\pi$ , of following the OL. Since it may not seem reasonable on a conceptual level to claim that adding another agent will reduce this uniform value across the group, we resort to a recent extension (Karge 2024) of the CJT model of Theorem 2. This extension generalizes the bound on the minimum probability of success for a uniform  $\pi$ -value to allow for an average probability of following the OL and also allows for a meaningful discussion of the potential increase or decrease in correlation within a group of agents. In the following, we first introduce this generalization, following the exposition of Karge (2024), and then use this bound to derive the desired threshold.

### 3.1 Extending the CJT to average opinion leader influence

As a first step towards this generalization, the original OL framework is extended to allow for a finite number of different  $\pi$ -values ( $\pi_1, \dots, \pi_k, \dots, \pi_s$ ), where  $1 \leq s \leq n$ . This means that at most each agent can have its own  $\pi$  value. In a next step, the whole electorate is divided into different subgroups, where each subgroup comprises exactly those agents that have the same  $\pi$ -value.

Each subgroup is denoted by  $\mathcal{G}_{\pi_k}$ , where  $\pi_k$  represents the exact  $\pi$ -value of that subgroup, such that

$$\mathcal{G}_{\pi_1} \cup \dots \mathcal{G}_{\pi_k} \dots \cup \mathcal{G}_{\pi_s} = \mathcal{N} = \{a_1, \dots, a_n\}.$$

This is illustrated in the following example.

**Example 4.** Let's consider ten agents ( $a_1, \dots, a_{10}$ ) and four different  $\pi$ -values ( $\pi_1 = 0.05, \pi_2 = 0.1, \pi_3 = 0.15, \pi_4 = 0.9$ ). One possible distribution of agents to subgroups could be:

$$\mathcal{G}_{\pi_1} = \{a_1, a_2, a_3\}, \mathcal{G}_{\pi_2} = \{a_4, a_5\}, \mathcal{G}_{\pi_3} = \{a_6, a_7, a_8, a_9\}, \mathcal{G}_{\pi_4} = \{a_{10}\}.$$

In this case, agents 1-3 follow the OL with probability 0.05, agents 4 and 5 with probability 0.1, and so on.

For each subgroup  $\mathcal{G}_{\pi_k}$ , the number of agents in that subgroup is referred to as  $|\mathcal{G}_{\pi_k}|$ . From this, it is straightforward to define the average  $\pi$ -value,  $\bar{\pi}$  as follows.

$$\bar{\pi} = \sum_{k=1}^s \frac{|\mathcal{G}_{\pi_k}|}{n} \pi_k$$

That is, the average probability of following the opinion leader across all subgroups is the sum of all  $\pi$  values, one for each subgroup, weighted by the number of agents in that subgroup.

Finally, this leads to a bound on the minimal success probability that accounts for average OL influence (Karge 2024).

**Theorem 3.** Consider an approval voting setting as described in Theorem 2, but allowing for different  $\pi_k$ -values as defined above. Then it is guaranteed that the success probability of the approval voting process is at least

$$1 - (m - 1) \left( \hat{p} \left( e^{-\frac{2}{4n} (\Delta p (\sum_{k=1}^s |\mathcal{G}_{\pi_k}| (1 - \pi_k)))^2} \right) + (1 - \hat{p}) e^{-\frac{2}{4n} (\sum_{k=1}^s |\mathcal{G}_{\pi_k}| (\Delta p (1 - \pi_k) - \pi_k))^2} \right).$$

Similar to Karge (2024), this bound can be shown to be equivalent in the worst case to the bound in Theorem 2. That is, if the uniformly distributed  $\pi$ -value is equal to the average  $\bar{\pi}$ , then the minimum probability of success is the same, denoted by  $P_{\min, \pi} = P_{\min, \bar{\pi}}$ . This result then allows to use the bound of Theorem 2 for actual calculations, as it is somewhat easier to work with, but also allows to discuss an increase or decrease of the correlation on a conceptual level when using Theorem 2.

**Theorem 4.** If  $\pi = \bar{\pi}$ , then  $P_{\min, \pi} = P_{\min, \bar{\pi}}$ .

**Proof.** The proof can be found in the appendix, section 1. The implications of this theorem are illustrated in the following example.

**Example 5.** Consider again the group of ten agents and four subgroups from the previous example. The average probability of following the OL is  $\bar{\pi} = 0.185$ . Suppose we

have a very competent OL and a highly competent group of agents voting on three alternatives:  $\hat{p} = 0.9, \Delta p = 0.7, m = 3$ . Due to Theorem 4, we may now invoke Theorem 2 to compute  $P_{min}$ , the minimal success probability. This yields that worst-case minimum probability of identifying the correct alternative according to Theorem 2 is then 0.55.

### 3.2 A diversity threshold

Next, we want to determine the threshold at which increased diversity increases the group's capabilities relative to a more competent group. The derivation for this threshold can be found in the Appendix in section 7.3. Intuitively, to determine this threshold we introduce the parameter  $\varepsilon$ , which represents the amount by which the average correlation is smaller in the more diverse group. Similarly,  $\lambda$  represents the increase in competency in the less diverse group. The more diverse group outperforms the more competent one as soon as the minimal success probability,  $P_{min}$  is greater. This is the case when the increase in  $P_{min}$  due to the increase in diversity exceeds the increase in  $P_{min}$  due to the increase in the competence of the agents. This is the case when :

$$\varepsilon \geq \frac{\lambda - \pi\lambda}{\Delta p + 1} \quad (1)$$

This directly yields the following corollary which specifies when exactly DTA in our model.

**Corollary 1.** *In an approval voting setting as described in Theorem 2, and provided diversity and ability are defined as in Definitions 1 and 2, DTA if inequality 1 is true.*

We illustrate this threshold for an example setting in Figure 1, where we have 100 fairly competent agents voting on five alternatives with a moderately influential OL, where the first group is slightly more competent and where we vary the increase in diversity for the second group, i.e.,  $n = 100, m = 5, \Delta p = 0.5, \pi = 0.2, \lambda = 0.1$ .

From the plot, we can see that in this particular setting, even a small increase in diversity (decrease in  $\pi$ ) causes the more diverse group to outperform the more competent group.

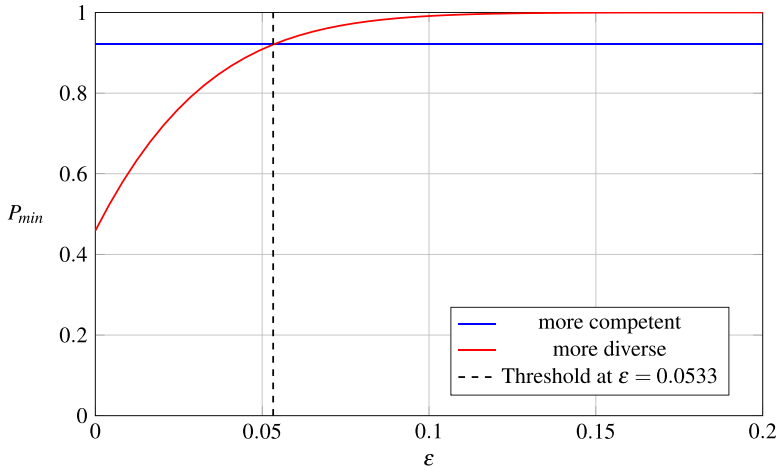
We further illustrate in Figure 2 for the same scenario, i.e.,  $n = 100, m = 5, \Delta p = 0.5, \pi = 0.2$ , a simultaneous increase in competence and diversity. In this plot, a value greater than 0 indicates that the more competent group outperforms the more diverse group (e.g., at  $\lambda = 0.2, \varepsilon = 0$ ), and conversely, a value less than 0 indicates that the more diverse group outperforms the more competent group (e.g., at  $\lambda = 0.1, \varepsilon = 0.2$ ).

Now that we have established a threshold for when diversity truly surpasses ability within the OL model, we turn to the next section to examine how this result fits within the broader research program of *epistemic democracy* and what contributions it offers to this field.

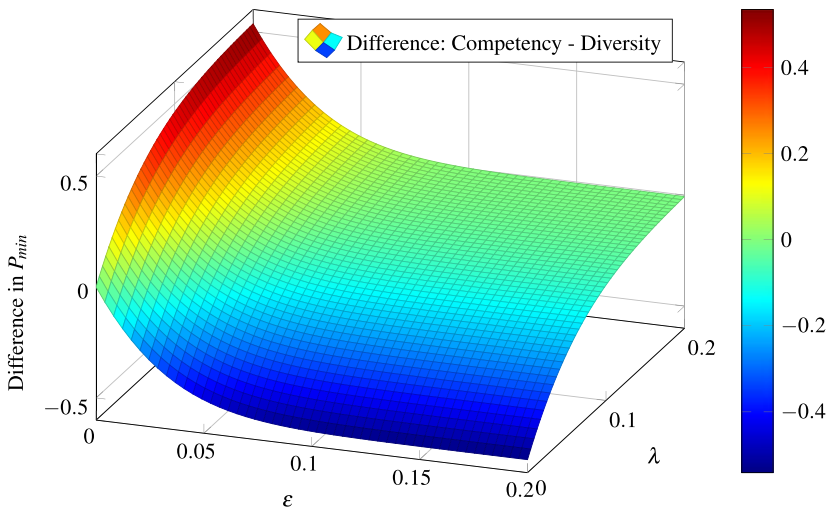
## 4 The bridge to epistemic democracy

In this section, we situate our results within the broader context of epistemic democracy. We begin with a brief overview of epistemic democracy and its long-standing connection to the CJT. Next, we summarize the CJT and its key generalizations. Finally, we revisit the DTA theorem and its implications for epistemic democracy.





**Figure 1.** Plot of success probability of a more competent group (blue) and a more diverse group (red) for varying  $\varepsilon$ .



**Figure 2.** Difference in  $P_{min}$  between the two functions as a function of  $\varepsilon$  and  $\lambda$ .

To begin, what is *epistemic democracy*? Broadly speaking, it is a normative theory asserting that democratic institutions or procedures *should* track the truth to be considered legitimate (Dietrich and Spiekermann 2021). In this context, *truth tracking* refers to epistemic arguments that justify democratic institutions by assuming the existence of some objective truth about political matters – one that can be approximated through democratic processes (Holst and Molander 2019).

#### 4.1 Epistemic democracy and the CJT

As outlined in the introduction, a key result in the truth-tracking literature is the CJT, which provides probabilistic guarantees that, under specific assumptions, an electorate



can correctly identify the best alternative through voting. This theorem is particularly relevant to epistemic democracy, as democratic institutions typically involve large groups of voters, thereby leveraging the *wisdom of the crowds* effect that underlies the CJT.

Analyzing democratic procedures from a truth-tracking perspective contrasts with their typical treatment in social choice theory, where voting rules are designed not to identify a correct alternative but to ensure equal treatment of all voters (Everaere et al. 2010; Dietrich and Spiekermann 2021). Nonetheless, the epistemic democracy framework has a long-standing tradition in philosophical discourse.

According to Schwartzberg (2015), references to the *wisdom of the crowds* argument can be traced back to Aristotle, who stated:

*There is this to be said for the many: each of them by himself may not be of a good quality; but when they all come together it is possible that they may surpass – collectively and as a body, although not individually – the quality of the few best, in much the same way that feasts to which many contribute may excel those provided at one person's expense* (Aristotle, Pol. 3.1281b).

Although generally skeptical of democratic ideals, Aristotle acknowledges a core principle of epistemic democracy: the epistemic advantage of collective deliberation. He argues that there is an epistemic benefit when notables and the populace deliberate together rather than when either deliberates alone (Bohman 2006). Unlike the CJT, however, Aristotle does not specify a precise mechanism – such as an opinion aggregation procedure – that could harness the epistemic benefits of the many (Bohman 2006). A first step in this direction can be found in Rousseau's Social Contract (1762, see Gourevitch (1997)), where he argues that through voting, people express their individual opinions, and the tally of these votes constitutes a declaration of the general will (Schwartzberg 2015). Rousseau thus not only identifies a specific aggregation procedure – voting – but also specifies the truth to be tracked: the general will.

As noted by Schwartzberg (2015), it was Cohen's (1986) reinterpretation of Rousseau through the lens of the CJT that sparked renewed interest in epistemic democracy. This led to a substantial body of literature on the CJT and epistemic democracy, culminating in the observation that the CJT has been referred to as the "jewel in the crown of epistemic democrats" (List and Goodin 2001), as highlighted by Schwartzberg (2015). While the CJT's importance to epistemic democracy is well established, it has also faced criticism (Schwartzberg 2015). Estlund (2008) argues that 'if democracy has any epistemic value, it is partly due to the sharing of diverse perspectives'. Consequently, the challenge of incorporating this diversity becomes paramount when employing the CJT to support democratic principles.

On the other hand, some scholars caution that an exclusive focus on diversity risks overlooking another key factor in democratic epistemic benefits: the role of experts (Holst and Molander 2019). Specifically, it has been argued that insufficient attention has been given to how expertise can be leveraged to enhance the epistemic quality of democratic decision-making (Holst and Molander 2019).

This leads to the first philosophical contribution of our results: we extend the long-standing discussion of the CJT in epistemic democracy by embedding formally precise definitions of diversity and expertise within the CJT framework. Specifically, we define a group as diverse if it has a low  $\pi$ -value, indicating a low probability of following an opinion leader. Similarly, we characterize a group as highly expert if it has a large  $\Delta p$ -value, representing a high average probability of selecting the correct alternative.

Moreover, by introducing a diversity threshold, we formalize the interplay between diversity and expertise, allowing us to determine when one factor outweighs the other.

#### 4.2 Generalizations of the CJT

To justify democratic procedures using the CJT, it is essential to develop generalizations that extend its core principles under more realistic conditions (Dietrich and Spiekermann 2021). Recall that the original CJT relies on several key assumptions: that all agents have equal competence (homogeneity), that they are more likely to select the correct alternative than an incorrect one (reliability), that their votes are cast independently, free from external influence (independence), and that they choose exactly one option (completeness) from a binary set of alternatives (dichotomy) in a majority vote.

In their seminal paper, Goodin and List (2001) extend the original CJT to plurality voting, allowing agents to select a single alternative from a finite set – thus relaxing the dichotomy assumption. This contribution continues a long-standing tradition of adapting the CJT to more realistic settings. Subsequently, this result was further generalized by removing the completeness assumption, permitting agents to vote for multiple alternatives under approval voting (Everaere et al. 2010).

Taking a different approach, Owen, Grofman, and Feld (1989) demonstrated that the CJT holds even under heterogeneous competence levels, provided that independence, completeness, and dichotomy are maintained. In this setting, the only requirement is that, on average, agents are more likely to select the correct alternative than any other.

When allowing for interdependent voting choices, various models of dependence can be considered. A common approach introduces the concept of an opinion leader (OL), representing an external influence – such as extreme environmental conditions in sensor fusion scenarios or human actors like lobbyists and pundits in political debates.

Previous work on generalizing the CJT under the influence of an opinion leader began with Boland, Proschan, and Tong (1989), who assumed homogeneous competence levels across agents and an equally competent OL in a dichotomous majority voting setting. Shortly after, Berg (1994) extended this framework to dichotomous weighted voting rules. Goodin and Spiekermann (2012) further generalized the dichotomous voting setting by allowing the OL's competence to differ from that of the agents. Additionally, they identified a specific threshold at which the asymptotic behavior of the CJT fails.

As outlined in the preliminaries, a recent generalization of the CJT successfully relaxes all its original assumptions simultaneously (Karge et al. 2024). This framework – underpinning the diversity threshold introduced in our study – permits agents to vote for any finite number of alternatives while accounting for heterogeneous competence levels and correlated voting behavior, modeled through an opinion leader. To the best of our knowledge, this represents the most general CJT result to date.

In essence, the CJT remains a foundational mechanism for justifying epistemic democracy (Schwartzberg 2015), as it formalizes the *wisdom of the crowds* effect. The ongoing development of CJT research aims to maintain this core principle under increasingly realistic conditions that reflect actual electorates.

This leads to a second key contribution of our diversity results to the epistemic democracy literature. By being embedded within the most general CJT framework, it makes the least restrictive assumptions about the electorate. Furthermore, by providing a precise bound for when diversity outweighs expertise, it advances CJT research in its pursuit of identifying the exact conditions under which the wisdom of the crowds effect is preserved.

### 4.3 Epistemic democracy and DTA

Beyond the CJT, a second central mechanism in epistemic democracy – widely regarded as highly promising – is the DTA theorem (Schwartzberg 2015). However, a key criticism of the DTA is that it fails to account for a fundamental epistemic challenge: it assumes that agents form their opinions independently, without external influences such as opinion leaders (Schwartzberg 2015). In addition, it has often been recognized that democracies can benefit epistemically from diversity among their citizens, but that it is challenging why this is the case (Bohman 2006).

This brings us to our final contribution to the research program of *epistemic democracy*. We establish a formal connection between the two primary mechanisms that underpin the wisdom of the crowds effect – the CJT and the DTA – by embedding the DTA within the CJT framework. Moreover, by explicitly modeling the influence of an opinion leader, we provide a formal explanation for the epistemic benefits of diversity: specifically, its role in reducing systematic bias introduced by external influences.

## 5 Summarizing the argument and addressing key concerns

In this section, we summarize our main argument and address potential concerns. We began by noting that the original DTA theorem plays a significant role across various disciplines but has also faced substantial criticism regarding its mathematical rigor (Thompson 2014). Additionally, discussions of the DTA in decision-making typically revolve around three key epistemic frameworks: search problems, deliberation, and voting (Niesen et al. 2024). Based on these observations, we can summarize our core argument as follows:

1. While the DTA is widely recognized for its significance, its mathematical foundations remain insufficiently rigorous. Thus, grounding it in a more robust mathematical framework is a worthwhile endeavor.
2. To ensure mathematical precision, we focus on one of the three primary epistemic frameworks – namely, voting.
3. Given that the CJT is the standard epistemic model for voting, we embed the DTA within the CJT framework, thereby clarifying the relationship between diversity and ability.

Building on these considerations, and our interpretation of *diversity* and *ability* within the CJT framework, we derived a mathematically precise bound that determines when diversity truly surpasses ability.

**Potential Concerns.** The structure of our argument is not without potential objections. Most notably, the CJT itself is subject to criticism. The original CJT relies on highly restrictive assumptions about the voting process – assumptions that are rarely met in practice. In particular, it presumes that voters are always more likely to choose the better of two alternatives and that their decisions are statistically independent. However, in reality, electorates often include incompetent or even malicious voters, and voting behavior is frequently correlated.

Given these limitations, one might raise the following concern: What justifies embedding an already debated theorem, the DTA, within another controversial framework, the CJT?

The response we have put forward in this paper is as follows: First, if we aim to make mathematically rigorous claims about the relationship between diversity and ability in voting, we must adopt a formal model of the voting process. This necessarily involves

probabilistic assumptions about voters, which, like any model, require some level of abstraction from reality. Second, rather than embedding the DTA into the original, highly restrictive CJT, we incorporate it into a generalized version (Karge et al. 2024), which, to our knowledge, is the least restrictive CJT framework available. Notably, this generalization accounts for unreliable agents and allows for correlations among voters, using the classical opinion leader model as established in the CJT literature.

A second line of criticism against the CJT and similar models of large elections has been raised by Barnett (2020), who challenges the common assumption that large elections can be accurately modeled using a binomial framework. His argument suggests that this approach fails to capture important empirical patterns in election outcomes. To address this concern, we first outline the core of his critique.

**Barnett's Argument and the Binomial Model.** In probabilistic models of large elections, it is common to assume a binomial structure, where each voter's choice can be seen as an independent coin flip: each agent votes for a particular alternative with some probability, and does not vote for it otherwise.

Barnett (2020) challenges this approach by arguing that it fails to explain a key empirical observation – large elections are frequently very close. If the binomial model were an accurate representation of voter behavior, it should naturally produce close elections under reasonable assumptions. However, Barnett constructs a simple counterexample demonstrating that when two candidates compete and one has even a slight advantage in voter preference, the probability of the less favored candidate winning rapidly approaches zero as the electorate grows. This suggests that, under the standard binomial model, competitive elections should be exceedingly rare, contradicting real-world patterns where close races are the norm. To illustrate his argument, Barnett (2020) presents a small-scale counterexample:

Consider an election with two candidates, Donald and Daisy. Suppose each voter has a 50.5% probability of voting for Daisy, giving her only a slight advantage over Donald. Intuitively, we would expect the election to be relatively close, as the margin favoring Daisy is minimal. However, under the binomial model, the probability that Donald wins or ties the election is approximately 0.00000000000078 – less than one in a trillion (Barnett 2020). This stark contrast between the model's prediction and our intuitive expectation highlights a potential flaw in using the binomial model to represent real-world elections.

Let us assume that Barnett's counterexample is valid and indeed challenges the binomial model as a framework for large elections. Even if we grant this, we argue that the underlying intuition behind his critique is not only unproblematic for our framework but actually reinforces it. While our model also represents an agent's final vote as a probabilistic event – determined by a mixture of their private approval and the opinion leader's choice – it crucially differs from the classical binomial model in one key aspect: the votes are not independent but correlated through the opinion leader.

By explicitly incorporating the opinion leader as a source of correlation, our model naturally accounts for the issue raised by Barnett. In fact, we can reformulate his counterexample within our framework as follows:

**Example 6.** Suppose we have two alternatives,  $m = 2$ , and a total of  $n = 500,000$  agents. Each agent has a 50.5% probability of voting for one alternative over the other, which corresponds to a  $\Delta p$ -value of 0.01. Since there are only two alternatives, the probability of voting for the less favorable one is  $1 - 0.505 = 0.495$ , and  $\Delta p$  represents the difference between these probabilities. Now, assume that the opinion leader (OL) is as competent as the agents, meaning  $\hat{p} = 0.505$ , and that she exerts only minimal influence over the electorate, with each agent following the OL with a probability of just 1%. Applying

Theorem 2, we can compute the minimal success probability for Daisy to win this election, which results in approximately 0.506, or just slightly above 50%.

This example demonstrates that by accounting for correlation among agents, our model not only avoids the flaw of the binomial model but also aligns with our intuition that the election should indeed be close.

## 6 Conclusion

In this paper, we brought together two foundational arguments in epistemic democracy: the DTA theorem and the CJT. By embedding the former within the framework of the latter, we provided a formal characterization of *diversity* and *ability* in voting contexts. This allowed us to derive a precise threshold for when diversity surpasses ability. Beyond establishing this result, we situated our findings within the broader discourse on *epistemic democracy*, highlighting its implications for the field. Finally, we addressed key concerns regarding our model.

We sincerely thank the two anonymous reviewers for their valuable feedback. In particular, we are grateful to the first reviewer for highlighting potential concerns regarding the structure of our argument and for drawing our attention to Barnett's work (2020). We also appreciate the second reviewer's insightful suggestions on improving the presentation of key sections and their encouragement to provide a stronger philosophical motivation for our core results. This work is partly supported by BMBF in DAAD project 57616814 (SECAI).

## References

- Aristotle** (1995). *The Politics*, transl. E Barker. Oxford, England: Oxford University Press.
- Barnett Z.** (2020). Why you should vote to change the outcome. *Philosophy & Public Affairs* 48(4), 422–446.
- Berg S.** (1994). Evaluation of some weighted majority decision rules under dependent voting. *Mathematical Social Sciences* 28(2), 71–83.
- Bohman J.** (2006). Deliberative democracy and the epistemic benefits of diversity. *Episteme* 3(3), 175–191.
- Boland P.J., Proschan F. and Tong Y.L.** (1989) Modelling dependence in simple and indirect majority systems. *Journal of Applied Probability* 26(1), 81–88.
- Christiano T. and Bajaj S.** (2024) Democracy. In Edward N. Zalta and Uri Nodelman, editors, *The Stanford Encyclopedia of Philosophy. Metaphysics Research Lab*, Stanford University, Summer.
- Cohen J.** (1986) An epistemic conception of democracy. *Ethics* 97(1), 26–38.
- Condorcet N.C.** (1785) *Essai sur l'application de l'analyse à la probabilité des décisions rendues à la pluralité des voix*. Imprimerie Royale, Paris.
- Dietrich F. and Spiekermann K.** (2021) Social epistemology. In Markus Knauff, Wolfgang Spohn, editors, *The Handbook of Rationality*, pp. 579–590. Cambridge, MA: The MIT Press.
- Estlund D.** (2008). Introduction: Epistemic approaches to democracy. *Episteme* 5(1), 1–4.
- Everaere P., Konieczny S. and Marquis P.** (2010) The epistemic view of belief merging: Can we track the truth? In Helder Coelho, Rudi Studer and Michael J. Wooldridge, editors, *Proceedings of the 19th European Conference on Artificial Intelligence (ECAI 2010)*, volume 215 of FAIA, pages 621–626. IOS Press.
- Genta B.S.** (2024). Formal models and justifications of democracy. *Synthese* 204(5), 138.
- Gourevitch V.** (1997). *The Social Contract and other later political writings*. Cambridge University Press.
- Holst C. and Molander A.** (2019). Epistemic democracy and the role of experts. *Contemporary Political Theory* 18(4), 541–561.
- Hong L. and Page S.E.** (2004) Groups of diverse problem solvers can outperform groups of high-ability problem solvers. *Proceedings of the National Academy of Sciences* 101 (46), 16385–16389.

- Karge J.** (2024). Taming dilation in imprecise pooling. In Ryuta Arisaka, Victor Sanchez-Anguix, Sebastian Stein, Reyhan Aydoğan, Leon van der Torre and Takayuki Ito, editors *International Conference on Principles and Practice of Multi-Agent Systems*, pp. 428–443. Cham: Springer Nature Switzerland.
- Karge J., Burkhardt J.-M., Rudolph S. and Rusovac D.** (2024). To lead or to be led: A generalized condorcet jury theorem under dependence. In *AAMAS '24: Proceedings of the 2024 International Conference on Autonomous Agents and Multiagent Systems*.
- Landmore H.** (2012). *Democratic reason: Politics, collective intelligence, and the rule of the many*. New Jersey: Princeton University Press. p. 304.
- List C. and Goodin R.E.** (2001). Epistemic democracy: generalizing the condorcet jury theorem. *Journal of Political Philosophy* 9(3), 277–306.
- Niesen P., Spiekermann K., Herzog L., Girard C. and Vogelmann F.** (2024). Does diversity trump ability? *Politische Vierteljahresschrift* 4, 1–21.
- Owen G., Grofman B. and Feld S.L.** (1989) Proving a distribution-free generalization of the Condorcet Jury Theorem. *Mathematical Social Sciences* 17(1), 1–16.
- Sakai R.** (2020). Mathematical models and robustness analysis in epistemic democracy: a systematic review of diversity trumps ability theorem models. *Philosophy of the Social Sciences* 50(3), 195–214.
- Schwartzberg M.** (2015). Epistemic democracy and its challenges. *Annual Review of Political Science* 18(1), 187–203.
- Spiekermann K. and Goodin R.E.** (2012). Courts of many minds. *British Journal of Political Science* 42(3), 555–571.
- Thompson A.** (2014). Does diversity trump ability? *Notices of the AMS*, 61(9):1024–1030.

## Appendix A

### A.1 Formal framework

Let  $\mathcal{W} = \{\omega_1, \dots, \omega_m\}$  denote a finite set of  $m$  alternatives. Moreover, we define a finite set  $\mathcal{N} = \{a_1, \dots, a_n\}$  consisting of  $n$  agents. We can then represent a single *approval voting (instance)* by

$$V \subseteq \mathcal{N} \times \mathcal{W}$$

where  $(a_i, \omega_j) \in V$  means that agent  $a_i$  approves choice  $\omega_j$ . Subsequently, we define the *score*  $\#_V \omega$  of some choice  $\omega \in \mathcal{W}$  as:

$$\#_V \omega = |\{a_i \in \mathcal{N}_n \mid (a_i, \omega) \in V\}|.$$

Finally, the winner of  $V$  is defined to be the alternative that receives a strictly higher score than any alternative:

$$\#_V \omega > \max_{\omega' \in \mathcal{W} \setminus \{\omega\}} \#_V \omega'.$$

**Example 7.** Suppose we have  $m = 3$  alternatives, i.e.,  $\mathcal{W} = \{\omega_1, \omega_2, \omega_3\}$  and that we have  $n = 2$  agents, i.e.,  $\mathcal{N} = \{a_1, a_2\}$ . Suppose  $a_1$  votes for the alternatives  $\omega_1, \omega_2$  and  $a_2$  votes only for  $\omega_1$ . Then  $\omega_1$  has a strictly higher score than either of the competing alternatives and wins the approval vote.

The described voting scenario is modeled by a random process that generates the correct alternative,  $\omega_*$ , the OL's choice as well as  $V$  and is governed by a joint probability distribution  $\mathbb{P}$  over the Bernoulli (i.e.,  $\{0, 1\}$ -valued) random variables  $X_*^{\omega_1}, \dots, X_*^{\omega_m}$ ,  $X_o^{\omega_1}, \dots, X_o^{\omega_m}$  as well as  $X_i^{\omega_1}, \dots, X_i^{\omega_m}$  for all agents  $1, \dots, i, \dots, n$  and all alternatives  $1, \dots, j, \dots, m$  such that the values taken by these random variables represent the outcome of a voting event as follows.

- $X_*^{\omega_j}$  is 1 if  $\omega_j$  is the true world state (i.e.,  $\omega_j = \omega_*$ ), else 0,
- $X_o^{\omega_j}$  is 1 if the OL approves  $\omega_j$ , and 0 otherwise,
- $X_i^{\omega_j}$  represents the private signal of the  $i$ th agent regarding his approval of the  $j$ th world state: it is 1 if  $a_i$  privately approves  $\omega_j$  and otherwise 0.

**Example 8.** Suppose, as in the previous example, that we have  $m = 3$  alternatives, i.e.,  $\mathcal{W} = \{\omega_1, \omega_2, \omega_3\}$  and that we have  $n = 2$  agents, i.e.,  $\mathcal{N} = \{a_1, a_2\}$ . Let  $\omega_1$  be the true state of the world, i.e.,  $X^{\omega_1} = 1, X^{\omega_2} = 0, X^{\omega_3} = 0$ . The OL approves options 1 and 2, that is,  $X_o^{\omega_1} = 1, X_o^{\omega_2} = 1, X_o^{\omega_3} = 0$ . Assume that  $a_1$  and  $a_2$  would vote as before if they were not influenced by the OL:  $X_1^{\omega_1} = 1, X_1^{\omega_2} = 1, X_1^{\omega_3} = 0, X_2^{\omega_1} = 1, X_2^{\omega_2} = 0, X_2^{\omega_3} = 0$ .

Given this joint distribution, we introduce the random variable  $V_i^{\omega_j}$ , which represents the final outcome of an agent's vote, i.e., after the OL's potential influence. According to our assumption,  $V_i^{\omega_j}$  is the probabilistic mixture of  $X_o^{\omega_j}$  with probability  $\pi$  and of  $X_i^{\omega_j}$  with probability  $1 - \pi$ . From this, we get for any  $x \in \{0, 1\}$  that

$$\mathbb{P}V_i^{\omega_j} = x = \pi\mathbb{P}X_o^{\omega_j} = x + (1 - \pi)\mathbb{P}X_i^{\omega_j} = x.$$

We denote by  $p_1^{\omega}, \dots, p_n^{\omega}$  the Bernoulli parameters of the “inner voice” random variables  $X_1^{\omega}, \dots, X_n^{\omega}$ , for all  $\omega \in \mathcal{W}$ , that is,  $p_i^{\omega_j} = \mathbb{P}(X_i^{\omega_j} = 1)$ . In a similar vein, for every  $\omega \in \mathcal{W}$ , we let  $\hat{p}^{\omega_1}, \dots, \hat{p}^{\omega_m}$  denote the Bernoulli parameters of the random variables  $X_o^{\omega_1}, \dots, X_o^{\omega_m}$ . Whether the OL approves the correct alternative, i.e., whether  $X_o^{\omega_*} = 1$ , is governed by the parameter,  $\hat{p} = \mathbb{P}(X_o^{\omega_*} = 1)$ . For convenience, the choice of the correct alternative, or correct world state, being unknown to the agents, will be abbreviated by  $[\omega_* = j]$ .

**Example 9.** We provide some example values for each parameter: Suppose that the OL has a rather high probability of approving the correct alternative  $\hat{p} = 0.6$  and that agents are relatively inclined to follow the OL  $\pi = 0.4$ . Suppose also that  $a_1$  has a high probability of voting for the first two alternatives when uninfluenced, but never votes for the third:  $p_1^{\omega_1} = 0.8, p_1^{\omega_2} = 0.75, p_1^{\omega_3} = 0$ .

In the following, we define the two central assumptions regarding the joint distribution. Conditioning upon the actual world state, we may define *private agent approval independence* as follows:

**Definition 3.** A joint distribution satisfies *private agent approval independence* if, conditioned on the actual world state, the private decision to approve any given  $\omega_j$  is made independently across all agents, i.e., for any  $\omega, \omega_j \in \mathcal{W}$  and any sequence  $v_1, \dots, v_n$  of values from  $\{0, 1\}$  the following holds:

$$\mathbb{P} \bigwedge_{i=1}^n X_i^{\omega_j} = v_i \mid [\omega_* = \omega] = \prod_{i=1}^n \mathbb{P}X_i^{\omega_j} = v_i \mid [\omega_* = \omega].$$

That is, conditional on the true state of the world, the joint probability of any given pattern of private approval decisions with respect to a given alternative can be computed by taking the product of the corresponding marginal probabilities.

Another central assumption concerns the “internal competence”  $p_k^{\omega}$  of the  $k$ th agent regarding his ability to identify the true world state among any number of alternatives if no influence is exerted. This assumption can be formalized as follows, where we denote the average of these “internal competencies” with

$$\bar{p}^{\omega} = \frac{1}{n} \sum_{k=1}^n p_k^{\omega}.$$

**Definition 4.** A joint probability distribution satisfies  $\Delta p$ -group reliability for some  $\Delta p > 0$ , if the probability, with respect to the agent's inner voice, to approve the true world state, averaged across all agents, is at least by  $\Delta p$  higher than the averaged probability for approving any other state, i.e., for every  $n$  and  $\omega_{\dagger} \in \mathcal{W} \setminus \{\omega_*\}$  holds  $\bar{p}^{\omega_*} \geq \Delta p + \bar{p}^{\omega_{\dagger}}$ .

## A.2 Proof of Theorem 4

*Proof.* The proof parallels that of Karge (2024) for a similar case. Assume,  $\pi = \bar{\pi}$ . Observe that both bounds are of the form  $1 - (m - 1)(x + y)$  where the factor  $(x + y)$  of Theorem 3 and Theorem 2 respectively consist of two parts: one where the OL endorses the correct alternative, and one where this is not the case. Consider the



first case in Theorem 3,  $e^{-\frac{2}{4n}(\Delta p(\sum_{k=1}^s |\mathcal{G}_{\pi_k}|(1-\pi_k)))^2}$ ). By definition of  $\sum_{k=1}^s |\mathcal{G}_{\pi_k}|$ ,  $\sum_{k=1}^s |\mathcal{G}_{\pi_k}| = n$ . Moreover, by

definition of  $\tilde{\pi}$ ,  $\sum_{k=1}^s \frac{|\mathcal{G}_{\pi_k}|}{n} \pi_k = \tilde{\pi}$ . Thus, as  $\pi = \tilde{\pi}$  by assumption,  $\sum_{k=1}^s \frac{|\mathcal{G}_{\pi_k}|}{n} \pi_k = \pi$ .

Finally,  $\sum_{k=1}^s |\mathcal{G}_{\pi_k}|(1-\pi_k) = n(1-\pi)$  when  $\sum_{k=1}^s \frac{|\mathcal{G}_{\pi_k}|}{n} \pi_k = \pi$  and  $\sum_{k=1}^s |\mathcal{G}_{\pi_k}| = n$ .

As  $\Delta p$  is fixed, we have that  $(\Delta p(\sum_{k=1}^s |\mathcal{G}_{\pi_k}|(1-\pi_k)))^2 = (n(1-\pi)\Delta p)^2$ .

By algebra, we obtain  $e^{-\frac{2}{4n}(\Delta p(\sum_{k=1}^s |\mathcal{G}_{\pi_k}|(1-\pi_k)))^2} = e^{-\frac{1}{2}n\Delta p^2(1-\pi)^2}$ . As an analogous argument can be made for the subcase where the OL is wrong, we conclude that  $P_{\min,\pi} = P_{\min,\tilde{\pi}}$ .

### A.3 Proof of Corollary 1

The proof is similar to a proof for a similar problem in Karge (2024).

*Proof.* To determine this threshold, we introduce the parameter  $\varepsilon$ , which represents the amount by which the average correlation is smaller in the more diverse group. Similarly,  $\lambda$  represents the increase in competency in the less diverse group. The more diverse group outperforms the more competent one as soon as the minimal success probability is greater, that is, as soon as:

$$\begin{aligned} 1 - (m-1) \left( \hat{p} e^{-\frac{1}{2}n(\Delta p + \lambda)^2(1-\pi)^2} + (1-\hat{p}) e^{-\frac{1}{2}n(\Delta p + \lambda)(1-\pi-\pi)^2} \right) \\ < 1 - (m-1) \left( \hat{p} e^{-\frac{1}{2}n\Delta p^2(1-(\pi-\varepsilon))^2} + (1-\hat{p}) e^{-\frac{1}{2}n(\Delta p(1-(\pi-\varepsilon))-(\pi-\varepsilon))^2} \right) \end{aligned} \quad (2)$$

Instead of determining the threshold for the general case, we observe that increasing  $\pi$  has the most detrimental effect on the group's ability to identify the correct alternative when  $\hat{p}$  is low. In the worst case,  $\hat{p} = 0$ . Therefore, to derive the desired threshold for when an increase in diversity outweighs an increase in competence for the competing group, we consider this worst case and assume  $\hat{p} = 0$ . In this scenario, the above expression simplifies to

$$1 - (m-1) e^{-\frac{1}{2}n(\Delta p + \lambda(1-\pi)-\pi)^2} < 1 - (m-1) e^{-\frac{1}{2}n(\Delta p(1-(\pi-\varepsilon))-\pi-\varepsilon)^2} \quad (3)$$

We further simplify this problem by noting that the two expressions differ only in their exponents. Since they are of the form  $e^{-x}$  and  $e^{-y}$ , respectively, they increase as  $x$  and  $y$  decrease. With an increasing factor of the form  $e^{-x}$ , the total expression of the form  $1 - (m-1)e^{-x}$  increases, and so does the minimum probability of success.

For the sake of simpler algebra, we check for when

$$-\frac{1}{2}n(\Delta p + \lambda(1-\pi) - \pi)^2 < -\frac{1}{2}n(\Delta p(1-\pi-\varepsilon) - \pi - \varepsilon)^2 \quad (4)$$

By solving for  $\varepsilon$ , we can derive that this inequality is true if

$$\varepsilon < \frac{\lambda - \pi\lambda}{\Delta p + 1}$$

From this, we can derive that

$$1 - (m-1) e^{-\frac{1}{2}n(\Delta p + \lambda(1-\pi)-\pi)^2} < 1 - (m-1) e^{-\frac{1}{2}n(\Delta p(1-\pi-\varepsilon)-\pi-\varepsilon)^2} \quad (5)$$

if

$$\varepsilon \geq \frac{\lambda - \pi\lambda}{\Delta p + 1} \quad (6)$$