

THE COSMIC DISTANCE SCALE

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ABSTRACT. The zero-point of the extragalactic distance scale, defined by about two dozens of nearby, late-type galaxies, has remained nearly unchanged for the last decade, in spite of the advent of new techniques and great efforts. The distances are essentially tied to trigonometric parallax stars and hence independent of the Hyades modulus; they are consistent with RR Lyr stars. The mean zero-point is therefore probably secure to better than 10%.

All known secondary distance indicators are still affected by zero-point errors, by problems in the definition of their relation between distance indicator and absolute magnitude (or linear size), and/or by selection bias. The effect of the very important selection bias (Malmquist effect), which causes a seemingly non-linear expansion field, is illustrated by two examples. To test for any true deviations from a linear expansion the Hubble diagram of nearly bias-free first-ranked cluster galaxies and supernovae Ia is shown; this imposes stringent limits on any non-linearity of the Hubble flow within $v < 5000 \text{ km s}^{-1}$.

After freeing the available distances of field galaxies from selection bias and after reducing them to a common zero-point, one finds $H_0 = 55-65$. Several distance indicators require a best Virgo cluster modulus of $(m-M) = 31.60$, which implies for the Coma cluster $(m-M) = 35.38$ and, with $v(\text{Coma}) = 7217 \text{ km s}^{-1}$, $H_0 = 60$. Supernovae Ia and first-ranked cluster galaxies out to large distances give $H_0(\text{global}) = 53$. Thus the evidence from clusters and field galaxies is best satisfied by $H_0 = 55$; the assigned mean error of ± 7 is to indicate a 3σ range of $35 < H_0 < 75$.

Purely physical methods to determine extragalactic distances have modest weight yet; they will contribute eventually much to the determination of H_0 .

If H_0 were as large as 100, several paradoxa would arise. The Milky Way would have a very high supernova frequency, our Galaxy and M31 would be oversized, the baryon

density would fall short to bind clusters, and Friedman universes were excluded.

Because all systematic errors have conspired and probably still conspire to measure H_0 too high, the true value could well be 40. Until new, decisive evidence becomes available, it is suggested for all practical purposes to use $H_0=50$.

I. INTRODUCTION

Distance determinations outside our Galaxy have still to rely on the true luminosity or on the true size of extragalactic objects, which have "identical" counterparts close enough that their distances are known by some method or another.

In spite of considerable efforts, the history of the extragalactic distance scale is beset by errors and setbacks. Much of the reason lies in the term "identical" counterparts. A priori they may not exist outside our Galaxy due to differences in the evolutionary history, metallicity etc. Even if they exist in first approximation, their properties must have some intrinsic scatter and the term "identical" must be replaced by "similar". This makes a fundamental difference, because as soon as distance indicators have intrinsic scatter of their luminosity or size, they are subject to selection bias: at large distances only overluminous or oversized objects tend to enter our catalogues. This tendency distorts systematically the distance scale. The problem becomes particularly severe for the determination of the global value of the Hubble constant H_0 [$\text{km s}^{-1}\text{Mpc}^{-1}$], because to find its true value freed of any local peculiar and streaming motions, one has to go to distances where the recession velocities are larger than at least $v_0=5000 \text{ km s}^{-1}$. Such distances are presently difficult to bridge in one step (the exception may be supernovae of type Ia [SNe Ia] at maximum light), and by the time one has patched up the distance ladder one may have accumulated several systematic errors. Because all systematic errors due to selection bias tend to underestimate the distances, the resulting, typically non-linear distance scale may be seriously compressed with a correspondingly, unrealistically high value of H_0 . The reliability of any distance indicator must therefore be thoroughly studied before it can be put to use. In many cases this can be achieved by consistency checks as shown below.

The development of extragalactic astronomy has been accompanied by only a slow grasp of the true size of the universe and its brightest/largest constituents. This is reflected in a continuous reduction of H_0 . Hubbles original value of $H_0=513$ (1929) or 526 (1936) had to be halved after

Baade's (1952) 1.5 mag. correction of the zero-point of the P-L relation of Cepheids. Sandage's (1958) reanalysis of the P-L relation, his application of the new magnitude scale by Stebbins, Whitford, and Johnson (1950) to the brightest stars in external galaxies, and his proof that Hubble's brightest stars were actually HII regions, led him to conclude that $50 < H_0 < 100$. In particular he reasoned that if the brightest blue stars have a - presently generally accepted - absolute magnitude of " $M_{pg}(\text{stars}) = -9.5$, then $H = 55$ ". The first suggestion that the best value of H_0 is as low as 50 is due to de Vaucouleurs (1970). Later work by many researchers has led to a bifurcation with values either near $H_0 = 100$ (sometimes called the short distance scale) or near $H_0 = 50$ (long distance scale). This discrepancy, corresponding to 1.5 mag. in the distance moduli, is not mainly due to uncertainties of the distances to very nearby galaxies, but it accumulates gradually as one goes to larger distances. The steady divergence of the two distance scales suggests that their main difference lies in the treatment of the already mentioned selection effects. The demonstration of this is in fact the main goal of the present review.

In Section II a brief discussion is given of the primary distance indicators. They provide distances to 24 nearby galaxies, which can be used to calibrate additional distance indicators which reach to larger distances; modern results for these galaxies are compiled in Section III and they are shown to have changed on average surprisingly little during the last twelve years. A selection of proposed secondary distance indicators is listed in Section IV and it is stressed that they must be subjected to a severe scrutiny of their reliability; in particular their possible vulnerability against the Malmquist effect is illustrated. As an example, the internal consistency of the distances derived from the luminosity index Λ_C (Section V) and from 21cm-line widths combined with infrared (IR) magnitudes (Section VI) is checked. The possibility of an overall non-linearity of the expansion field out to $v_0 \sim 5000 \text{ km s}^{-1}$ is rejected in Section VII. The calibration of the luminosity of SNe Ia at maximum light is discussed in Section VIII. Section IX compiles the evidence for the distance of the Virgo cluster. A compromise global value of H_0 is derived in Section X. Conclusions are drawn in Section XI.

II. PRIMARY DISTANCE INDICATORS

Classical Cepheids are ideal extragalactic distance indicators because as supergiants they are luminous enough to be observed from the ground out to $\sim 7 \text{ Mpc}$, and because they follow the period-luminosity-color (P-L-C) relation with very little scatter (Sandage and Tammann, 1971), which

makes them insensitive to selection bias. They show considerably more scatter in a simplified P-L relation, but the known intrinsic width of the relation offers also here a handle against selection bias. A disadvantage of the optical P-L-C and P-L relation is that they yield too large distances for low-metallicity Cepheids. This difficulty is minimized by the IR P-L relation (for references see Table 1). The IR relation has also the advantage that it has a reduced width and that it is insensitive to internal absorption. - The zero point of the optical and IR P-L relation rests on the Cepheids which are members of galactic clusters. The distances of the latter are generally tied to the Hyades modulus. When the classical Hyades modulus of $3^m.03$ was revised upwards by $\sim 0^m.25$ the question arose whether all Cepheid distances had to be increased by the same amount. Van den Bergh (1977) warned that the correction may be compensated by the over-metallicity of the Hyades. Indeed, tying the Cepheid-bearing clusters M25 and NGC 6087 to parallax stars, which are reduced to the same metallicity, yields cluster moduli (Pel 1985; Cameron 1985a, b; Turner 1986) only $0^m.05$ larger on average than those derived on the basis of a Hyades modulus of $3^m.03$ and no metallicity correction (Sandage and Tammann, 1969). The two cases suggest that the revision of the Hyades modulus affects the extragalactic distance scale by a barely significant amount of only $0^m.05$. It has therefore been decided here to fit the Cepheid-bearing clusters, whose metallicity is yet unknown, to a formal Hyades main-sequence with $(m-M)=3^m.03$. The remaining zero-point error of the Cepheids is estimated to $\pm 0^m.10$. Eventually it will become possible to base the zero-point of the P-L relation on Cepheid distances derived from the purely physical Baade-Becker-Wesselink method, but at present its accuracy is not compatible; the main reason is that no models of moving atmospheres are yet available (cf. Gautschy 1986).

RR Lyr stars are powerful distance indicators, because they follow a tight P-L relation (Sandage 1981). Their zero-point of $\langle M_V \rangle = 0^m.6 \pm 0^m.1$ is widely accepted. This zero-point cannot yet be based on the Baade-Becker-Wesselink method, because the problem of non-static atmospheres is here still more severe than for Cepheids, as evidenced by the appearance of emission lines and shock fronts. The disadvantage of RR Lyr stars is that they are intrinsically much fainter than Cepheids, and that their luminosity is a function of metallicity. The internal accuracy of statistical parallaxes is not yet sufficient to determine the sign of the metal correction (Strugnell, Reid, and Murray, 1986; Barnes and Hawley, 1986), but from cluster variables it follows that they become fainter with decreasing metallicity (Sandage 1970; Carney 1980).

Spectral types of sufficiently bright stars in external galaxies have been used to determine distances. The assumption here is, of course, that the empirical relation between spectral type and absolute magnitudes, derived in our Galaxy, holds also in other galaxies. In view of the width of the relation, the sampling of the extragalactic stars poses a severe problem. Sampling from bright magnitudes into a galaxian population tends to distort the sample and to underestimate the mean stellar magnitudes and hence the distances. This trend can be traced historically through the literature for instance in the case of LMC.

Color-magnitude diagrams (CMD) of brightest stars in external galaxies are subject to the same selection problems as spectral types. Evolved and untypically bright stars enter the sample first. In addition blends with very faint stars tend to brighten and redden the sample stars (unless the "sky" is measured near to the stars), which leads to an overestimate of their absorption and underestimate of their distance.

If only the brightest part of the CMD can be observed, brightest blue stars, Hubble-Sandage variables, and brightest red stars can be used as distance indicators. Unfortunately the luminosity of the brightest blue stars correlates with the size of the parent galaxy, yet the latest calibration of the mean of the three brightest blue stars shows a useful plateau at $M_B(3) = -10^m$ for galaxies with $M_B < -20^m$ (Sandage 1986c). The same source shows the dependence of the brightest red stars on the galaxy size to be shallower; they have correspondingly higher weight as distance indicators.

Mira variables have been shown by M. Feast and his collaborators at the Cape to follow a P-L relation. The uniqueness of this relation is, however, not yet established (Menzies and Whitelock, 1985). The applicability of Miras is in any case yet restricted to LMC (Feast 1984, 1986).

Since Novae exhibit a correlation between absolute magnitude and decline rate they may be useful distance indicators. In principle their Galactic zero-point can be found from expanding nova shell parallaxes, which makes them independent of all other distance scales. An old zero-point was provided by T. Schmidt-Kaler, but the results were not very encouraging (van den Bergh 1977; Tammann 1977; de Vaucouleurs 1978). A new start has been made by Cohen (1985) and van den Bergh and Pritchett (1986a). The method is of course also vulnerable to selection effects due to the still not well known intrinsic scatter of the absolute magnitude-decline rate relation. Moreover in distant galaxies there is the danger that the nova sample is biased by objects whose decline rate is so steep that they fall below the detection limit before their decline rate can be determined. This would result in a sample of novae which are overly bright at

discovery and at the reference time (or magnitude) where their decline rate is determined.

Supernovae of type Ia as distance indicators are discussed in Section VII and VIII.

Table 1: True Distance Moduli to Nearby Calibrating Galaxies as Adopted by Sandage and Tammann (1974) and from New Evidence.

	ST (1974)	new	(m-M)	Sources
LMC	18.59	18.50	-0.09	1
SMC	19.27	18.85	-0.42	2
M31	24.16	24.2	+0.04	3
M33	24.56	24.4	-0.16	4
NGC 6822	23.95	23.4	-0.55	5
IC 1613	24.43	24.1	-0.33	6
NGC 300	-	26.0	-	7
NGC 247	-	26.7	-	8
NGC 253	-	27.5	-	9
NGC 7793	-	27.5	-	9
NGC 2403*	27.56	27.8	+0.24	10
NGC 3031	27.56	28.7	+1.14	11
M 101 **	29.2	29.2	0	12
			$\overline{-0.01 \pm 0.17}$	

* Other group members: NGC 2366, NGC 4236, IC 2574, HoII, HoI, HoIX

** Other group members: NGC 5204, NGC 5474, NGC 5477, NGC 5585, HoIV

The adopted "new" distances are based on the following determinations:

1. Feast (1986) gives as the best mean from optical and infrared observations of Cepheids, from RR Lyr stars, Mira variables, and clusters, including the extensive work at the Cape, 18.5 ± 0.1 (cf. also Stothers, 1983). Infrared observations of Cepheids alone give 18.45 (Welch et al., 1985) and 18.50 ± 0.07 (McAlary and Welch, 1985). Aperture photometry at $1.05 \mu\text{m}$ of Cepheids yields 18.56 ± 0.07 (Visvanathan, 1985, after reducing the value by $0^{\text{m}}.26$ to agree with the presently adopted Hyades zero point for Cepheids in Galactic clusters). ZAMS fitting of LMC clusters tends to yield small moduli of $18.1-18.42 \pm 0.2$ (Andersen et al., 1985; Schommer et al., 1984; Walker, 1986). However, numerical experiments moving Galactic clusters to roughly the LMC distance and calculating the effect of resulting blends shows that the method yields systematically too low moduli

- (Brodbeck, 1986). The outlying cluster NGC 1841 at 18.7 (Andersen et al., 1985) may not be a member (or may suffer less blends?). Chiosi and Pigatto (1986) have shown that the inclusion of convective overshooting increases the ZAMS fitting distances by typically $0^m.2$. Conti and Garmany (1986) found from O,B stars 18.3 ± 0.3 , as compared to 18.63 ± 0.2 by Crampton (1979). Shobbrock (1986) finds from $H\beta$ photometry of B supergiants 18.8 ± 0.3 . The adopted value of 18.50 ± 0.10 appears to be a good compromise.
2. A review by Feast (1986) gives 18.8. Eggen (1977) derived from optical observations of Cepheids 18.95. BVI (Caldwell and Coulson, 1985) and JHK (Laney and Stobie, 1986) photometry of Cepheids shows SMC to be more distant than LMC by $0^m.30 \pm 0^m.06$ and $0^m.32 \pm 0^m.04$, respectively. Infrared photometry of Cepheids by Welch et al. (1985) and McAlary and Welch (1985) shows this difference to be $0^m.38_5$, while Visvanathan (1985) obtained $0^m.29$ at $0.05 \mu\text{m}$ and Madore (1986) $0^m.50$ in the infrared. RR Lyr stars give 0.35 (Graham, 1975). Assuming $(m-M)=0.35$ (cf. Tammann et al., 1980), a true modulus of 18.85 ± 0.15 is adopted.
 3. Infrared observations of Cepheids give 24.26 ± 0.08 (Welch et al., 1986). From the photometry of giant branch Pop.II stars follows 24.4 ± 0.25 (Mould and Kristian, 1986). Assuming $M_B(\text{RR Lyr})=1.02$ and $A_B=0.32$ van den Bergh and Pritchett (1986) found from three RR Lyr stars 24.16 ± 0.18 . Novae in M31 yield, with the same value of A_B , 24.03 ± 0.20 (Cohen, 1985). Baade and Swope's (1963) value of 24.20 from Cepheids was revised by Sandage (1983a) to 24.11, if $(m-M)^{\text{Hyades}}=3.03$ and $A_B=0.64$ for Cepheids is assumed. A value of 24.2 is consistent with all of these determinations.
 4. Blue photometry of Cepheids gives $(m-M)_{A_B}=25.35$ (Sandage and Carlson, 1983; Sandage 1983a), or $0^m.08$ less if $(m-M)^{\text{Hyades}}=3.03$. With $A_B=0.8$ from Freedman (1986) a true modulus follows of 24.47 ± 0.15 . Infrared Cepheids yield 24.1 ± 0.1 (Freedman, 1986) or 24.17 ± 0.15 (McAlary and Welch, 1985). Halo giants in M33 indicate 24.8 ± 0.3 (Mould and Kristian, 1986). The value of 24.4 is the best compromise.
 5. Infrared observations of Cepheids give 23.47 ± 0.11 (McAlary et al., 1983), 23.30 ± 0.13 (McAlary and Welch, 1985), 23.4 ± 0.2 (Freedman, 1986), and 23.5 (Madore, 1986). The adopted mean value of 23.4 is $0^m.55$ smaller than the value adopted from blue Cepheid magnitudes

(Sandage and Tammann, 1974) presumably due to intrinsic absorption and/or low metallicity of NGC 6822.

6. Cepheids in the infrared yield 24.08 ± 0.14 (McAlary and Welch, 1985), 24.0 ± 0.2 (Freedman, 1986), and 24.2 (Madore, 1986). Conservatively a mean value of 24.1 is adopted, rather than the considerably higher value of 24.43 from blue Cepheid magnitudes (Sandage, 1971), which may be similarly affected as in NGC 6822.
7. Graham (1982) found from old red giants 25.8 ± 0.5 . Blue magnitudes of Cepheids give 26.09 ± 0.2 (Graham, 1984) and infrared magnitudes 25.9 ± 0.2 (Freedman, 1986). Planetary nebulae suggest 25.85 ± 0.34 (Lawrie and Graham, 1983).
8. From resolution into brightest stars, which is intermediate between NGC 300 and NGC 253/NGC 7793 (Sandage, 1986b). The value agrees within $0^m.2$ with that given by de Vaucouleurs (1975).
9. From resolution into brightest stars (Sandage, 1986a).
10. Infrared magnitudes of Cepheids yield 28.15 ± 0.20 (McAlary and Madore, 1984), 28.09 ± 0.21 (McAlary and Welch, 1985), and 27.5 ± 0.2 (Freedman, 1986). Sandage (1984b) obtained 27.66 (with $A_B = 0^m.24$) from Cepheids and the C-M diagram of the brightest stars.
11. Brightest blue and red stars, blue irregular Hubble-Sandage variables, and Cepheids require (with $A_B = 0^m.10$) 28.7 (Sandage, 1984a).
12. Brightest blue and red stars, blue irregular Hubble-Sandage variables, and the absence of Cepheids require ≥ 29.2 (Sandage, 1983b). This is compatible with 28.9 ± 0.3 from M supergiants (Humphreys and Strom, 1983). The discovery of Cepheids at very faint levels suggests 29.3 (Cook, Aaronson, and Illingworth, 1986).

III. DISTANCES TO NEARBY CALIBRATING GALAXIES

The distance indicators, which can be calibrated in our Galaxy and which are briefly discussed in the previous Section, have been used by many authors to determine distances to galaxies within ~ 7 Mpc. In Table 1 in the column headed "new" the mean distance moduli from many such determinations since about 1980 are given. The multitude of sources, as discussed in the footnotes to Table 1, should minimize any personal bias, and the entries - although of

different weight - are intended to represent generally acceptable compromises.

The distance moduli available in 1974, essentially based on observations of Cepheids in optical wavebands, are also listed in Table 1 (Sandage and Tammann, 1974a; 1974b, cf. also Tammann, Sandage, and Yahil, 1980, Table 9). A comparison of the 1974 and "new" distances shows for individual galaxies revisions by up to $1^m.1$, but the impressive result is that the mean zero-point of the calibrating galaxies has hardly changed at all.

The fact that the zero-point of the extragalactic distance scale has essentially remained unchanged during an interval of a decade, in spite of the advent of new techniques and the enormous efforts of many researchers, gives considerable confidence that the mean zero-point as defined by the galaxies in Table 1 can be trusted at the $0^m.1$ - $0^m.2$ level.

Because several recent distance determination methods have based their results relative to an adopted LMC modulus of $18^m.5$, the latter value carries considerable weight. Recent infrared work on LMC Cepheids has shown that this value may be low by $0^m.2$ - $0^m.3$ (Laney and Stobie, 1986). This suggests that the adopted "new" zero-point is, if anything, somewhat faint.

The number of 13 calibrating galaxies in Table 1 can be increased by members of the NGC 2403 group and of the M101 group. They are listed at the foot of Table 1; they bring the total number of calibrators to 23. There are in addition a number of small and very small irregulars whose distances are known from Cepheids and/or brightest stars (Sandage 1986d). We will return to these nearby objects in Section X, but so far dwarf galaxies have not been applied as stepping stones of the extended distance ladder.

It should be noted that the calibrating galaxies are either of spiral or later type. Some distances of dE galaxies in the Local Group are known from RR Lyr stars, but a real E galaxy is missing among the calibrators, unless one wants to accept M32, an exceptionally faint E-type companion of M31, to be representative for its much brighter fellow galaxies.

IV. PROPOSED DISTANCE INDICATORS BEYOND ~ 7 MPC AND THE EFFECT OF SELECTION BIAS

The calibrating galaxies in Table 1 offer a data base from which either the intrinsic size/luminosity of particularly conspicuous objects or the global properties of galaxies can be determined. The calibration can then, hopefully, be applied to more distant galaxies. A multitude of objects and

global parameters have been proposed for this purpose. Some of the more important ones are listed in the following.

Among individual objects, HII regions can be identified out to large distances. Their size (Sandage and Tammann, 1974a), H α flux (Kennicutt 1981), and velocity dispersion (Melnick et al., 1986) have been applied for deriving distances. Another example is the peak of the luminosity function of globular clusters (van den Bergh, Pritchet, and Grillmair, 1986).

Among the observable global parameters of galaxies which correlate with the galaxian luminosity, and which hence may be distance indicators, are

- the surface brightness/luminosity relation of spirals (Holmberg 1958) and of dE galaxies (Binggeli, Sandage, and Tarenghi, 1984),
- the luminosity classification of spiral galaxies according to the "beauty" of the spiral structure (van den Bergh 1960; Sandage and Tammann, 1974c; however also Kraan-Korteweg, Sandage, and Tammann, 1984),
- the luminosity index Λ_c as a derivative of the luminosity classification (de Vaucouleurs 1979),
- the color/luminosity relation of E (Visvanathan and Sandage, 1977) and spiral (Visvanathan and Griensmith, 1977) galaxies,
- the 21cm line width/luminosity relation of spiral galaxies in optical (Tully and Fisher, 1977) and infrared (Aaronson, Huchra, and Mould, 1979) wavelengths,
- the velocity dispersion/luminosity relation of E galaxies (Faber and Jackson, 1976) and of the spheroidal components of spirals (Whitmore, Kirshner, and Schechter, 1979; 1981),
- the brightness distribution/luminosity relation of E galaxies (Kormendy 1977),
- look-alike galaxies ("sosies") as standard candles (Paturel 1984; Bottinelli et al., 1985),
- the metallicity/luminosity relation of E galaxies, particularly the Mg₂ index (Dressler 1984),
- the velocity dispersion/magnitude-related diameter relation of E galaxies (Burstein et al., 1986; Djorgovski and Davis, 1986), and
- first-ranked cluster galaxies as standard candles (Sandage 1972).

Before these or any other distance indicators can be relied upon it is essential to gain full control of the following points:

(1) The shape of the distance indicator/luminosity (or size) relation must be known, unless the distance indicator is treated as a standard candle. The number of calibrating galaxies and their absolute-magnitude range prove frequently insufficient to yield a reliable shape. The shape must therefore typically be based on members of a cluster, whose

number is necessarily also limited, or on the assumption of an ideal (or corrigible) Hubble flow of field galaxies.

(2) The zero-point of the relation must be well known, which requires a sufficient number of applicable calibrators.

(3) The intrinsic scatter of the relation between observable and luminosity (or size) is of overwhelming importance for the distance scale and will therefore be discussed below.

(4) The parameter space in which the distance indicator can be applied must be well tested. A distance indicator may be sensitive to the type of galaxy, it may be reliable only within a certain luminosity interval etc. Of particular importance is here the question of field (or group) and cluster galaxies. Given the possibility that the intrinsic properties change between the two kinds of galaxies, one normally depends on an assumption when the calibrating galaxies, which are typical for the field and group population, are directly compared with cluster galaxies.

So far none of the listed secondary distance indicators satisfies all four points. Either the shape of the distance indicator/luminosity relation is uncertain or no zero-point is available. The latter problem is particularly severe for E galaxies as discussed above. While these difficulties are hardly controversial, there is considerable disagreement on the importance of the intrinsic scatter. Because I believe this problem to be the main reason for the present existence of two distance scales, the short scale with $H_0 \sim 100$ and the long scale with $H_0 \sim 50$, a somewhat more extended explanation is in place.

To illustrate the last point the Spaenhauer diagram is repeated here in Fig.1 (cf. Sandage 1987, Fig.11). It represents in the upper panel the distribution in absolute magnitude of galaxies which scatter by an intrinsic value σ_M about a mean value. Because the number of objects increases with the volume surveyed, one reaches more and more objects as the distance increases. The eventually large number of objects causes the appearance of "improbable" objects, i.e. their absolute magnitudes deviate from the mean by $2\sigma_M$, $3\sigma_M$ or even $4\sigma_M$ in either direction. The character of the diagram is universal if only $\sigma_M \neq 0$; it holds equally for E galaxies, for galaxies of given luminosity parameter, or for any other subsample of galaxies.

The lower panel of Fig.1 repeats the upper panel, but now an apparent-magnitude limit is introduced (here at $m = 13^m$). The devastating effect can be clearly seen, which the observational limit causes on the sample. At larger distances only "improbably" luminous objects are retained, while their underluminous counterparts are eliminated. Therefore the mean luminosity of the objects increases with distance, and one must not expect that a distant sample complies with the zero-point determined from nearby objects. There is an additional effect, which enhances the problem.

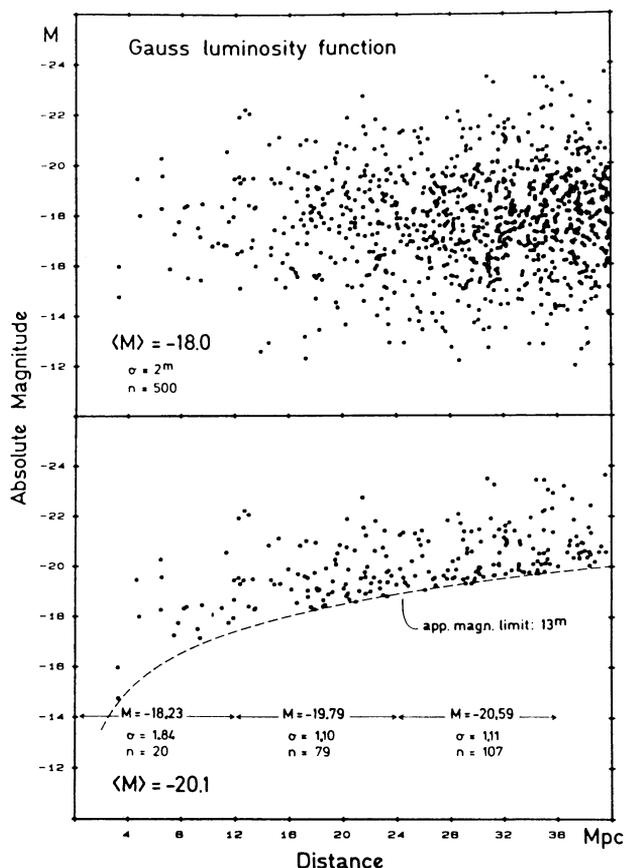


Fig.1. Upper panel: Monte Carlo distribution in distance and absolute magnitude of 500 galaxies within 38 Mpc. Constant space density and a mean absolute magnitude of $\langle M \rangle = -18^m$ with a Gauss standard deviation of $\sigma_M = 2^m$ are assumed. Lower panel: The same sample cut by an apparent-magnitude limit of $m = 13^m$. Note the increase of the galaxian luminosities with increasing distance and the small effective (observable) scatter σ_M within individual distance intervals.

The observable "effective" scatter is much smaller than the intrinsic scatter. One can therefore easily be misled to conclude from the observed scatter that the bias is

negligible. In fact the bias is determined by the intrinsic scatter, which in most practical cases is very difficult to determine.

The lower panel of Fig.1 is an illustration of a very general case. Almost all galaxy catalogs are limited by apparent magnitude and are therefore subject to the "Malmquist bias". An exception are first-ranked cluster galaxies, which are identified, irrespectively of any overluminosity, in independently discovered clusters. Also all SNe Ia, due to their extreme luminosity, can be found within a reasonable redshift limit; they should carry therefore a minimum bias. In general the bias is the less severe the smaller σ_M (intrinsic).

In practice it is not possible to correct analytically for the bias. Its size does not only depend on σ_M , but also on the shape of the true luminosity distribution of the objects considered, and on their space density \mathfrak{g} , which realistically is variable and unknown as long as no correct distances are known. Malmquist (1920) has derived an explicit correction for the idealized case of a Gaussian luminosity distribution and $\mathfrak{g} = \text{const}$; this correction, however, implies an infinite sample in space, which in the presence of the K-term is highly unrealistic for recessing galaxies.

There are tactics to compensate the Malmquist in first approximation. A possibility is to shift the limiting magnitude to fainter values as one goes to larger recession velocities. Strictly speaking this requires still a well understood expansion field and $\mathfrak{g} = \text{const}$. The method has been applied for distances from luminosity classes (Sandage and Tammann, 1975), but for most purposes the distant galaxies must be sampled to unpractically faint limits. Another possibility is to rely on an empirical and in each case newly determined correlation between luminosity and recession velocity (Sandage, Tammann, and Yahil, 1979). Yet another possibility is not to treat the luminosity indicator (e.g. the 21-cm line width) as the independent variable, but rather the absolute magnitude, which can be predetermined except for a constant term from any arbitrary value of H_0 , if a perfect Hubble flow is assumed (Schechter 1980). This requires, however, that the sample is unbiased with respect to the distance indicator, an assumption which is not generally fulfilled (Kraan-Korteweg, Cameron, and Tammann, 1986). Finally the Malmquist bias can be excluded if one samples by a fixed magnitude interval into the population of clusters at different distances.

Neglect of the Malmquist bias always leads to too short a distance scale. However, the error is not easily recognizable, because the artificially decreased luminosity scatter and the a priori unknown space density of galaxies will still allow a seemingly consistent model. To test for

the presence of a Malmquist bias somewhat more sophisticated methods have to be applied. They involve the fundamental linearity of the expansion field. How these tests can be performed is illustrated in the next two Sections for two different distance indicators, i.e. the luminosity index Λ_C and the infrared Tully-Fisher relation.

V. THE LUMINOSITY INDEX Λ_C AS DISTANCE INDICATOR

Distances derived from the luminosity index Λ_C were published for 309 non-Local Group, Shapley-Ames spiral galaxies (de Vaucouleurs 1979). Their recession velocities v_{220} corrected for the Virgocentric infall component following Kraan-Korteweg (1985), are plotted in a double-logarithmic diagram (Fig.2). De Vaucouleurs has concluded from these data that $H_0=100$ (e.g. de Vaucouleurs and Corwin, 1986), and he has stressed that the diagram proved the Λ_C distance scale to be linear. However, the eye is insensitive against

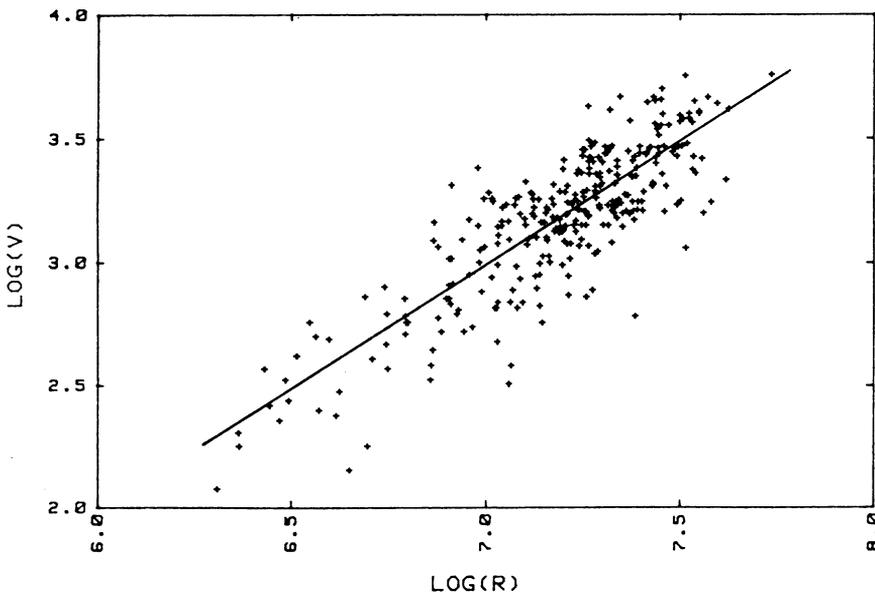


Fig.2. A plot of the logarithm of the corrected recession velocity versus the logarithm of the published Λ_C distance (de Vaucouleurs 1979) of field spiral galaxies. The data seem compatible with $H_0=100$ (full line).

systematic effects in a log-log plot. Therefore the data of Fig.2 are repeated in Fig.3, but now the numeric values of distance R and velocity v_{220} are plotted. The diagram reveals a strong non-linearity with the velocities increasing faster than the Λ_C distances. With an ansatz of $R = a \cdot v_{220}^b$ one finds from a least-squares solution $a = 0.227$ and $b = 0.589$. This seems to require an increasing value of the Hubble constant, viz. $H_0 = 57$ at $v_{220} = 500 \text{ km s}^{-1}$, $H_0 = 100$ at $v_{220} = 2000 \text{ km s}^{-1}$ and $H_0 = 146$ at $v_{220} = 5000 \text{ km s}^{-1}$; the asymptotic (global) value of H_0 would then have to be > 150 . A much more attractive interpretation is, however, to explain the non-linearity by the expected Malmquist bias.

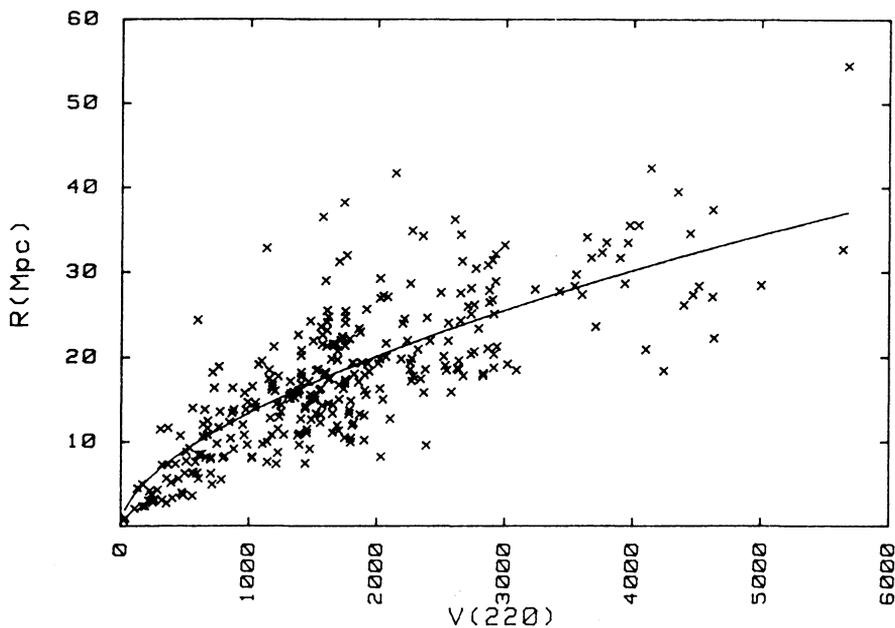


Fig.3. A plot of the Λ_C distances, as published by de Vaucouleurs (1979), against the recession velocity (corrected for a Virgocentric infall model with a local infall velocity of $v_{vc} = 220 \text{ km s}^{-1}$). Note the non-linearity of the relation.

The question is then, how the best, unbiased value of H_0 can be derived from the Λ_C distances. A possibility is to determine a Hubble constant $H_i = v_{220}(i)/R_i$ for each galaxy i . The resulting values are plotted against the velocity v_{220} in Fig.4. Here the increase of H_i with velocity (i.e.

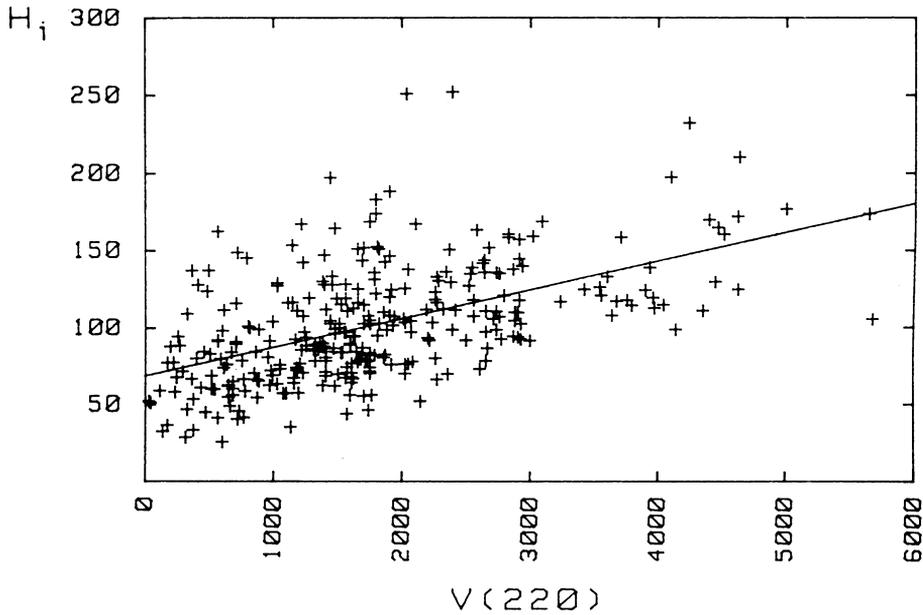


Fig.4. Hubble ratios H_i for individual galaxies with published Λ_c distances plotted against the corrected velocity v_{220} . Due to the Malmquist effect the values of H_i increase with velocity/distance.

Table 2: de Vaucouleurs' Calibrators

Galaxy	$(m-M)_{dev}^0$	$(m-M)_{new}^0$	$(m-M)$
LMC	18.31	18.50	-0.19
M31	24.07	24.2	-0.13
M33	24.30	24.4	-0.10
NGC 2403	27.10	27.8	-0.70
NGC 3031	27.7	28.7	-1.00
NGC 4236	27.7	27.8	-0.10
M101	28.5	29.2	-0.70
NGC 5474	28.5	29.2	-0.70
NGC 5585	28.5	29.2	-0.70
NGC 247	27.0	26.7	+0.30
NGC 253	27.0	27.5	-0.50
NGC 300	27.0	26.0	+1.00
NGC 7793	27.0	27.5	-0.50
			-0.31 ± 0.15

distance) becomes very clear, in fact a least-squares solution gives

$$H_1 = 68.0 + 0.019v_{220}. \quad (1)$$

From eq.(1) the best value of H_0 is judged to be $H_0=68$. This should not be interpreted as the Hubble constant at zero velocity/distance, because eq.(1) is still well determined if one excludes the nearby galaxies with $v_{220} < 1000 \text{ km s}^{-1}$. Rather the value $H_0=68$ should be taken as the best estimate at zero bias. The conclusion from this is that the Λ_C distances do not require $H_0=100$, but much more likely $H_0=68$.

The value of $H_0=68$ is still based on de Vaucouleurs' (1979) calibrators. They define, as Table 2 shows, a somewhat fainter zero-point than the "new" calibrators of Table 1. The zero-point difference of $-0^m.31$ requires an increase of the Λ_C distances by a factor of 1.15. Including this correction, the best estimate of the Hubble constant, derived from Λ_C distances, is $H_0=59$. It is impossible to assign a formal error to this value. The Λ_C distances contain internal inconsistencies, e.g. one obtains different values of H_0 if the galaxies are subdivided into absolute magnitude or type bins (Tammann and Sandage, 1983). Moreover, the numerical result depends on the linear correlation between H_1 and velocity, which was assumed in eq.(1). Any other, non-linear relation would have led to a different, zero-bias value of H_0 . These remaining uncertainties introduce an estimated error of about $\pm 20 \text{ km s}^{-1} \text{ Mpc}^{-1}$, i.e. $H_0=59 \pm 20$ from the Λ_C distance indicator.

VI. 21cm-LINE WIDTHS AND INFRARED MAGNITUDES AS DISTANCE INDICATORS

21cm-line widths Δv_{21} of spiral galaxies, corrected for inclination, and (nearly absorption-free) IR magnitudes m_{IR} have been used to derive a Hubble constant of $H_0=90$ (Aaronson and Mould, 1986, and references therein). In order to test whether this distance scale is compatible with a linear expansion field, the same procedure has been employed as in Section V. For this purpose, IR magnitudes and inclination-corrected 21cm-line widths are used of 308 field spirals outside the Local Group (17 of which are certain or probable Virgo cluster members) from Aaronson et al. (1982). A quadratic relation between absolute magnitude M_{IR} and Δv_{21} has been prescribed by Aaronson et al. (1986). Following strictly these precepts, the distance moduli $m_{\text{IR}} - M_{\text{IR}}$ and the corresponding linear distances R have been calculated for all 308 spirals. Their observed recession velocities have been corrected for the effect of a selfconsistent Virgo-

centric infall model with a Local Group infall of $v_{\text{LG}}=220 \text{ km s}^{-1}$ (following Kraan-Korteweg, 1985). If one plots $\log R$ versus $\log v_{220}$ one finds a seemingly linear relation (analogously to Fig.2). A more revealing plot of the numeric values of R versus v_{220} , however, discloses a definite non-linearity (Fig.5), which can be expressed in the form $R = a \cdot v_{220}^b$ with $a=0.0430$ and $b=0.811$. This non-linearity, requiring an asymptotic value of $H_0 > 110$, is naturally explained by the Malmquist effect. The bias is here somewhat milder than for the Λ_C distances, as can be seen from a comparison of the exponents b . The procedure followed here to correct in first approximation for the bias is the same as in Section V. Individual Hubble ratios H_i are determined for each galaxy and these are plotted versus v_{220} (Fig.6). A linear regression then yields

$$H_i = 78.8 + 0.0117v_{220}. \quad (2)$$

This suggests a zero-bias value of $H_0=79$, which is based, however, on only three calibrating galaxies, viz. M31, M33,

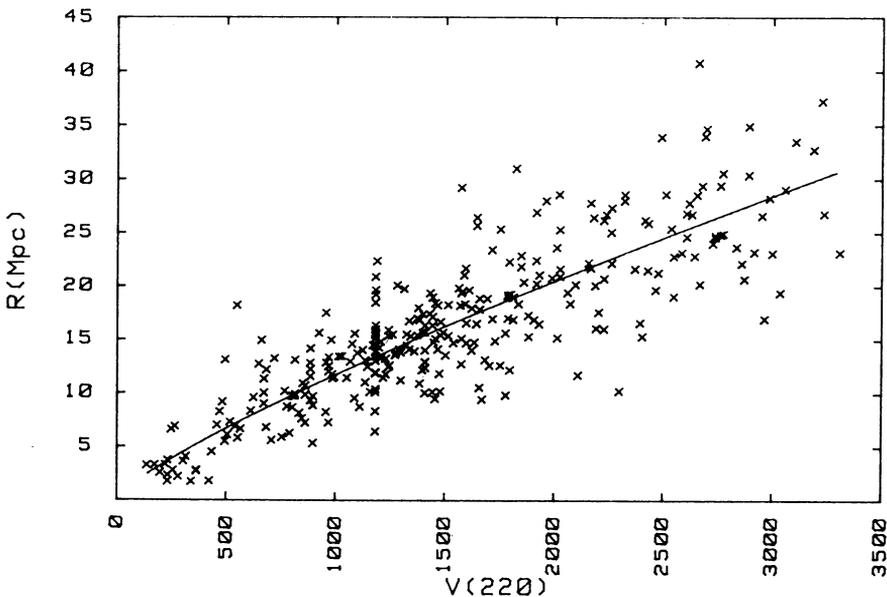


Fig.5. A plot of the IR 21cm-line width distances of 308 field and Virgo cluster spirals against the corrected recession velocity v_{220} . The distances are calculated following the precepts of Aaronson et al. (1986). Note the non-linearity of the relation.

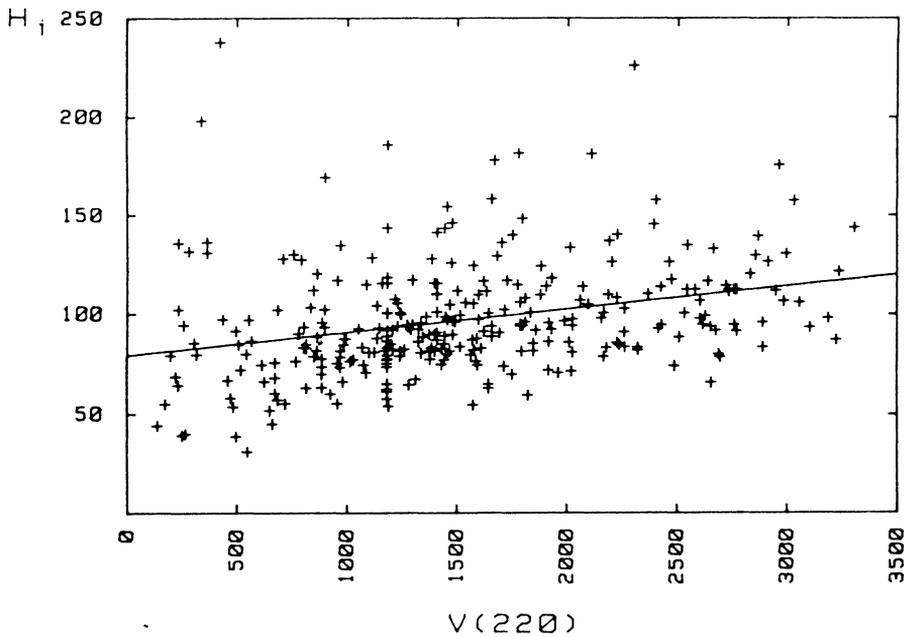


Fig.6. The Hubble ratios H_i for individual galaxies with known distances from IR magnitudes and 21cm-line widths. Due to the Malmquist effect the values of H_i increase with velocity/distance.

and NGC 2403 (Aaronson et al., 1986). One of these calibrators, M31, may have an unreliable IR magnitude (Manousoyanniki and Chincarini, 1986). A safer procedure is to use as many calibrators as possible. In Table 3 the distance moduli $(m-M)_{Aa}$ are listed as obtained from the precepts of Aaronson et al. (1986) and they are compared to the new distance moduli repeated from Table 1. The mean difference of $(m-M) = -0^m.75$ requires a stretching factor of 1.4, i.e. $H_0 = 79:1.4 = 56$.

The details of this discussion may be debated. For instance, one could criticize that the quadratic $M_{IR}/\Delta v_{21}$ relation should not be used for the intrinsically fainter galaxies but rather the linear relation of Aaronson et al. (1982a). If this second possibility is followed up one finds a zero-bias value of $H_0 = 72$ and after correction to the "new" zero-point $H_0 = 60$. Another objection might be that only calibrators should be used with $\Delta v_{21} > 200 \text{ km s}^{-1}$; the nine remaining calibrators would then give $H_0 \sim 70$.

Table 3: The Calibrators of Aaronson et al.(1986)

Galaxy	$(m-M)_{Aa}$	$(m-M)_{new}$	$(m-M)$
M31	<u>24.12</u>	24.2	-0.08
M33	<u>24.17</u>	24.4	-0.23
NGC 247	27.05	26.7	+0.35
NGC 253	27.22	27.5	-0.28
NGC 7793	27.61	27.5	+0.11
NGC 2366	26.20	27.8	-1.60
NGC 2403	<u>27.57</u>	27.8	-0.30
NGC 3031	<u>27.57</u>	28.7	-1.13
IC 2574	26.22	27.8	-1.58
NGC 4236	27.58	27.8	-0.22
NGC 5204	27.16	29.2	-2.04
HoIV	26.69	29.2	-2.51
NGC 5585	28.92	29.2	<u>-0.28</u>
			-0.75±0.25

Note: Only the underlined distances are used by Aaronson et al. (1986) as calibrators, but the distances of the remaining galaxies are implied by their calibrated eq.4.

These remarks already show that the $M_{IR}/\Delta v_{21}$ data leave considerable margin for interpretation. A compromise value may be $H_0=65\pm 15$. If anything, this margin is increased by the impossibility to apply a rigid correction for the Malmquist bias; that the optical and IR Tully-Fisher relations have considerable intrinsic scatter (viz. $\sigma_M=0^m.7$ according to Rubin et al., 1985), and must therefore be vulnerable to the Malmquist effect, is not new (cf. Bottinelli et al., 1986; Kraan-Korteweg et al., 1986; Giraud, 1985, 1986). Other unsolved problems of the Tully-Fisher relation concern its quadratic or linear shape and its observed type dependence (Roberts, 1978; Giraud, 1985; Rubin et al., 1985; Kraan-Korteweg et al., 1986). To take the Tully-Fisher relation as the one reliable distance indicator would therefore be a misrepresentation. The method will have to be replaced eventually by physically more meaningful kinematical para-meters (Persic and Salucci, 1986).

The IR Tully-Fisher relation has also been applied to ten clusters beyond the Virgo cluster (Aaronson et al., 1986). The resulting distances relative to Virgo carry less of a Malmquist bias, i.e. they define a more nearly linear velocity field beyond Virgo, because the distant clusters were sampled to fainter magnitudes than the Virgo cluster. It was mentioned in Section IV that a consistent change of the apparent limiting magnitude is indeed a powerful remedy against Malmquist bias.

VII. THE LINEARITY OF THE LOCAL EXPANSION FIELD

In the two previous Sections the demonstration of the Malmquist effect relied on the systematic deviations from a linear expansion field. This made the tacit assumption that the true expansion field is linear. The evidence for a truly linear Hubble flow at a scale of $v < 5000 \text{ km s}^{-1}$ is given here.

The linearity of the expansion field can be tested with relative distances only, i.e. with standard candles for which an arbitrary absolute magnitude can be adopted. SNe Ia at maximum light and first-ranked galaxies in groups are ideal for this purpose, because they have not only small intrinsic scatter σ_M , but their discovery is also nearly independent of their luminosity as argued in Section IV, making them insensitive to Malmquist bias.

For 49 SNe Ia with $v_{220} < 5500 \text{ km s}^{-1}$ blue maximum magnitudes are known (Cadonau 1986). They are corrected for Galactic absorption and, in the case of the 38 SNe Ia in S/Im galaxies, for intrinsic absorption in the parent galaxy. For the latter correction a preliminary absorption law of $A_B = E(B-V)$, as evidenced by IR data, was adopted. No absorption correction within the parent galaxy was applied for the 11 SNe Ia in E/S0 galaxies. Plotted into a Hubble diagram the resulting values of $m_B^0(\text{max})$, combined with their respective corrected recession velocities v_{220} , define a Hubble line of the form

$$m_B^0(\text{max}) = 5 \cdot \log v_{220} - 3.32. \quad (3)$$

It was seen before that the (logarithmic) Hubble diagram is insensitive to a linearity test. An arbitrary absolute magnitude of SNe Ia at maximum of $M_B(\text{max}) = -20^m0$ was therefore adopted, and the corresponding linear distances R were calculated. They are plotted versus v_{220} in Fig.7.

Corrected magnitudes m_V are published for 38 first-ranked galaxies in groups and clusters with $v_{220} < 5500 \text{ km s}^{-1}$ (Sandage 1975). They define a Hubble line of

$$m_V(1) = 5 \cdot \log v_{220} - 6.82 \quad (4)$$

A comparison of eq.(3) and (4) shows that $M_V(1)$ is 3^m50 brighter than $M_B(\text{max})$ for SNe Ia, for which an arbitrary value of -20^m0 was adopted. If one therefore adopts for first-ranked galaxies $M_V(1) = -23^m50$, it will put them on the same (arbitrary) distance scale. With this precept linear distances were calculated for the first-ranked galaxies and they were added to Fig.7.

Inspection of Fig.7 puts stringent limits on any systematic deviations from a linear expansion field. This proves that the local Hubble flow is linear and justifies

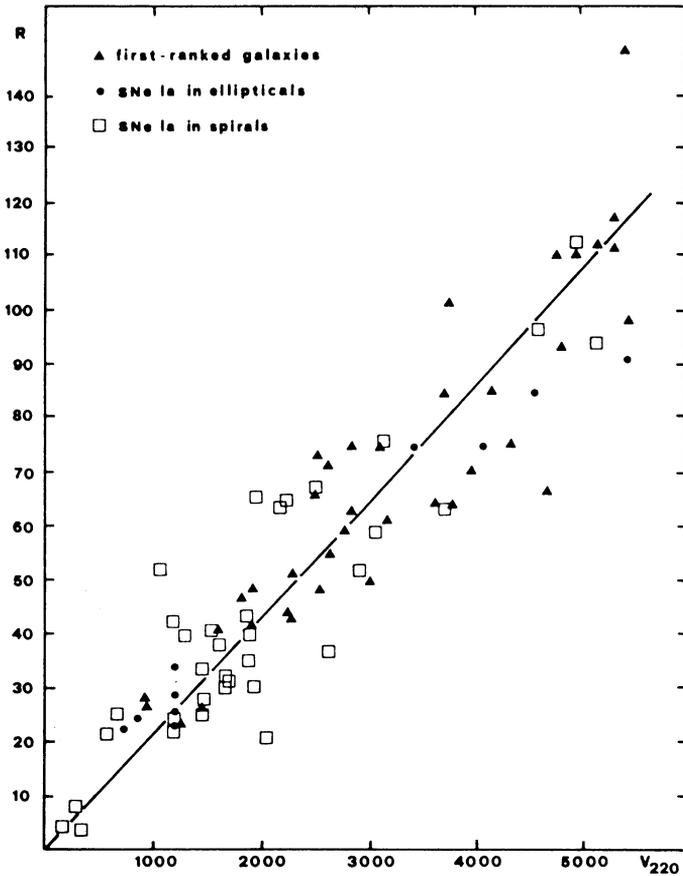


Fig.7. The distance-velocity diagram of SNe Ia and first-ranked cluster galaxies in groups and clusters. The full drawn line corresponds to an arbitrary Hubble constant of 46.1, which follows from the choice of $M_B(\text{max}) = -20^m.0$ for SNe Ia and correspondingly of $M_V(1) = -23^m.50$ for first-ranked galaxies. The diagram puts strong limits on any non-linearity of the local expansion field.

the procedure in Sections V and VI, where the deviations from linearity were charged to the Malmquist bias.

In passing it is noted that the random scatter in Fig.7 is considerable. This scatter can be interpreted as a scatter in distance or in velocity or as a combination of both. If the scatter is read in distance it corresponds to a mean error of $\Delta r/r \sim 0.25$ or $\sigma(\text{mag}) = 0^m.5$, which can be due to

luminosity scatter of the standard candles and/or to random photometric errors. If on the other hand, one assumes (unrealistically) that the total scatter is caused by peculiar motions of the individual objects, it follows $\sigma(v_{220})=550 \text{ km s}^{-1}$. This value is an upper limit of the mean one-dimensional peculiar motion of field galaxies, groups and clusters. The corresponding three-dimensional value of $\sigma(v_{220}) < 950 \text{ km s}^{-1}$ can be compared with the absolute peculiar velocity of 600 km s^{-1} of the Local Group with respect to the microwave background. This suggests then, that our peculiar motion may be quite typical.

VIII. THE ABSOLUTE MAGNITUDE OF SNe Ia AT MAXIMUM

The evidence for the absolute magnitude $M_B(\text{max})$ of SNe Ia has been recently compiled by Cadonau, Sandage, and Tammann (1985). Therefore only a brief, slightly updated review is given here.

For two SNe, presumably of type Ia, distances are known from the brightest stars of their parent galaxies (Sandage and Tammann, 1982). Their somewhat revised data are given in Table 4.

Table 4: Data for two SNe Ia with known distances of their parent galaxies

SN	Galaxy	(m-M)	$m_B(\text{max})$	(B-V)	E_{B-V}	$m_B^O(\text{max})$	$M_B^O(\text{max})$
1937c	IC 4182	28.21	8.79	+0.02	0.29	8.50	-19.71
1954a	N 4214	28.92	9.79	-0.03	0.24	9.55	<u>-19.37</u>
							-19.54

The apparent magnitudes $m_B(\text{max})$ and colors (B-V) are taken from Cadonau (1986), as well as the intrinsic color $(B-V)_{\text{max}} = -0^m.27$. As in Section VII it was assumed that the resulting reddening E_{B-V} is equal to the blue absorption A_B from within the parent galaxy. Note that this absorption correction is very conservative; a conventional law of $A_B=4 E_{B-V}$ would bring the mean absolute magnitude from $M_B(\text{max}) = -19^m.54$ to $-20^m.34$. In addition, if one or both SNe in Table 4 had not been of type Ia, this type, as the brightest known, could only be still brighter.

The absolute magnitude of two historical SNe, Tycho and Kepler, was estimated to be $-20^m.0 \pm 0^m.6$ (Cadonau et al., 1985). This implies distances of the SN remnants of 4.4 and 3.6 kpc, respectively, as can be seen from data given elsewhere (Tammann 1982). These distances lie within the limits of current determinations, which disagree however by factors of almost 2. The calibration from historical data is

therefore still weak. However, in principle this calibration sets a lower limit, because if the Tycho and/or Kepler SNe had not been of type Ia, they could yield only too low a luminosity.

From the angular extent of the supernova remnant of the type II SN 1979c, measured with VLBI radio techniques, and the optical expansion velocity Bartel et al. (1985) have determined a Virgo modulus of 31.4 ± 0.8 . The large error is due to the difficulty to combine the radio and optical data, but the distance has been closely confirmed by Chevalier und Fransson (1985) who studied the SN interaction with a circumstellar wind. Because the seven SNe Ia in Virgo have $\langle m_B(\max) \rangle = 12^m.24$, a value of $M_B(\max) = -19^m.2 \pm 0^m.8$ follows.

A Baade-Becker-Wesselink parallax of a SN Ia yields $M_B(\max) = -20^m.5$ with an internal error of $\pm 0^m.6$ (Branch et al., 1983). This value should be made brighter by $\sim 0^m.5$, because the canonical time between explosion and B maximum of 15 days should now be revised to > 20 days (Cadonau, 1986). From a type II SN a Virgo cluster distance of 23 ± 3 Mpc was found consistently by different authors (Panagia et al., 1980; Branch et al., 1981; Kirshner 1985), which implies from the argument in the foregoing paragraph $M_B(\max) = -19^m.6$ for SNe Ia. The quantitative exploitation of SN expansion parallaxes poses still considerable difficulties, but a mean value and error of $M_B(\max) = -20^m.2 \pm 0^m.7$, stemming from two types of SNe, may be realistic (cf. also Branch 1985).

Physically determined distances of SNe Ia follow also from deflagrating C/O White Dwarf models producing radioactive ^{56}Ni . The models of several authors give $M_B(\max) = -19^m.6$ with larger margins toward brighter magnitudes than vice versa (Sutherland and Wheeler, 1984; Arnett, Branch, and Wheeler, 1985; Nomoto 1986). Converting the bolometric luminosity at maximum of $2.5 \cdot 10^{43} \text{ erg s}^{-1}$ by Woosley, Taam, and Weaver (1986) into B magnitudes according to Arnett et al. (1985) gives $M_B(\max) = -20^m.2$ ($-0^m.3$; $+0^m.8$). Thus the deflagration models seem to require $M_B(\max) = -19^m.8 \pm 0^m.6$.

The five routes to $M_B(\max)$ of SNe Ia are all still inflicted by considerable uncertainties, but it is encouraging in view of their complete autonomy that they are fully consistent with a single value of $M_B(\max) = -19^m.7 \pm 0^m.4$. We can now return to eq. (3) to find that a SN Ia has, e.g. at $v_{220} = 5000 \text{ km s}^{-1}$ an apparent magnitude of $m_B(\max) = 15^m.17$, corresponding to a distance modulus of $(m-M) = 34.87$ or 94.4 Mpc. All existing evidence of SNe Ia, including Baade-Becker-Wesselink parallaxes of SNe II, points therefore to a global (i.e. large-scale) value of $H_0 = 53 \pm 11$.

IX. THE DISTANCE OF THE VIRGO CLUSTER

Simple arguments suggest that the Virgo cluster is roughly three times more distant than the M101 group (Sandage and Tammann, 1976), i.e. at a distance of ~ 20 Mpc. The arguments involve, besides 21cm-line widths, the brightest stars, H II region sizes, luminosity classes, galaxian magnitudes and diameters, as well as the velocity ratio. Indeed from more modern data the ratio $v_{220}(\text{Virgo})/v_{220}(\text{M101 group})$ becomes 2.9 (Kraan-Korteweg 1985). This points to a Virgo distance of 16-26 Mpc if allowance is made for a peculiar group velocity of $\pm 100 \text{ km s}^{-1}$.

On the basis of recent data the blue-magnitude/21cm-line width relation gives a Virgo modulus of 31.70 (Richter and Huchtmeier, 1984) and 31.40 (Sandage and Tammann, 1984). These values may be low by $\sim 0^m.25$, because the Virgo spirals may have been overcorrected for intrinsic absorption (van den Bergh, 1984). Visvanathan (1983), however, confirms the conventional absorption corrections by using Tully-Fisher relations for optical magnitudes and for a magnitude measured at $1.05 \mu\text{m}$. He finds a Virgo modulus of 31.33. - The three distance moduli in this paragraph are all reduced to the new zero-point defined in Table 1. The resulting mean of $(m-M)_{\text{Virgo}} = 31.5$ is in principle a lower limit because no Malmquist correction has been applied.

A bias-free distance of Virgo can be derived in the following way from IR magnitudes and 21cm-line widths. For all 15 Sbc, Sc galaxies with inclinations $i > 45^\circ$ from the Revised Shapley-Ames Catalog (Sandage and Tammann,; RSA), which are certain Virgo cluster members, IR magnitudes and 21cm-line widths Δv_{21} are available. The sample is therefore unbiased in Δv_{21} (but of course not in M_{IR}). Treating M_{IR} as the independent variable should therefore lead to an unbiased Virgo distance, which in particular should be free of any Malmquist bias. The field galaxies with IR magnitudes (Aaronson et al. 1982) and 13 of the calibrators of Table 1 determine the following inverse Tully-Fisher relation

$$\log \Delta v_{21} = -0.0780 M_{\text{IR}} + 0.820 (\pm 0.017). \quad (5)$$

This relation is not strictly correct, because the field galaxies constitute a sample which is still biased in Δv_{21} (Kraan-Korteweg et al., 1986), and this will influence the slope of eq. (5). With a blind eye for the remaining bias, we derive with the 15 Virgo spirals a cluster modulus of 31.57 ± 0.25 . This somewhat crude application of the inverse Tully-Fisher relation shows, that the IR magnitudes and 21cm-lin widths do not lead to low Virgo cluster distances, but rather that they yield a marginally higher modulus than the blue Tully-Fisher relation.

It was stated before that the seven SNe Ia in the Virgo cluster have $\langle m_B(\max) \rangle = 12^m.24 \pm 0^m.19$. Combining this with the calibration in the previous Section gives a cluster modulus of 31.94 ± 0.44 .

Globular clusters have resisted for a long time to yield a meaningful Virgo distance. However, the recent discovery of the turnover of the magnitude distribution of the globulars in M87 at $B = 25^m.0 \pm 0^m.3$ leads, together with the assumption that the turnover lies at the same luminosity as in the Local Group, to a Virgo modulus of 31.43 ± 0.3 (van den Bergh, Pritchett, and Grillmair, 1986).

Very recently Dressler (1986) has discovered that the relation between a photometric diameter and the central velocity dispersion, known for ellipticals, can be extended to the bulges of early-type spirals. From this relation he finds a Virgo modulus of 31.64 ± 0.3 if M31 is used as the sole calibrator, or $31.5 < (m-M) < 32.2$ if M31 and M81 are used for the calibration. In the latter case the lower modulus corresponds to the conventional assumption that M81 lies at the distance of NGC 2403, while the higher modulus follows from Sandage's (1984a) larger M81 distance.

The foregoing distance determinations are in excellent agreement with an adopted Virgo cluster modulus of $(m-M) = 31.6 \pm 0.3$ or $r = 20.9 \pm 3.0$.

The unquestionable Virgo members define a mean velocity of $v_o = 976 \pm 67 \text{ km s}^{-1}$; higher values in the literature are due to outlying galaxies which belong to the background as judged from their morphological appearance (Binggeli, Sandage, and Tammann, 1987). Increasing this value by our Virgocentric infall velocity of $v_{vc} = 220 \pm 50 \text{ km s}^{-1}$ (Tammann and Sandage, 1985) we obtain $v_{220}(\text{Virgo}) = 1196 \pm 84 \text{ km s}^{-1}$. From this follows a Hubble constant at the distance of the Virgo cluster of $H_o = 57 \pm 9$.

The Virgo cluster is quite rigidly tied into the global expansion field by relative distance indicators. From this it was concluded that $H_o = (50 \pm 7)(21.6/r_{\text{Virgo}})$ (Tammann and Sandage, 1985). With the present distance of $r_{\text{Virgo}} = 20.9$ one finds $H_o = 52 \pm 10$.

To illustrate this point, one may use for instance the well determined relative distance between the Virgo cluster and the Coma cluster (Table 5) to obtain $r_{\text{Coma}} = 119 \pm 18 \text{ Mpc}$ and with $v_{220}(\text{Coma}) = 7217 \pm 400 \text{ km s}^{-1}$ a large-scale Hubble ratio of $H_o = 61 \pm 10$. The error of the Coma cluster velocity is to allow for a possible one-dimensional peculiar velocity of the cluster of 400 km s^{-1} .

Distances relative to Virgo are also available for ten clusters with IR and Δv_{21} data (Aaronson et al. 1986). With the present Virgo distance they require $H_o = 65$. This is rather an upper limit to H_o because the clusters were not sampled to the same absolute magnitude, as mentioned before, and some remaining Malmquist bias is to be expected.

Table 5: The Modulus Difference Coma-Virgo

(m-M)	Author(s)	Remarks
3.87±0.13	Dressler (1984)	1
3.90±0.10	Tammann and Sandage (1985)	2
3.69±0.12	Aaronson et al. (1986)	3
3.80±0.06	Vader (1986)	4
3.60±0.10	Lynden-Bell (1986)	5
3.81±0.12	Lucey (1986)	6
3.78±0.05		

Remarks:

1. Mean of luminosity/velocity dispersion and luminosity/Mg index relations.
2. Mean from various authors and methods (excluding Dressler 1984).
3. From IR Tully-Fisher relation.
4. From multivariate analysis of E galaxies.
5. From velocity dispersion/diameter relation.
6. From central-luminosity/velocity dispersion relation.

Along still another route the adopted Virgo and Coma distances define the absolute magnitude of first-ranked cluster galaxies to $M_V(1) = -23^m.37$ and $-23^m.56$, respectively, using apparent magnitudes from Sandage (1975). Combined with eq. (4) the mean value of $M_V(1) = -23^m.56 \pm 0^m.4$ yields $H_0 = 45 \pm 9$. Although the value relies on only two calibrators, it is in principle a very powerful determination, because eq. (4) is defined out to $v \sim 10\,000 \text{ km s}^{-1}$ (Sandage 1975) and must describe the truly global expansion.

X. THE GLOBAL VALUE OF H_0

Field galaxies out to $v \sim 3000\text{--}5000 \text{ km s}^{-1}$ give an estimate of the Hubble constant of 59 ± 20 and 65 ± 15 from the Λ_c distance indicator and from the IR Tully-Fisher relation, respectively, if one allows in first approximation for Malmquist bias (cf. Sections V and VI).

An impartial test for the presence of a Malmquist bias is provided by the inverse Tully-Fisher relation, as mentioned before. It is uncontested that the regression of Δv_{21} on M is independent of the sample selection in M ; it requires, however, that the sample is unbiased in Δv_{21} . The practical difficulty is to define objectively a sample of field galaxies with complete Δv_{21} data. An approximation is provided by the 147 Sbc, Sc galaxies with $m_B < 12^m.0$ and inclination $i > 45^\circ$ from the RSA; for 136 of these galaxies 2lcm-line

widths are available and they form hence an almost complete sample, which is hopefully unbiased in Δv_{21} (cf. Kraan-Korteweg et al., 1986). The 136 galaxies define a regression

$$\log \Delta v_{21} = -0.082M_B + 0.88, \quad (6)$$

if $H_0 = 55$ is assumed. Ten calibrating RSA galaxies from Table 1 require a constant term of 0.862 ± 0.025 , which translates into $H_0 = 50 \pm 8$. However, five of the calibrators are of type Sd; if they should be excluded, the constant term becomes 0.906 ± 0.022 and $H_0 = 64 \pm 8$. These results are in satisfactory agreement with the de-biased values above.

SNe Ia in field galaxies out to $v \sim 5000$ km s⁻¹ give an independent and unbiased determination of $H_0 = 53 \pm 11$ (cf. Section VIII).

The combined evidence from field galaxies is therefore well satisfied by $H_0 = 55 \pm 10$.

The distance scale building on clusters yields a Hubble constant at the Virgo cluster distance of $H_0 = 57 \pm 9$ and a value of $45 < H_0 < 65$ for clusters out to $v \sim 10\,000$ km s⁻¹.

The agreement of the distance scales from field galaxies and from cluster galaxies is very satisfactory. It suggests that no major systematic errors dominate the solution. Errors due to selection effects propagate differently through the field and cluster distance scales, because field galaxies (mainly from the RSA) and cluster galaxies (from specific catalogs) are typically sampled to different apparent-magnitude limits.

The impossibility of the short distance scale with $H_0 \sim 100$ becomes most obvious just from the discrepancy between clusters and field galaxies. Even if the cluster scale may appear as linear, and to define a single value of H_0 , the value of H_0 from field galaxies agrees at one specific distance only, being lower at smaller distances and higher at larger distances.

With a consistent, linear distance scale at hand, which reaches out to $v = 5000 - 10\,000$ km s⁻¹ and defines $H_0(\text{global}) = 55 \pm 10$, a rather unexpected result emerges. If one determines the Hubble constant from the calibrating galaxy M101 alone (Table 1; $v_{220} = 415$ km s⁻¹), one finds $H_0 = 60$. In a detailed analysis of the local expansion field, including several additional dwarf galaxies with known distances, Sandage (1986d) has drawn the same conclusion, viz. $H_0(\text{local}) = 55$. It is surprising that in the presence of large galaxy clusterings and of considerable streaming velocities the local value of H_0 is the same as $H_0(\text{global})$ within the observational errors.

XI. CONCLUSION

The available data, involving various methods and different authors, impose consistently a Virgo cluster modulus of 31.6 ± 0.3 and a global value of the Hubble constant, derived independently from clusters and field galaxies, of $H_0 = 55 \pm 7$. The mean errors quoted here are estimates, because in the presence of possibly remaining systematic errors they cannot be calculated in a formal way. The error of the Hubble constant defines a probably realistic 99% confidence interval of $35 < H_0 < 75$.

The so-called short distance scale with $H_0 \sim 100$ implies discordant scales for clusters and field galaxies, which is obviously the result of a well understood selection effect (Malmquist bias). A first-order correction for this bias removes the discrepancy from the above results.

The short distance scale requires all distant extra-galactic objects to be fainter by $\sim 1^m.5$ than presently assumed. This difference is not caused by major differences of the adopted local calibrators. The bulk of the difference builds up between the local calibrators and the Virgo cluster, which the short distance scale locates almost 1^m nearer than in the case of $H_0 = 55$. Taken at face value the short distance scale would create a number of artificial problems, e.g.:

- It would imply $M_B(\text{max}) = -18^m.3$ for SNe Ia. In that case at least four historical SNe would have occurred within 2.5 kpc during the last millennium (cf. Tammann 1982, Table 4), which corresponds for the whole Galaxy to an unparallelled frequency of 1 SN per < 10 years.
- The Galaxy and M31 would have the largest diameters among the galaxies in the Local Supercluster (van der Kruit 1986).
- Primordial nucleosynthesis is most easily understood with a baryon density of $\Omega_0 \sim 0.1 (50/H_0)^2$ (Rees 1986). With $H_0 = 100$ one obtains $\Omega_0 = 0.025$, which is about four times less than required to bind clusters of galaxies (Sandage and Tammann, 1984b). In that case the clusters were bound by mainly non-baryonic matter.
- The maximum Friedman time would be $1/H_0 = 10^{10}$ years, i.e. less than the age of globular clusters (Sandage 1982; Renzini 1986). Therefore $H_0 \sim 100$ excludes a priori all Friedman models.

No such problems arise with $H_0 \sim 50$. It seems to require superluminal velocities up to $\sim 10c$ in some radio sources (Porcas 1985), but a value of $5c$ in the case of $H_0 \sim 100$ poses in principle similar constraints on the models.

As the historical evidence shows, it is much easier to measure too high a value of H_0 than too low a value. Also future corrections will probably tend to lower H_0 even further. Very little work has been done to determine an

absolute lower limit of H_0 , but if $H_0 < 40$ our present understanding of galaxian parameters and SN luminosities would rather be hindered than helped. For all practical purposes the convenient number of $H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$ is recommended; it will probably take a long time before a significantly different value becomes necessary.

ACKNOWLEDGEMENTS. Because this Conference coincides within a few weeks with Allan Sandage's 60th birthday it is appropriate to recollect here how much he has contributed to our present topic. I could accompany him for a part of his work on the distance scale for almost 25 years; for this privilege I am deeply grateful.

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DISCUSSION

M.Aaronson: I have a comment and a question. First the comment: It might have been fair for you to point out that our reason for collecting the nearby field galaxy data was not to study H_0 but to analyze deviations from uniform Hubble flow. I believe our analysis of potential bias in the latter problem was correct. As far as H_0 goes, we have always emphasized working with clusters, as this avoids difficulties in knowing how to properly treat magnitude bias effects or bias effects from streaming motions within the Local Supercluster. (Nevertheless, one would expect the field and cluster samples to yield consistent results. In fact they do! Both sets of data give a similar value for Virgocentric motion). Now the question: Could you tell me what paper of mine you purported to have taken distances from in the table you showed?

G.A.Tammann: It is, of course, interesting for me to learn about the reason for your collecting the data. But since you have published them, and since you have published formulae to derive absolute magnitudes from them, I deemed it fair to do just this. I chose your quadratic equation (Aaronson et al., 1986, *Ap.J.*302, 536, eq.4); I will try in addition your earlier linear version in the written version of my talk. I hope it will not make too much difference, because otherwise the IR Tully-Fisher method would depend on details which are difficult to control. - You cannot test the linear distance scale by the Virgocentric motion because it depends only on relative distances. Finally you mention a central point of our disagreement: I cannot see how nearby calibrating galaxies and distant Virgo cluster galaxies, all obeying the same apparent-magnitude limit of the Shapley-Ames catalog, could have the same mean properties.

M.Aaronson: The reason for the big discrepancies with the low luminosity galaxies in the table you showed was a result of your extrapolation of the quadratic form of the relation down to the low luminosity regime. As we discussed in our last paper, where we introduced this quadratic form, this extrapolation should not be made. (One should also confine the calibrating galaxies to the same velocity width range as the more distant cluster galaxies, as we have done).