ECONOMIC ASPECTS OF SECURITIZATION OF RISK

BY

SAMUEL H. COX, JOSEPH R. FAIRCHILD AND HAL W. PEDERSEN [1]

ABSTRACT

This paper explains securitization of insurance risk by describing its essential components and its economic rationale. We use examples and describe recent securitization transactions. We explore the key ideas without abstract mathematics. Insurance-based securitizations improve opportunities for all investors. Relative to traditional reinsurance, securitizations provide larger amounts of coverage and more innovative contract terms.

KEYWORDS

Securitization, catastrophe risk bonds, reinsurance, retention, incomplete markets.

1. INTRODUCTION

This paper explains securitization of risk with an emphasis on risks that are usually considered insurable risks. We discuss the economic rationale for securitization of assets and liabilities and we provide examples of each type of securitization. We also provide economic axguments for continued future insurance-risk securitization activity. An appendix indicates some of the issues involved in pricing insurance risk securitizations. We do not develop specific pricing results. Pricing techniques are complicated by the fact that, in general, insurance-risk based securities do not have unique prices based on axbitrage-free pricing considerations alone. The technical reason for this is that the most interesting insurance risk securitizations reside in *incomplete* markets.

A market is said to be *complete* if every pattern of cash flows can be replicated by some portfolio of securities that are traded in the market. The payoffs from insurance-based securities, whose cash flows may depend on

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[1] Please address all correspondence to Hal Pedersen.

hurricanes, earthquakes and so on, cannot be closely approximated by a portfolio of the traditional assets that are already traded in the market such as stocks and bonds. This is because there are states of the world reflected in insurance-based securities that are not reflected by the existing traditional securities. In a complete market a new security can always be priced relative to existing securities by finding a replicating portfolio and pricing it. The no-arbitrage property implies that the new security and the replicating portfolio must have the same price because they have the same payoffs. However, if the market is incomplete a replicating portfolio may not exist and arbitrage considerations alone may not determine a unique price. The appendix describes a method for dealing with incompleteness. In the main body of the paper we do not discuss arbitrage based pricing theory further but the reader who will ultimately be involved in the pricing of these products should bear in mind that there are fundamental practical differences between products to be valued in complete markets and products that are valued in incomplete markets. Incompleteness is one of the unusual characteristics of insurance-based securities relative to many other securitizations. It is very interesting to note that the fundamental reason insurance risk securitizations tend to reside in incomplete markets – namely that states of the world reflected in insurance based securities are not reflected by the existing traditional securities, is also the fundamental reason why these securities provide diversification of investment risk and thereby make these attractive investments for many portfolio managers. Although we will not explicitly use the notion of incompleteness in the main body of the paper because the focus of this paper is not on technical valuation, an actuary involved in these securitization deals must be aware of these fundamental pricing issues.

Two actuarial principles, diversification and contractual risk transfer, play important roles in most securitizations, yet relatively few actuaries work in the securitization business. It seems that the opportunities for actuaries in securitization will increase and we may see more actuaries working in this field in the future.

We begin this paper with an idealized catastrophe property risk securitization. This example illustrates the key ideas without abstract mathematical or financial theories. We hasten to emphasize that although the key ideas of securitization can be illustrated without these theories, the practical implementation of a securitization deal requires financial theory for pricing and risk measurement. As a broad definition, securitization means "the bundling or repackaging of rights to future cash flows for sale in capital markets." In all the cases we mention here, and more generally in all of the deals we know of, the repackaging provides a more efficient allocation of risk. This process can be costly, but evidently the reallocation is valuable enough to make it worthwhile.

After describing this simple example, we turn to the common features of securitization and then review some recent catastrophe risk securitizations. We compare catastrophe risk securitizations with the asset securitizations: bond strips, mortgage-backed securities, life insurance policyholder loans, and life insurance premium loadings. The Chicago Board of lyade offers options based on property insurance loss ratios. We mention them only to contrast them with catastrophe risk securitizations. We discuss some possible future uses of securitization of insurance risks. The paper ends with a discussion of the economics of securitization. We offer a discussion of the reasons for these transactions and attempt to answer the questions:

- Why do investors buy insurance-based securities?
- Why do insurers use securitizations to cover insurable risks?

2. Securitization of Catastrophe Risk

We will give a simple illustrative idealization of catastrophe risk bonds – customarily referred to as cat bonds. During 1997 and 1998 there were successful catastrophe risk bond issues by USAA, Swiss Re, Winterthur, St. Paul Re, and others. Later we will provide an economic rationale for the supply (why to insurers sell cat bonds?) and demand (why do investors buy cat bonds?). For now we focus on the mechanics of these transactions.

We illustrate the model with two examples, first a single-period model and second a two-period model. In each example catastrophe risk has a binomial structure. There is no interest rate risk in either example. The market interest rate on risk-free securities is a constant 8% per year. The probability of a catastrophe that triggers a "default" is a constant 3% per year [1]. These values are merely to illustrate the mechanics of the transactions. In practice we would use the prevailing interest rate term structure and a model for insurance losses to determine the probabilities. Embrechts and Meister take this approach to develop a valuation model for exchange-traded insurance options [12].

Example 1. The first example is similar to the USAA bonds. The face amount is 100 and the annual coupon rate is 12%. Coupon and principle are at risk. This means that the principal and coupon are paid only if no catastrophe occurs during the period [0, 1]. The total principal and coupon 112 is paid at time 1 only if no catastrophe occurs during the period [0, 1]. The catastrophe states and probabilities, along with the corresponding cat bond cash flows are shown in Figure 1. The positive cash flow is paid to the bondholders; the negative cash flow is the price the bondholders pay to obtain the rights to the future cash flow.

[1] Failure to pay a coupon or to repay the principal because a catastrophe occurs is not a default in the legal sense. The catastrophic event is well-defined in the bond indenture and buyers and sellers understand the circumstances under which coupons and principal will not be paid. Nevertheless, it is convenient to refer to this event as a default.

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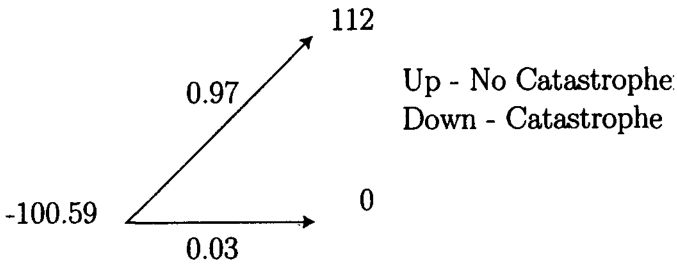

FIGURE 1: One-Period Catastrophe Risk Bond Cash Flow.

The expected bondholder payments, averaged over the catastrophe distribution, are $\bar{c}(1) = 112(0.97) + (0)(0.03) = 108.64$. The discounted expected value, using the constant 8%, is the price of the cat bond:

$$\frac{1}{1.08}[108.64] = 100.59$$

Consider a bond that has the same prospective cash flow (*i.e.*, 12% coupon), but no possibility of default. This is called a *straight bond.* The price of the straight bond at the time the cat bond is issued is found by discounting the cash flow:

$$\frac{1}{1.08}[112] = 103.70$$

The cash flows of the straight bond are shown for comparison to the cat bond in Figure 2.

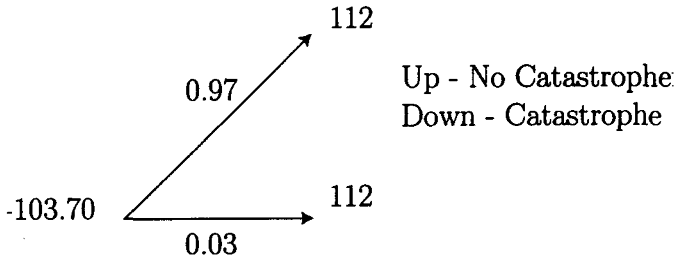

FIGURE 2: One-Period Straight Bond Cash Flow.

Suppose an insurer (like USAA) issues the cat bond and simultaneously buys the straight bond. The straight bond is more expensive. The trades cost the insurer 3.11 per 100 of face value (ignoring transactions costs). What does the insurer get in return? If there is no catastrophe, the insurer's net cash flow is zero because it receives the straight bond coupon and pays the cat bond coupon. However, if there is a catastrophe, it still receives the straight bond coupon and principal (112), but does not pay the corresponding cat bond cash flow. In effect, the insurer has purchased a one-year catastrophe reinsurance contract which pays 12 in case a catastrophe occurs during the period. This increases the insurers capacity to sell insurance for one year (just as a traditional reinsurance does) by 112 at a cost of 3.11 per 100 of bond face value. The rate on line [1] for this "synthetic" reinsurance is $100 \times 3.11/112 = 2.78$ per 100 of coverage per year. The net cash flow is shown in Figure 3.

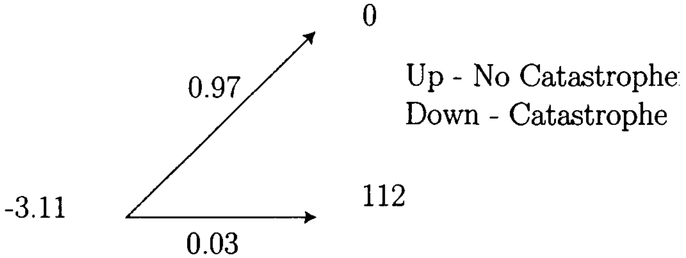

FIGURE 3: One-Period Net Cash Flow: Long Straight Bond and Short Cat Bond.

There are several multiple period cat bonds. The majority are essentially extensions of the concept illustrated in Example 1 in that the bond "defaults" as soon as a catastrophe occurs, regardless of when the catastrophe occurs. The bond indenture may specify that future coupon and principal payments to bondholders are forfeited as soon as a catastrophe occurs. Alternatively it may specify that coupons only are at risk or that coupons and a fraction of the principal is at risk. USAA actually issued one series with coupon only at risk and another with principal and coupon at risk. The Swiss Re [20] and Yasuda Marine [29] bonds have a single limit applicable over several years. The Winterthur bonds take yet another form allowing the limit to be reset each year. Our second example is like the bond Winterthur issued in 1997[2].

Example 2. Coupons only are at risk. This means that the principal of 100 is paid to the bondholder at $k = 2$ with probability one. A coupon of 12 is paid at $k = 1, 2$ provided no catastrophe occurs during the period $[k-1, k]$. The catastrophe states and probabilities, along with the corresponding cat bond cash flows are shown in Figure 4. The positive cash flows are paid to the bondholders, the negative cash flow is the price the bondholders pay to obtain the rights to future cash flows.

[1] For a one-year policy, *rate on line* is the ratio of premium to coverage layer, usually multiplied by 100. The concept is not usually applied to multiple year policies.

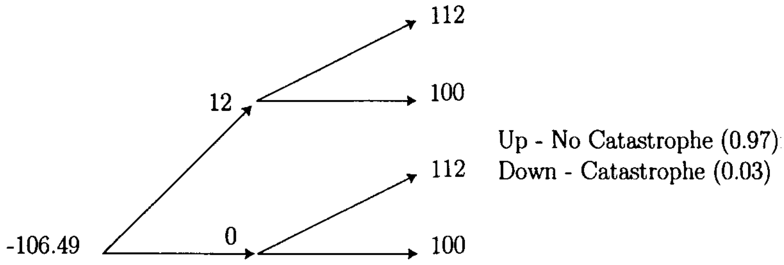

FIGURE 4: Two-Period Catastrophe Risk Bond Cash Flow.

As in the first example, the expected bondholder payments are $\bar{c}(1) = 12(0.97) = 11.64$ and $\bar{c}(2) = 100 + 11.64 = 111.64$. The discounted expected value is the price of the cat bond:

$$\frac{1}{1.08}\left[11.64 + 111.64\left(\frac{1}{1.08}\right)\right] = 106.49$$

Consider a bond that has the same prospective cash flow (i.e., 12% coupons), but no possibility of default. This is called a *straight bond.* The price of the straight bond at the time the cat bond is issued is found by discounting the cash flows:

$$\frac{1}{1.08}\left[12 + 112\left(\frac{1}{1.08}\right)\right] = 107.13$$

The cash flows of the straight bond are shown for comparison to the cat bond in Figure 5.

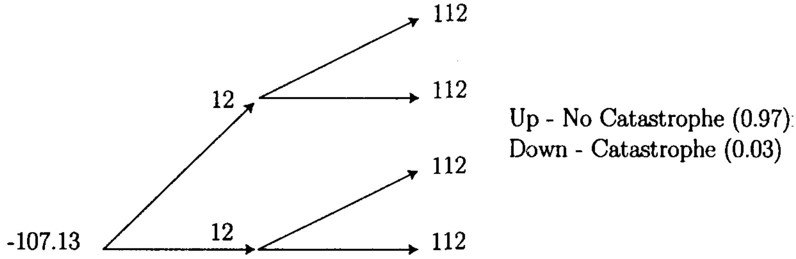

FIGURE 5: Two-Period Straight Bond Cash Flow.

As before, suppose an insurer (like Winterthur or Swiss Re) issues the cat bond and simultaneously buys the straight bond. The trades cost the insurer 0.64 per 100 of bond face value and provide 12 units of coverage per period. The "rate on line" is $100 \times 0.64/12 = 5.33$, but one must keep in mind that this is the rate paid once at the beginning of the policy period for a two year cover. If we must compare this to a one year policy, we should divide by two: $5.33/2 = 2.66$. In each of the two future periods, if there is no catastrophe, the insurer's net cash flow is zero because it receives the straight bond coupon and pays the cat bond coupon. However, if there is a catastrophe in either period, it still receives the straight bond coupon (12), but does not pay the cat bond coupon. In effect, the insurer has purchased a two year catastrophe reinsurance contract which pays 12 in case a catastrophe occurs during either period. This increases the insurers capacity to sell insurance for each of the next two years by 12 at cost of 0.64 per 100 of face value (or 5.33 single premium per 100 of coverage for a two year cover). The net cash flow is shown in Figure 6.

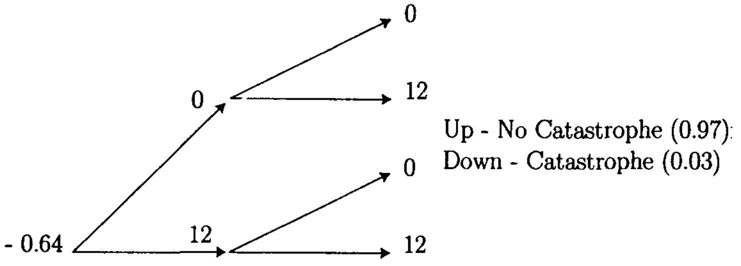

FIGURE 6: Net Cash Flow: Long Straight Bond and Short Cat Bond.

The actual deals we have described all increase the bond issuer's capacity. The technology required to issue cat securities is being developed and refined and thus the transactions costs of these deals will probably decrease in the future. Moreover, investors are becoming more familiar with the product which will have a further tendency to render future deals relatively less costly. Lastly, as others have pointed out [13], the insurance industry would be strained by a $50 billion hurricane loss, but the capital markets could withstand it with relative calm. Catastrophe bonds may become a routine method of transferring catastrophe risk. Practical considerations and economic theory would both predict this outcome.

It should be emphasized that the line of insurance is immaterial to the capital market – it does not have to be catastrophe risk. We will show later that investors will demand these bonds because their returns have low correlation with stock returns. There may be many kinds of insurance risks that have low covariance with the stock market. At the 1997 Swiss Actuarial Summer School held at the University of Lausanne we heard from Winterthur actuaries of a proposal to issue bonds which would transfer mortality risk to bondholders [1]. It seems intuitively clear to us that mortality risk has low covariance with the stock market and thus we expect these bonds would be attractive to investors. As we understand it, Winterthur has long term annuity liabilities and as a result faces the risk of unexpected improvement in beneficiary mortality. A security with bondholder cash flows

[1] In late 1999 we learned that three large international insurers are considering securitization of mortality risk.

tied to a mortality index would provide Winterthur with very long term coverage that is not available in the traditional reinsurance market. In the United States some companies offer very attractive term life insurance rates on selected lives in a very competitive market. There is little experience to indicate what the ultimate mortality will be for these select lives. Securitization would allow very long term coverage of the risk that ultimate mortality will diverge greatly from projected mortality for the selected lives.

3. Structure of Securitization

The securitization technology applies to many kinds of risk, not merely catastrophe risk. In asset and liability securitizations the common structure typically involves four entities: retail customers, a retail contract issuer, a special purpose company, and investors. In the case of catastrophe risk bonds, the four entities are as follows:

(1) Homeowners who buy policies from an insurer.
(2) The insurance company that issues the homeowners policies (*i.e.*, the retail contracts) and buys reinsurance from a special purpose reinsurer (*i.e.*, the special purpose company).
(3) The special purpose reinsurer that issues the reinsurance and sells bonds.
(4) Investors who buy the bonds.

Figure 7 illustrates the direction and timing of cash flows to and from each entity involved in or related to a securitization.

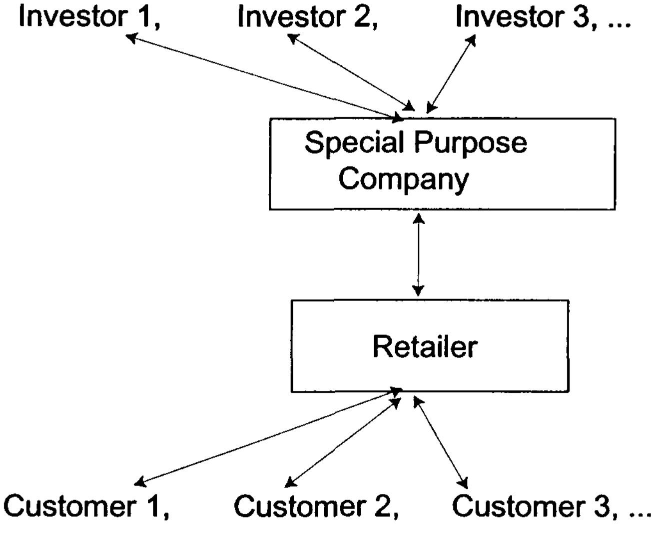

FIGURE 7: Securitization Components.

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Each of the arrows denotes an exchange of cash corresponding to a contract. The timing varies with the application. For example, in the case of homeowners insurance, the customers pay a cash premium to the insurer and get a contract (the homeowners policy) in exchange. Later the cash flows the other way for those customers who suffer losses and obtain insurance benefits. The insurer pays a premium initially to the special purpose reinsurer and gets a reinsurance policy in exchange. Later, the cash may flow the other way if the catastrophic event or events occur. The investors initially pay cash to the special purpose company and get bonds in exchange. Later they receive coupons and principal, provided no catastrophes occur. The special purpose company invests the combined premiums and proceeds from the sale of the bonds in default free securities.

These transactions provide a structure for which the price of the bonds (paid by the investors), the reinsurance premium (paid by the retailer) and investment income are adequate to cover the catastrophe loss with certainty. Tilley [25, 26] refers to this as a fully collateralized transaction since the special purpose insurer cannot default on the reinsurance contract. By collateralizing the transaction the risk of default, called *counter-party risk*, is eliminated [1]. The ability to eliminate counter-party risk is an advantage of securitization relative to traditional reinsurance.

Insurance risk securitizations present a moral hazard problem that has to be addressed. The insurer has an incentive to apply the coverage to a loss so it will not have pay a coupon, so the investors will want to see that the terms of the coverage are applied properly. We are aware of two methods for resolving the problem that have been used in practice.

Method (1)
The security can be written in terms of an independently determined loss ratio. This takes determination of the security's coverage out of the hands of the insurer, solving the problem, but introducing *basis-risk* – the contract covers industry losses, not the insurer's own losses.

Method (2)
An independent firm is hired to provide claims services.

We now turn our attention to some recent catastrophe risk bond deals.

USAA hurricane bonds. USAA is a personal lines insurer based in San Antonio. It provides financial management products to current or former US military officers. *Business Insurance* [27] in reporting on the USAA deal, described USAA as "over exposed" to hurricane risk due to its personal

[1] Counter-party risk is the risk that the other party will fail to pay as required by the contract. This can be a significant risk in a reinsurance contract, but it is nil in securitizations as we have described them.

automobile and homeowners business along the US Gulf and Atlantic coasts. In June 1997, USAA arranged for its captive Cayman Islands reinsurer, Residential Re, to issue $477 million face amount of one-year bonds with coupon and/or principal exposed to the risk of property damage incurred by USAA policyholders due to Gulf or East coast hurricanes. Residential Re issued reinsurance to USAA based on the capital provided by the bond sale. USAA sold $450 million of similar bonds again in 1998 according to an article in the *Financial Times* [l].

The 1997 bonds were issued in two series (also called tranches), according to an article in *The Wall Street Journal* [22]. In the first series only the coupons are exposed to hurricane risk – the principal is guaranteed. The return of principal will be at the end of the first year if there is no loss (described below), but the return will be at the end of ten years if a loss occurs. For the second series both coupons and principal are at risk. The risk is defined as damage to USAA customers on the Gulf or East coast during the year beginning in June due to a Class-3 or stronger hurricane. The coupons and/or principal will not be paid to investors if these losses exceed one billion dollars. That is, the risk begins to reduce coupons at $1 billion and at $1.5 billion the coupons in the first series are completely gone (and the principal repayment delayed nine years) and in the second series the coupons and principal are lost. The coupon-only tranche has a coupon rate of LIBOR plus 2.73%. The principal and coupon tranche has a coupon rate of LIBOR + 5.76%. The press reported that the issue was "oversubscribed," meaning there were more buyers than bonds, *i.e.*, demand exceeded supply. The press reports indicated that the buyers were life insurance companies, pension funds, mutual funds, money managers, and, to a very small extent, reinsurers. As a point of reference for the risk involved, we note that industry losses due to hurricane Andrew in 1992 amounted to $16.5 billion and USAA's Andrew losses amounted to $555 million. Niedzielski reported in the *National Underwriter* that the cost of the coverage was about 6% rate on line plus expenses [1]. According to Niedzielski's (unspecified) sources the comparable reinsurance coverage is available for about 7% rate on line. The difference, however, may or may not be completely offset by the expenses related to establishing Residential Re and the fees to the investment bank for issuing the bonds. One might argue that the higher cost of securitization is justified by lower *counter-party* risk. The rate on line refers only to the cost of the reinsurance. The reports did not give the sale price of the bonds, but the investment bank probably set the coupon so that they sold at face value.

[1] As we noted earlier, rate on line is the ratio of premium to coverage layer. The reinsurance agreement provides USAA with 80 percent of $500 million in excess of $1 billion. The denominator of the rate on line is (0.80)(500) = 400 million, so this implies USAA paid Residential Re a premium of about (0.06)(400) = 24 million.

As successful as this issue turned out, it was a long time coming. Despite advice of highly regarded advocates such as Morton Lane and Aaron Stern [13, 14, 19], catastrophe bonds have developed more slowly than many experts expected. According to press reports, USAA has obtained 80% of the coverage of its losses in the $1.0 to $1.5 billion layer with this deal. On the other hand, we have to wonder why these are one year deals. Perhaps it is a matter of getting the technology in place, forcing reinsurers to lower prices on future deals, related US tax code issues, *etc*. The off-shore reinsurer is re-usable and the next time USAA goes to the capital market investors will be familiar with these exposures. If the traditional catastrophe reinsurance market gets tight, they will have a capital market alternative. The cost of this issue is offset somewhat by the gain in access to alternative sources of reinsurance. The 1998 issue was more favorable to USAA; it a reported a yield to the bondholders of LIBOR + 4% [1].

Winterthur Windstorm Bonds. Winterthur is a large insurance company based in Switzerland. In February 1997, Winterthur issued three year annual coupon bonds with a face amount of 4700 Swiss francs. The coupon rate is 2.25%, subject to risk of windstorm (most likely hail) damage during a specified exposure period each year to Winterthur motor insurance customers. The deal was described in the trade press and Schmock has written an article in which he values the coupon cash flow [21]. The deal has been mentioned in US and European publications (for example, *Investment Dealers Digest* [18] and *Euroweek* [2]). If the number of motor vehicle (automobile and motorcycle) windstorm claims during the annual observation period exceeds 6000, the coupon for the corresponding year is not paid. The bond has an additional financial wrinkle. It is convertible at maturity; each face amount of CHF 4700 plus the last coupon is convertible to five shares of Winterthur common stock at maturity. Furthermore, due to the merger of Winterthur Insurance and Crédit Suisse Group on December 15, 1997, investors can now convert into 35.5 Crédit Suisse Group registered shares at maturity of the WinCat bond [1].

Swiss Re California Earthquake Bonds. The Swiss Re deal is similar to the USAA deal in that the bonds were issued by a Cayman Islands reinsurer, evidently created for issuing catastrophe risk bonds, according to an article in *Business Insurance* [28]. However, unlike USAA's deal, the underlying California earthquake risk is measured by an industry-wide index rather than Swiss Re's own portfolio of risks. The index was developed by Property Claims Services. The bond contract is written on the same (or similar) California index underlying the Chicago Board of Trade (CBOT) Catastrophe Options. The CBOT options have been the subject of numerous scholarly and trade press articles [8, 10, 11, 12]. As described above, in a

[1] We learned of this from one of the ASTIN referees.

securitization of insurance risk there is a moral hazard problem that has to be addressed in the contract defining the contingent events covered by the security. The investors demand that the losses be reported accurately and in accordance with the contract. The Swiss Re bonds are written in terms of the PSC index, neatly solving the moral hazard problem, although it introduces basis-risk. In this case, basis-risk is the risk that the actual Swiss Re losses differ from industry losses, evidently acceptably small.

Zolkos reported details on the Swiss Re bonds in *Business Insurance*. There were earlier reports that Swiss Re was looking for a ten year deal. This deal is not it and perhaps they are still looking for such a ten year deal. According to Zolkos, SR Earthquake Fund (a company Swiss Re set up for this purpose) issued Swiss Re $122.2 million in California reinsurance coverage based on funds provided by the bond sale.

Yasuda Fire and Marine Bonds. Finally we note that recently Aon Capital Markets structured and marketed a catastrophe bond providing windstorm coverage to Yasuda Fire and Marine Insurance Company [29]. Munich Re "validated the transaction from the perspective of investors" and will provide claims services. Evidently, the moral hazard problem we mentioned earlier is resolved in this case by using Munich Re's claims services. No investment banks were mentioned in the reports because Aon Capital Markets acted as and is registered as its own investment bank. This is an example of how brokers and reinsurers have reacted to securitization – they are acquiring the skills needed to enter the business and marketing services explicitly. The coverage is long term, provides Yasuda with dual "trigger" options (we discuss these in detail in another paper [9]), and makes use of the reputation and administrative services of an established reinsurer. In the next section we review securitization of assets.

4. Asset Securitizations

We are going to describe five examples: stripping coupons, mutual funds, mortgage-backed securities, life insurance policyholder loans, and life insurance premium loadings.

Stripping Coupons. Merrill Lynch and other investment banks create default free zero coupon bonds by means of an asset securitization. This is an example of securitization of securities – repackaging and reselling securities. The resulting securities are called T-bond-backed securities. The bank buys U.S. Treasury bonds. It issues its own zero coupon bonds based on the cash flow from its pool of coupon bearing bonds. In this case, the "customers" are all the same entity: the U.S. government. The retailer and the special purpose company are the same, the bank. The investors buy the zero coupon bonds from the bank. The zero coupon bonds are issued by a private corporation but the bond covenant conveys the pooled Treasury bond cash flow to the zero coupon bondholders. Therefore, the bank's bonds are default-free. The popularity of zero coupon bonds led the U.S. and Canadian governments to assign registration numbers to coupons of some bonds when they are issued. This allows the coupons to be traded directly without securitization. Nevertheless, securitization is still used to create zero coupon bonds. The actuarial textbook [4, page 73] has a simple numerical illustration and the investments textbook [3, page 414] describes some of the marketing aspects of this securitization.

T-bond securitization is a simple asset securitization example, but it illustrates the essential components and principles of these deals. The reason for this securitization is that the demand for default-free zero coupon bonds exceeds the supply provided by the government. A 30-year coupon bearing bond exposes its owner to changes in interest rates corresponding to maturities over the 30-year term of the bond. A zero coupon bond is sensitive only to the interest rate corresponding to its only payment. Therefore, this securitization divides the pooled cash flow into pieces that better meet the needs of some investors and provide a preferable (or more efficient) allocation of interest rate risk. It is an illustration of the use of contracts to transfer and reallocate risk.

Mutual Funds. Pooling also underlies mutual funds and mortgage backed securities (MBS). A mutual fund purchases assets, such as stocks or bonds. The fund sells securities (or shares) that provide the owner a proportional share of the market value of the pool. In this way, an investor receives the average return of the pooled assets without buying shares in each individual asset. Fund managers issue shares in the mutual fund to the investors in exchange for cash and the fund managers have a contractual obligation to buy individual stocks. Owners are entitled to a proportionate share of the fund, less operating fees and commissions. Why would investors prefer to buy a mutual fund rather than the individual shares? Under "perfect market" assumptions, the absence of transactions costs, perfect divisibility of shares, *etc.*, investors would *not* buy mutual funds as they could do for themselves exactly what the mutual fund does for them. However, the real world is not perfect and mutual funds exist because of market "imperfections." First consider transactions costs. Trading stocks is costly because stock brokers charge commissions, but the commission rates are less for those who make large trades on a regular basis. Therefore, a mutual fund has an advantage relative to individual investors because it will have lower transactions costs. A second imperfection is lack of divisibility. An individual may want to buy a stock with a high price per share. Berkshire Hathaway is trading for about $52,000 per share (January 2000). Some investors might want to have some Berkshire Hathaway shares, but buying as few as 10 shares might be impossible. On the other hand, the same individual may have shares in a mutual fund that can easily own 100 or more shares, providing the individual a fraction of Berkshire Hathaway's value. A third consideration is the cost of information acquisition. Under the conditions of "perfect markets" all investors have access to the same information – an assumption that is clearly violated in the real world. Information acquisition is expensive, but a mutual fund applies the same information on behalf of all of its owners, providing an economy of scale.

Finally we consider the diversification of risk. We begin with a brief discussion of the Markowitz [16] risk-return model in order to illustrate diversification. Later we will use the same model to determine the effect of adding insurance-based securities to a portfolio. We will follow the Luenberger's exposition [15]. A different but equivalent approach appears in [4, Chapter 8]. Luenberger shows how to use the model, with some additional assumptions, to describe the effect of diversification. This is a one period market model, focused on the first two moments of the joint distribution of return random variables R_1, R_2, ..., R_n, namely

- the expected returns $\mu_i = E[R_i]$ and
- the covariance matrix $\Sigma = [\sigma_{ij}]$ where $\sigma_{ij} = \mathrm{Cov}(R_i, R_j)$.

These moments can be estimated by observing return outcomes over several time periods, assuming stationarity. Statistics derived from the observations estimate the risk versus return relation in the future for portfolios of assets.

Following Luenberger's discussion of diversification [15, page 200], let us assume that we can write the return of each asset in terms of a single factor: $R_i = a_i + b_i F + \varepsilon_i$ where a_i and b_i are constants, F is a random variable (the single factor), and the ε_i are random error terms. Assume that the following relations hold:

$$E(\varepsilon_i) = 0, \quad E(\varepsilon_i \varepsilon_j) = 0 \text{ for } i \neq j, \quad \mathrm{Cov}(F, \varepsilon_i) = 0,$$

and their variances have a common bound $\mathrm{Var}(\varepsilon_i) \leq s^2$.

A portfolio is constructed from the n given assets by specifying the percentage of the value of the portfolio which is invested in each asset. Under the assumptions commonly used, the scale of investment does not affect the percentages in the sense that investors with the same risk-return preferences will select the same portfolios regardless of the size of their investments. Hence in specifying a portfolio, we need only specify the percentage invested in each security. We let w_i denote the percentage invested in the i-th asset; it is called the *weight* of asset i in the portfolio.

For a "well diversified" portfolio, we can assume that each weight is about $1/n$. The portfolio return is $R_w = \sum_{i=1}^{n} w_i r_i = a + bF + \varepsilon$ where

$$a = \sum_{i=1}^{n} w_i a_i, \quad b = \sum_{i=1}^{n} w_i b_i \quad \text{and} \quad \varepsilon = \sum_{i=1}^{n} w_i \varepsilon_i.$$

Under the assumptions we made above, the variables ε and F are uncorrelated so $\mathrm{Var}(R) = b^2 \mathrm{Var}(F) + \mathrm{Var}(\varepsilon)$. Since the errors ε_i are uncorrelated, $\mathrm{Var}(\varepsilon) = s^2/n$ and as n increases this term tends to zero. Diversification eliminates this component. The other component does not tend to zero because b is the average of the b_i. This term represents *non-diversifiable risk*. The diversification principle is familiar to actuaries from its application to pools of insurance policies.

In summary, mutual funds exist because they provide greater efficiency, overcome some of the effects of market imperfections, and provide diversification of risks more efficiently than individual investors can achieve on their own.

Mortgage-Backed Securities (MBS). A mortgage is a loan requiring periodic payments of principal and interest with real estate as collateral [1]. The mortgage may be for a residence or for commercial real estate. We limit our discussion to US residential mortgages [2]. They are commonly issued with a fixed interest rate for a period of 15 to 30 years and require level monthly payments of interest and principal. Fixed-rate mortgages carry substantial interest-rate risk for the lender, especially in volatile economic times. For example, when interest rates fall, borrowers may re-finance their mortgages, returning the principal to the lender at a time when interest rates are lower than the rate at which the mortgage was issued. There are costs to re-financing, but when rates fall enough, borrowers have financial incentives to refinance. Mortgage securitization shifts the interest rate risk to investors through the securities market.

For mortgage-backed securities the components of the securitization are easy to identify: The customers are the mortgage borrowers. Initially the borrowers obtain cash and in exchange provide the lenders with a contractual obligation to repay the loan. The lenders convey their rights to a trust in exchange for cash. The trust issues securities based on the pooled mortgage contracts. The securities can take a variety of different forms.

One purpose of mortgage securitization (re-packaging) is to allow for a more efficient allocation of interest rate risk. Primary mortgage lenders (*e.g.*, banks and thrifts) usually have short-term demand deposits as liabilities, so for most of them mortgage assets are not well matched to their liabilities. On the other hand, life insurers, with long term liabilities, may desire to have mortgage-backed securities in their asset portfolios. We discuss two mortgage-backed securities: pass-through securities and stripped mortgage-backed securities. Several other forms exist, but these illustrate the basic ideas.

First we discuss *pass-through* mortgage-backed securities. With pass-through securities, mortgage borrowers make their monthly payments to the pool administrator. The pool collects the cash, deducts administrative fees,

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[1] This section relies on [24, Chapter 6].

[2] Mortgage terms and lending practices are different in other countries. For example, in Canada, mortgages are typically written for 5 or 10 years with a balloon payment (which is often refinanced) and no prepayment option. The Canadian practices put the interest rate risk on the borrower, lenders bear none, and there is no need for reallocating the lender's interest rate risk – and no mortgage-backed securities.

and passes the remaining cash to the security owners on a pro-rata basis. Thus, if a pool issues ten securities, each security owner receives one-tenth of the aggregate monthly cash flow, less fees. If a mortgage is repaid during the month, the repaid principal is paid to the security owners along with the monthly cash flow. Thus, the security owners bear the prepayment risk. Valuation of a pass-through security requires knowing the rates and maturities of the pooled mortgages. This and other information is provided to potential purchasers. An actuarial approach would involve modeling the "life" of a mortgage and considering the cash flow to be a cash-refund annuity. The difficulty, and the distinction from mortality-dependent cash flow, is that the mortgage life depends of the interest rate environment. All mortgage-backed securities present these same valuation problems.

A *stripped* mortgage-backed security divides the payments from pooled mortgages into classes with each class's security holder receiving income only from its portfolio, instead of distributing it on a pro-rata basis. For example, consider a stripped security with two classes: interest only and principal only. The interest-only class receives the interest paid on the pooled mortgages each month. The principal-only class receives each month's principal payments. Suppose that a representative mortgage in the pool carries an outstanding principal of \$90,000, an interest rate of 6 percent, and a level monthly payment of \$600. Ignoring fees, the interest-only class would be allocated \$450 ($\$90,000 \times 0.06/12$) this month from this mortgage. The principal-only class receives the principal paid with respect to the illustrative pool mortgage; that is, \$150 (\$600 − \$450) if the mortgage is not repaid during the month. If the illustrative mortgage loan is repaid during the month, the principal-only class receives \$90,000. The two classes receive similar payments from each mortgage with an outstanding balance at the beginning of the month.

The stripped pass-through security owners bear the interest rate risk of the pool, but it is allocated differently than it is for straight pass-throughs. The interest-only class receives interest until all the mortgages are repaid. Refinancing activity increases with falling interest rates, so the downside for interest-only security owners arises with declining interest rates. The principal-only class benefits from a decline in interest rates because refinancing means principal-only security owners receive their principal sooner. Thus, the stripped pass-through divides the cash flow pool into segments that give a pure reflection of the result of an increase or decrease in interest rates. This is more flexible than a straight pass-through mortgage-backed security and will appeal to many investors. After all, an investor who wants a straight pass-through could simply buy shares of both interest-only and principal-only classes.

Securitization of the mortgage industry has allowed investors to enter the mortgage market without having to be (or own) a mortgage originator. Insurance companies and pension funds have become substantial investors in MBS. Thus, securitization has allowed for a better allocation of interest rate risk and provided a more efficient way for capital to enter the home financing industry. The securitization technique is important for actuaries because the resulting products are used by insurance companies, the technique can be applied to other asset classes, and, perhaps most important, the expertise required to design and value these securities is fundamentally actuarial in nature. Let us illustrate this claim with the following idealized model.

Suppose that we are interested in a pass-through MBS for which the contractually specified monthly payments for the mortgage borrowers (in the absence of additional cash flows due to prepayment) per \$1 of face amount of the mortgage is denoted by c. Let the contractually specified effective monthly interest rate on the mortgage be denoted by r. In the absence of prepayment risk, level monthly payments are made over the entire term of the mortgage and the present value of these payments is equal to the face amount of the mortgage pool. In practice, mortgage borrowers will prepay with varying intensity and this rate of prepayment could depend on a variety of economic variables. For the sake of this illustration, let us assume that the rate of prepayment depends on the time since the issue of the mortgage (this makes an allowance for the average time a home is owned) and an annualized key interest rate level (for example, the 10-year yield rate on US treasury bonds, which makes an allowance for the cost of refinancing) denoted i. Providing the actuary has access to sufficient data, he would then estimate a two-dimensional table of prepayment rates. Let $q(t, i)$ denote the amount prepaid over month t to month $t+1$ per dollar of principal remaining when the key rate is equal to i. Let ℓ_t denote the amount of principal remaining in the mortgage pool at the end of the t-th month after the mortgage is issued. The total cash flow to the mortgage pool over month t to month $t+1$ is

$$\ell_t c + (\ell_t - [\ell_t c - \ell_t r])q(t, i).$$

In words, this monthly cash flow is the ordinary payment of interest and principal – namely $\ell_t c$, plus the amount of the remaining principal that is prepaid [1] – namely $(\ell_t - [\ell_t c - \ell_t r])q(t, i)$. This is a stochastic cash flow that depends on the key rate history. The evolution of the remaining principal in the mortgage pool can be determined recursively through the equation

$$\begin{aligned} \ell_{t+1} &= \ell_t - [\ell_t c - \ell_t r] - (\ell_t - [\ell_t c - \ell_t r])q(t, i) \\ &= [1 - q(t, i)](\ell_t - [\ell_t c - \ell_t r]). \end{aligned}$$

This is a stochastic equation for the evolution of the outstanding principal in the mortgage pool. The actuary can then value the MBS using stochastic cash flow valuation techniques from financial economics. The MBS market is a complete market and the valuation will be done in the context of a

[1] Note that $\ell_t c - \ell_t r$ is the amount of the regular payment that is applied to principal reduction that month prior to the prepayment amount being applied.

complete term structure model. Although the estimation of the prepayment rates and the definition of the MBS cash flows are fundamentally actuarial, the actuary must also be able to use tools from modern financial economics to complete his calculation of the value of the MBS and to assess the risks in the MBS.

Policy-Loan-Backed Securities. The laws of the United States and some other countries require certain life insurance policies to have cash values (savings). In still other countries, cash values are not legally required, but are commonly provided. In general, cash values emerge when the expected value of future benefits promised under a policy exceed the expected value of future (adjusted) premiums. In lay terms, cash values emerge when policyholders prepay future mortality costs. Cash values can be thought of as a type of savings within a life insurance policy that is available when a policyholder terminates (surrenders) his or her policy.

Economically, cash values are policyholder assets in the custody of the insurance company. Rather than surrendering their policies to obtain funds, policyholders may elect to borrow an amount not greater than the cash value from the insurance company on the security of their cash values. In the United States and some other countries, cash value policies are required to allow such borrowing privileges. Of course, the policyholder pays interest on the loan. Traditionally, U.S. insurers offered fixed-rate policy loans, but as interest rate volatility increased in the 1970s and 1980s, most companies began issuing policies with an indexed loan interest rate. When the interest rate is fixed, the policy loan provision is an interest rate call option. The value of the option increases with the volatility of interest rates.

Policy loans are carried on insurers' financial statements as assets. Securitization of a portfolio of policy loans allows the company to sell them. One reason for doing this is to reduce the cash strain induced by policy loan activity. Also, there may be a tax advantage when the loans are sold at a loss relative to their statement value. These reasons led to a large securitization of policy loans by the Prudential Insurance Company of America in 1987.

Policy loan interest and principal payments formed the cash flow to support the securities that were sold to investors as private placement policy-loan-backed securities. A special purpose corporation (SPC) was formed to issue the securities and simultaneously purchase the loan cash flow from Prudential, similar to collateralized mortgage obligations, as discussed in the preceding section. While the Prudential securitization borrowed concepts from the securitization of mortgage loans, it also employed new features. Since policy loan securitization was new, security buyers had no experience with loan repayment rates. To reduce the repayment risk to security owners, the securities provided for a minimum and maximum repayment schedule. If actual repayments fell behind the minimum schedule, Prudential promised to advance the needed cash to meet the required payments to security owners. (Cash flow simulations indicated that this was highly unlikely.) If repayments proved more rapid than the maximum, the SPC would invest the excess cash flow in a guaranteed investment contract (GIC). The SPC bought a 54-year GIC from a AAA-rated Swiss bank to provide security owners with evidence that the SPC would be able to perform on these promises.

The circumstances surrounding this transaction may be comparatively rare. Perhaps high interest rates might make them attractive again someday. On the other hand, as the value of the loan option in newer U.S. policies is nil, the magnitude of the problem created by increased exercise activity is steadily decreasing. Also, the costs of a policy loan securitization are substantial. Therefore, it may be a long time before we see another policy loan securitization in the United States.

Loadings in Premiums. In January 1997 the US life insurer American Skandia Life Assurance Corporation securitized mortality and expense risk fees that it will collect in the future from a portfolio of its variable annuity (VA) policies [6].

When a company issues a VA it pays a commission to an agent or financial advisor. Profits develop later. Thus issuing a VA requires cash, in contrast to other types of business that merely require setting up a reserve that may be financed with a non-cash asset or reinsurance. The faster the company grows the greater the need for cash. Skandia's fast growth led it to supplement traditional methods of financing growth (retained earnings, surplus notes, bank loans and reinsurance) with a securitization of the future fees Skandia will collect from a block of policies.

According to Connolly, the mortality and expense risk fees were taken from a block of approximately 33,000 American Skandia variable annuities, net of reinsurance, issued during the period between January 1, 1994 and June 30, 1996. The rights to the fees for a specified period of time were sold to American Skandia Investment Holdings, American Skandia Life Assurance Corporation's immediate parent, which transferred them to a trust, collateralized them and sold them to two investors, TIAA-CREF and Prudential Insurance Company. A total of eight insurers expressed interest in the offering.

Skandia managers think the costs of securitization will decrease as the process becomes more efficient and, ultimately, it should be cheaper than financing growth with reinsurance.

In this example, the customers are the variable annuity policyholders. Skandia is the retailer and the trust is the special purpose company. The investors are TIAA-CREF and Prudential. The actuarial modeling developed for traditional purposes (designing, pricing, cash flow testing, *etc.*) can be used in the securitization process. Since the buyers are also life insurers, they should have the expertise to evaluate the future fee cash flows. In general, a securitization of insurance risks would probably require independent consulting actuaries to resolve this moral hazard problem. In general, investors are not likely to have the expertise and they are not likely to accept the retailer's analysis without independent corroboration.

We have not seen an increase in life insurance securitizations. Mutual companies have more difficulty than stock companies in raising capital and cash. In the US many of them are electing to demutualize but at least one UK company is using securitization as an alternative.

5. The Demand for Insurance-Based Securities

Why do investors buy catastrophe risk bonds? The demand for securities based on insurance risk can be justified by the Markowitz mean-variance model. As we mentioned earlier, this is a one period market model. The assets returns over the period are random variables R_1, R_2, ..., R_n with means and covariances assumed to be know and denoted by $\mu_1 = E(R_i)$ and $\Sigma = [\sigma_{ij}]$ where $\sigma_{ij} = \mathrm{Cov}(R_i, R_j)$.

The n by n matrix $\Sigma = [\sigma_{i,j}]$, called the covariance matrix, is symmetric and the diagonal elements are simply the variances. We assume it is invertible.

A portfolio is constructed from the n given assets by specifying the percentage of the value of the portfolio which is invested in each asset. As stated earlier, we assume that the scale of investment does not affect the percentages in the sense that investors with the same risk-return preferences will select the same portfolios regardless of the size of their investments. Hence in specifying a portfolio, we need only specify the percentage invested in each security. We let w_i denote the percentage invested in the i-th asset; it is called the weight of asset i in the portfolio.

The return on the portfolio specified by the vector

$$w^{\mathsf{T}} = [w_1, \ w_2, \ ..., \ w_n]$$

is denoted by

$$R_w = \sum_{i=1}^{n} w_i R_i.$$

The portfolio return is the weighted average of the individual security returns. Thus the expected portfolio return, $\mu_w = E[R_w]$, and variance, $\sigma_w^2 = \mathrm{Var}[R_w]$, can be calculated in terms of the weights and the statistics of the individual securities as follows:

$$\mu_w = \sum_{i=1}^{n} w_i E[R_i] = \mu^{\mathsf{T}} w$$

$$\begin{aligned}\sigma_w^2 &= \sum_{i=1}^{n} w_i \sum_{j=1}^{n} w_j Cov(R_i, R_j)\\ &= w^{\mathsf{T}} \Sigma w\end{aligned}$$

The portfolio variance is a function of the vector of weights $w^T = [w_1, w_2, \ldots, w_n]$ and the covariance matrix $\Sigma = [\sigma_{i,j}]$.

An efficient portfolio is defined to be one which is not dominated by another portfolio. It is a portfolio for which there is none other with lower variance [1] and an equal or higher expected return. Figure 8 illustrates the concept of efficiency and the associated notion of portfolio dominance. Note that portfolio B dominates portfolio A since it offers the same variance but has a higher expected return. Similarly, portfolio B dominates portfolio C since it offers the same expected return but a lower variance. The basic portfolio problem is to find the maximum portfolio return for a given portfolio variance or the minimum portfolio variance for a given portfolio return. These optimal portfolios are said to be mean-variance efficient portfolios.

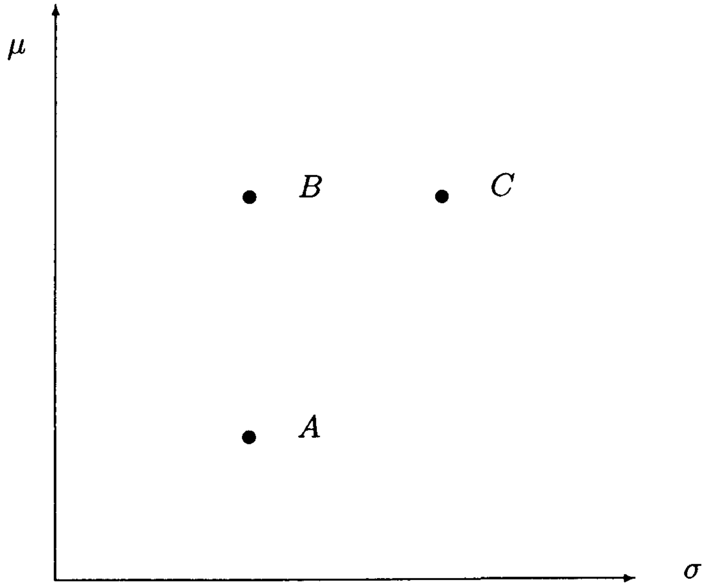

FIGURE 8: Risk and Return Relations.

There are a number of variants of the general portfolio problem. The following formulation of the standard version comes from [4]: Given the investor's required portfolio expected return $r > 0$ and a set of n securities

1 Either portfolio variance σ_w^2 or standard deviation σ_w can be used to measure risk. In the graphs we follow the usual practice of plotting expected return μ_w on the vertical axis and risk represented by standard deviation σ_w on the horizontal axis.

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with expected returns vector $\mu^{\mathsf{T}} = [\mu_1, \mu_2, \ldots, \mu_n]$ and covariance matrix Σ, determine the portfolio weights w in order to minimize the variance $\sigma_w^2 = w^{\mathsf{T}}\Sigma w$ subject to two constraints:

$$\sum_{i=1}^{n} w_i = 1$$

and

$$\mu_w = \mu^{\mathsf{T}} w = r.$$

The first constraint simply requires that the portfolio be 100% invested in the n risky securities being considered for inclusion in the optimal portfolio. It is convenient to introduce the n-vector $e^{\mathsf{T}} = [1, 1, \ldots, 1]$. The first constraint can be written compactly as $w^{\mathsf{T}}e = 1$. The second constraint selects the portfolio return to meet the investor's requirement. Of course there is a potentially different efficient portfolio for each target return we might select. In fact, we can graph an entire set of efficient portfolios, plotting the points (σ_w, r) by solving the portfolio problem for different values of σ_w corresponding to a range of values of target expected returns r. This graph is called the *efficient frontier* for the given n assets. The efficient frontier can be completely defined in terms of two efficient portfolios. This is the "two fund theorem," described by Luenberger [15, page 163] as follows.

The objective function, augmented with Lagrange terms corresponding to the constraints, is

$$\frac{1}{2} w^{\mathsf{T}}\Sigma w + \lambda(w^{\mathsf{T}}\mu - r) + \nu(w^{\mathsf{T}}e - 1).$$

The factor $\frac{1}{2}$ in the variance term is for convenience only. The objective is quadratic in the unknown weights w and linear is the Lagrange multipliers λ, ν, so the first order conditions for a minimum form a system of $n + 2$ linear equations:

$$\sum_{j=1}^{n} \sigma_{i,j} w_j + \lambda\mu_i + \nu = 0 \quad \text{for } 0 \le i \le n$$

$$w^{\mathsf{T}}\mu = r$$

$$w^{\mathsf{T}}e = 1$$

Write this as a single matrix equation

$$\Sigma^{\text{aug}}[w, \lambda, \nu]^{\mathsf{T}} = [0, \ldots, 0, r, 1]^{\mathsf{T}} \tag{1}$$

where Σ^{aug} is the result of augmenting the matrix Σ with two rows and columns:

$$\Sigma^{\text{aug}} = \begin{bmatrix} \sigma_{1,1} & \sigma_{1,2} & \dots & \sigma_{1,n} & \mu_1 & 1 \\ \sigma_{2,1} & \sigma_{2,2} & \dots & \sigma_{2,n} & \mu_2 & 1 \\ & & \dots & & & \\ \sigma_{n,1} & \sigma_{n,2} & \dots & \sigma_{n,n} & \mu_n & 1 \\ \mu_1 & \mu_2 & \dots & \mu_n & 0 & 0 \\ 1 & 1 & \dots & 1 & 0 & 0 \end{bmatrix}$$

In addition to assuming that the covariance matrix Σ is invertible, we also assume the expected return vector μ is not a multiple of e. This just means that the last two columns of the augmented matrix Σ^{aug} are linearly independent. Clearly, each of the first n columns of Σ^{aug} is linearly independent of each of the last two. Because Σ is invertible, the first n columns of Σ^{aug} are linearly independent. Because of the independence of μ and e, the last two columns of Σ^{aug} are linearly independent. Therefore, the columns of Σ^{aug} are linearly independent, it is invertible, and there is a unique solution for the weights w and the multipliers λ, ν.

Let (σ_B, r_B) denote the risk and expected return of a minimum variance portfolio. By this we mean an efficient portfolio with minimum variance among all efficient portfolios for various values of r. In general, the minimum variance could be zero corresponding to a market with a risk free security. However we assume that at this point we are considering only risky assets and $\sigma_B > 0$. We might think of this as a portfolio of corporate bonds; they are risky but not so risky as equity securities. Let w_B, λ_B, ν_B denote the corresponding weights and multipliers. Of course $\sigma_B = \left((w_B)^{\mathsf{T}}\Sigma w_B\right)^{1/2}$ and $r_B = \mu^{\mathsf{T}} W_B$.

Select any other efficient portfolio with weights w_S, multipliers λ_S, ν_S, return $r_S = \mu^{\mathsf{T}} w_S$ and risk $\sigma_S = \left((w_S)^{\mathsf{T}}\Sigma w_S\right)^{1/2}$ with $r_S > r_B$ and $\sigma_S > \sigma_B$. While the lower risk fund (σ_B, r_B) intuitively represents a bond fund, the more risky fund (σ_S, r_S) represents an equity portfolio.

Given any point (σ, r) on the efficient frontier, form the portfolio with weights w and multipliers λ, ν satisfying

$$[w,\ \lambda,\ \nu]^{\mathsf{T}} = (1-a)[w_B,\ \lambda_B,\ \nu_B]^{\mathsf{T}} + a[w_S,\ \lambda_S,\ \nu_S]^{\mathsf{T}}$$

where $a = (r - r_B)/(r_S - r_B)$. Now since $\Sigma^{\text{aug}}[w_B,\ \lambda_B,\ \nu_B] = [0,\ 0,\ \dots,\ r_B,\ 1]^{\mathsf{T}}$ and $[w_S,\ \lambda_S,\ \nu_S] = [0,\ 0,\ \dots,\ r_S,\ 1]^{\mathsf{T}}$, then $\Sigma^{\text{aug}}[w,\ \lambda_1,\ \lambda_2] = [0,\ 0,\ \dots,\ r,\ 1]$.

The solution is unique so we have

$$r = \mu_W = (1-a)r_B + ar_S$$

and

$$\begin{aligned}\sigma^2 = \sigma_w^2 &= \mathrm{Var}[(1-a)R_B + aR_S] \\ &= (1-a)^2\sigma_B^2 + 2a(1-a)\rho\sigma_B\sigma_S + a^2\sigma_S^2\end{aligned}$$

where we have abbreviated the notation with $R_B = \mu^{\mathrm{T}} W_B$ and $R_S = \mu^{\mathrm{T}} W_S$. Also we wrote the covariance term as

$$\mathrm{Cov}(R_B,\ R_S) = \rho\sigma_B\sigma_S,$$

where the correlation coefficient is ρ. In effect every point on the efficient frontier can be obtained as a weighted average of the two fixed portfolios W_B and W_S. This is what Luenberger calls the two fund theorem. Figure 9 illustrates the two fund theorem, showing two frontiers that differ only in that the solid frontier has a greater value of ρ than the dashed frontier. This illustrates that if nothing changes except the correlation is reduced, then the frontier pushes out to the left for those points between B and S.

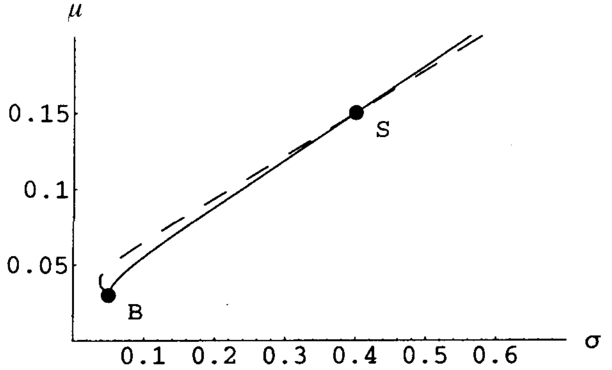

FIGURE 9: Efficient Frontiers – Two Fund Theorem.

Now we add to the investment opportunity set two new securities. The first is a risk-free bond. It has return r_f, zero variance, and zero covariance with every other security. Every investor is better off (or no worse off) as a result of this expanded opportunity set. This is illustrated by the "one fund theorem" described by Luenberger [15, page 168]. There is an efficient portfolio M of risky assets with weights $w_M^{\mathrm{T}} = [w_1, \ldots, w_n]$ such that any efficient portfolio can be constructed as a combination of M and the risk-free bond. This is represented graphically in Figure 10. The equation of the line is

$$r = r_f + \frac{r_M - r_f}{\sigma_M}\sigma.$$

This is called the capital market line (CML).

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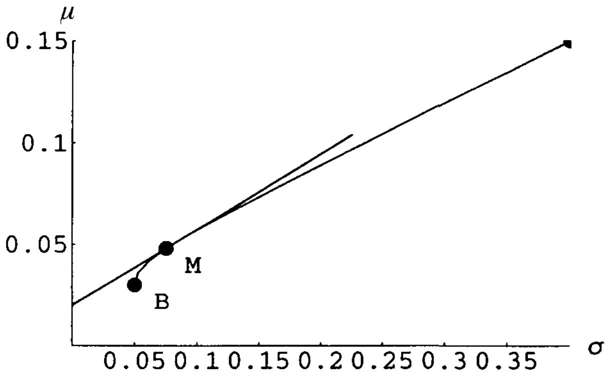

FIGURE 10: Efficient Frontiers.

As before, the efficient frontier before introducing the risk-free bond is the curved line. Any point (σ, r) on the CML can be obtained by investing the proportion $a = (\sigma_M - \sigma)\sigma_M^{-1}$ in the risk-free bond and $1 - a$ in the fund M. The capital market line lies above the original efficient frontier, except at M where they are equal. All investors hold a portfolio of the form $ar_f + (1 - a)r_M$ for some a, given this opportunity set. That is, all mean-variance optimizing investors will demand a portfolio on the capital market line [1]. Luenberger shows how to solve for the weights defining the portfolio M, which we will refer to as the *market portfolio*.

Now we introduce an insurance-based security C with high expected return, correspondingly high variance, but relatively low correlation with other risky assets. C could be a cat bond. At least for the case that the underlying insurance risk is catastrophe property loss, there is evidence that the return has zero correlation with the market portfolio M [5, 13]. The new asset has risk and return parameters σ_C and r_C and its correlation with the market $\rho_{C,M} = \dfrac{\text{Cov}(R_C,\ R_M)}{\sigma_C \sigma_M}$ is relatively small.

The portfolio returns obtained as linear combinations of R_M and R_C

$$R_a = aR_M + (1 - a)R_C$$

correspond to points $(\sigma_a,\ r_a)$ where

$$r_a = a\text{E}[R_M] + (1 - a)\text{E}[R_C] = ar_M + (1 - a)r_C$$

and

$$\sigma_a^2 = a^2\sigma_M^2 + 2a(1 - a)\rho_{C,M}\sigma_C\sigma_M + (1 - a)^2\sigma_C^2.$$

[1] The one-fund and two-fund theorems are valid whether markets are complete or not. Individual investor risk preferences are reflected in the choice of the factor a, but they nevertheless choose positions on the CML.

The dashed curve in Figure 11 illustrates the graph of the parametric equations for the points $\{(\sigma_a, r_a)|0 \leq a \leq 1\}$ for the case that $\sigma_C > \sigma_F$ and $\mu_C > \mu_F$. The following argument shows that so long as $\rho_{C,M} < \sigma_M/\sigma_C$, the curve joining C and M has a negative slope at M (where $a = 1$) and so it punches through the CML. As a consequence the new CML, determined after investors take into account the new security must have a greater slope than the original. This means all investors are better off.

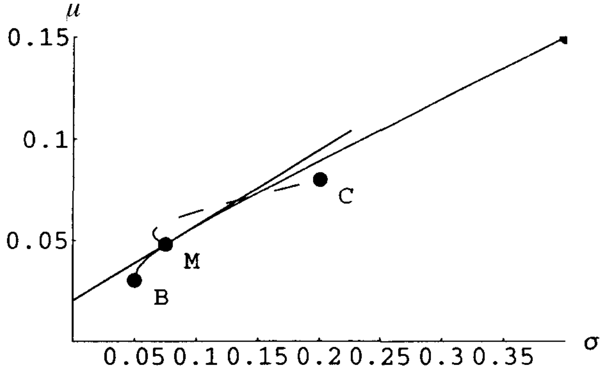

FIGURE 11: Efficient Frontiers.

In order for the curve to push up and to the left relative to the CML, it is sufficient that the slope of the curve at M be negative. Calculation of the slope goes like this:

$$
\begin{aligned}
2\sigma_a \frac{\partial \sigma}{\partial a} &= \frac{\partial \sigma^2}{\partial a} \\
&= 2a\sigma_M^2 + 2(1-2a)\rho_{C,M}\sigma_M\sigma_C - 2(1-a)\sigma_C^2.
\end{aligned}
$$

For $a = 1$, we find that

$$
\left.\frac{\partial \sigma}{\partial a}\right|_{a=1} = \sigma_M - \rho_{C,M}\sigma_C.
$$

The slope of the curve at M, therefore, is

$$
\frac{\frac{\partial r_a}{\partial a}}{\frac{\partial \sigma}{\partial a}} = \frac{r_M - r_C}{\sigma_M - \rho_{C,M}\sigma_C}.
$$

In the case illustrated in Figure 11, $r_C > r_M$ and the slope is negative provided only that $\rho_{C,M} < \sigma_M/\sigma_C$. We think that this describes the

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recently observed market for cat bonds. The correlation does not have to be zero. All investors are better off when catastrophe-based securities are introduced.

In the case that $r_C < r_M$, adding C also expands the efficient frontier provided that the slope of the (C, M)-curve is positive at M. This leads to the same condition, $\rho_{C,M} < \sigma_M/\sigma_C$, on the correlation coefficient, as illustrated in Figure 12. Our conclusion is that adding a security with nonnegative relatively low correlation (or negative correlation of any magnitude) with the market results in a new market equilibrium in which all investors have improved opportunities.

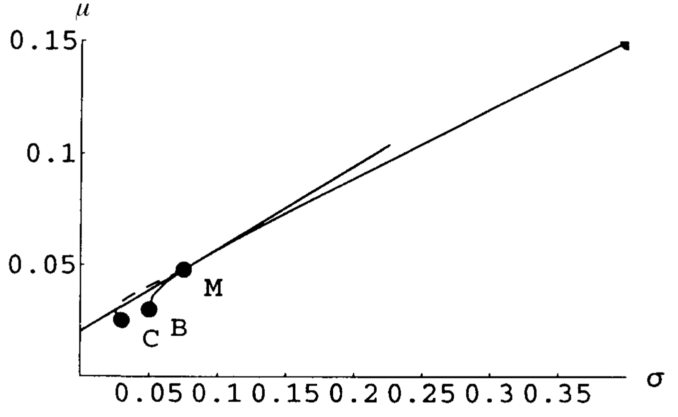

FIGURE 12: Efficient Frontiers.

We have shown that for the investment opportunities to improve it is sufficient that the covariance of the new security's returns with existing assets is relatively small in absolute value or negative. For example, it seems likely that long term bonds with coupons based on a mortality index would also improve investment opportunities, even if the risk and return were below equity levels. Thus the mean-variance model provides a rational for the demand for new insurance based securities. That is, all investors will now demand portfolios on the new capital market line [1]. The insurance press reports that investors (so far) like cat bonds. Some issues have been described as over-subscribed. This behavior seems to be consistent with the model.

In our construction we assumed the original opportunity set of n risky assets had an invertible covariance matrix, which means that no single asset is a linear combination of the other $n-1$ assets. We assumed also that transaction costs were zero, all available information was revealed to all investors instantaneously, and other market imperfections such as taxes were not present. We call these the perfect market assumptions.

[1] This holds regardless of individual investor risk preferences.

In the usual construction, the original n risky assets contain firm specific risks that we have now assumed to be engineered into cat bonds. This risk is usually assumed to be costlessly diversified away. Given the assumptions on transactions costs, information, *etc.*, the equilibrium that obtains would not be altered by the introduction of cat bonds or other such securities. The introduction of such securities does not change the CML.

Our argument is that in actual, imperfect markets, the introduction of such securities results in the market being more efficient. Their introduction allows investors to construct portfolios consistent with their preferences for less costs. The more efficient distribution of capital over risks results in a new equilibrium in which all investors are better off.

Our construction was designed to show that by adding such securities the market is pushed closer to the idealized perfect markets equilibrium. This is done by increasing the present value of profits of the firm via this activity in ways investors cannot on their own account (increasing efficiency of the firm) and by packaging the risks (that are assumed to be in the original n risky securities) and issuing them to the market in such a way that investors can distribute capital over these risks more efficiently than they could when they were contained in the original n risky securities (due to increased efficiencies such as lower bid-ask spreads, information acquisition costs, and so on).

We argue that this is the economic justification for this activity and, correspondingly, it should continue to be observed in the capital markets as long as securitization improves efficiency.

6. The supply of insurance-based securities

Why do insurers and reinsurers securitize insurance risks? Capacity to handle very large losses is frequently mentioned as a motive for catastrophe risk securities [7, 13]. We note also that many of the catastrophe risk deals provide long term coverage, in contrast to traditional reinsurance which is normally issued for a one year term. What about other insurance risks? As we described earlier, there have been few securitizations of mortality risk. This makes sense economically. Securitization brings more capital to cover risks that would not be covered otherwise. There seems to be a need for even more capital as economies develop and more property is insured. Securitization of insurance risk is expensive compared to an asset securitization such as a T-bond securitization or traditional reinsurance. Some of the additional cost is due to costs of measuring the risks and explaining them to investors – resolving the moral hazard problem. However, we expect these costs will decline as investors become more familiar with the risks. Perhaps securitization will always be more expensive than reinsurance, but we expect it will continue to be used for these reasons:

Securitization often provides innovative contract terms such as larger amounts of coverage (catastrophe property risk), coverage of risks not provided by traditional reinsurance (long term mortality risks), or unusual risks.

Counter-party risk is eliminated with securitization.

Securitizations may provide more favorable tax treatment. The special purpose reinsurer is usually located in a jurisdiction which allows favorable tax treatment of reserves.

The question (why do insurers buy reinsurance?) is interesting because in an ideal world – one with no taxes, transactions costs, or other "imperfections" the insurance company shareholders would not compensate managers for managing a risk they can diversify on their own behalf in the capital markets.

For example consider the risk of fire damage to the corporation's property. A shareholder with X dollars invested in the corporation will suffer a loss if the property burns. However, the investor can find a second company and invest $X/2$ in each company. This diversifies the shareholder's fire risk. Further diversification reduces the fire (and other risks) even more. This does not cost shareholders anything, so they will direct managers to retain diversifiable risks, rather than insuring them. This suggests corporations should not buy insurance, yet they buy a lot. Mayers and Smith [17] offer answers that can be summarized as follows: real-world imperfections make insurance a rational corporate purchase.

The rationale for reinsurance purchases and securitizations of insurance risk is analogous. The demand for securitizations will persist as long as it has an advantage in addressing the imperfections we described earlier.

7. The Role of Actuaries in Securitization

So far actuaries have been on the sidelines with a few exceptions. Of course, actuaries were involved in Prudential's securitization of policy loans and Skandia's securitization of premium loadings. In addition James Tilley developed the concept of a catastrophe risk bond [25] in connection with Morgan Stanley's effort to help fund the California Earthquake Authority. Prakash Shimpi is leading a Swiss Re subsidiary dedicated to trading insurance risks [23]. These are important developments, but we should see many more actuaries working in the field. The role of the actuary should go well beyond modeling loss distributions. The actuary has the skills to see the big picture as well as the technical details. There is an opportunity to contribute to contract design, security valuation, investor communications, etc.

8. Summary

There are four components of securitization. A retailer bundles customer risks and passes them as a group to a special purpose company. The special purpose company issues securities based on the pool. The process can be used to reallocate risk or rearrange cash flows to better suit the needs of investors. When applied to insurance risk, the process is costly but costs may decline to some extent but will likely remain more expensive than traditional reinsurance. The additional cost may be the price to be paid to overcome counter-party risk. The securitization business will grow since it provides access to large amounts of capital and it allows for innovative contracting, relative to traditional reinsurance. Introducing insurance-based securities into the capital market improves opportunities for all investors provided the underlying insurance risk is not correlated with existing market risk. This provides a rationale for the demand for such securities.

Appendix – On the Role of Incompleteness in Insurance Risk Securitization

In this appendix we attempt to give the reader a "feel" for the notions of completeness and incompleteness and why they are relevant and so important in insurance risk securitization. We begin by providing some general intuition on these concepts.

Some Intuition. It can be difficult to differentiate between insurance markets which are complete and those which are incomplete. Table 1 provides some examples of insurance products each with its embedded insurance risk and the type of market (complete or incomplete) which the product resides in.

TABLE 1

Some examples of insurance products and the type of market they reside in

Insurance Product	*Nature of Risk*	*Market Type*
Variable Annuities	Mortality Risk	Complete
Catastrophe Risk Bonds	Catastrophe Risk	Incomplete
Mortality Risk Bonds	Mortality Risk	Incomplete
Equity-Indexed Annuities	Market Risk	Complete

We offer a brief rationale why each of these products resides in a complete or incomplete market.

Variable Annuity: We shall assume that the primary risk in issuing variable annuities is that a contract holder dies during the contract period and the insurance company must honor the minimum investment return guarantee. Since the investments are made in standard securities such as S&P 500 index funds, providing the mortality of the contract pool follows a deterministic mortality pattern, this investment risk can be fully hedged. Investors who purchase variable annuities are generally not purchasing portfolio diversification. Instead, they are seeking tax advantages and protection of principal. These products offer little additional diversification of risk as the assets they are invested in are already available in the market.

Catastrophe Risk Bonds: The primary risk in cat bonds is the occurrence of a catastrophe that triggers the loss of principle. Since there are no securities, other than cat bonds, whose payoffs are contingent on the occurrence of catastrophes, cat bonds cannot be priced in terms of a portfolio of the assets that are already traded and priced in the market. Therefore, cat bonds reside in an incomplete market. Furthermore, cat bonds provide investors diversification of risk because the payoffs from these bonds are contingent on states that are not picked up by existing securities.

Mortality Risk Bonds: When mortality is assumed to follow a deterministic life table, the payments from traditional insurance products follow a fixed and known pattern. When deterministic mortality is assumed, even life insurance products whose benefits are contingent on the stock market or interest rates reside in a complete market. However, if there are substantial fluctuations in mortality experience across all policies then this risk cannot be hedged using existing securities because there are no existing securities whose payoffs are contingent on the mortality fluctuations. Consequently, mortality risk bonds reside in an incomplete market.

Equity-Indexed Annuities: Equity-indexed annuities have characteristics similar to variable annuities. For this reason, they too reside in a complete market.

A Simple One-Period Example. We now consider the concepts of complete and incomplete markets in the context of a simple model involving ordinary default-free bonds.

Let us consider a single-period model in which two bonds are available for trading, one of which is a one-period bond and the other a two-period bond. For convenience we shall assume that both bonds are zero coupon bonds. We further assume that the financial markets will evolve to one of two states at the end of the period, "interest rates go up" or "interest rates go down" and that the price of each bond will assume to behave according to the binomial model depicted in Figure 13.

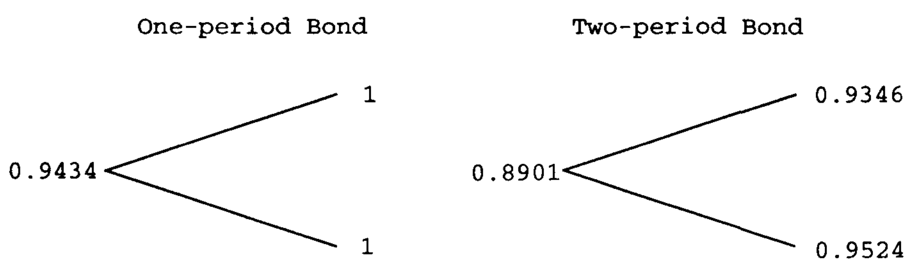

FIGURE 13: Payoffs from one-period and two-period bonds.

The bond prices for this model could be derived from a specification of risk-neutral probabilities and one-period rates but for simplicity we merely display these prices and note that we obtained them from an arbitrage-free model.

Suppose that we select a portfolio of the one-period and two-period bonds. Let us denote the number of one-period bonds held in this portfolio by n_1 and the number of two-period bonds held in this portfolio by n_2. This portfolio will have a value in each of the two states at time 1. Let us represent the state dependent price of each bond at time 1 using a column vector. Then we may represent the value of our portfolio at time 1 by the following matrix equation.

$$\begin{bmatrix} 1 & 0.9346 \\ 1 & 0.9524 \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \tag{2}$$

The cost of this portfolio is given by

$$0.9434n_1 + 0.8901n_2. \tag{3}$$

The 2×2 matrix of bond prices at time 1 appearing in equation (2) is nonsingular. Therefore, any vector of cash flows at time 1 may be generated by forming the appropriate portfolio of these two bonds. For instance, if we want the vector of cash flows at time 1 given by the column vector,

$$\begin{bmatrix} c^u \\ c^d \end{bmatrix} \tag{4}$$

then we form the portfolio

$$\begin{bmatrix} n_1 \\ n_2 \end{bmatrix} = \begin{bmatrix} 1 & 0.9346 \\ 1 & 0.9524 \end{bmatrix}^{-1} \begin{bmatrix} c^u \\ c^d \end{bmatrix}$$

at a cost of $0.9434n_1 + 0.8901n_2$. Carrying out the arithmetic, one finds that the price of each cash flow of the form (4) is given by [1] the expression

$$0.4717c^u + 0.4717c^d. \tag{5}$$

Since every such set of cash flows at time 1 can be obtained and priced in the model we say that the one-period model is *complete*. The notion of pricing in this complete model is justified by the fact that the price we assign to each uncertain cash flow stream is exactly equal to the price of the portfolio of one-period and two-period bonds that generates the value of the cash flow stream at time 1.

Let us see how the model changes when catastrophe risk exposure is incorporated as part of the information structure. Suppose that we have the

[1] With the rounding errors introduced in the calculations the reader who performs this calculation will probably obtain the expression $0.4716c^u + 0.4718c^d$ instead.

framework of the previous model with the addition of catastrophe risk. Furthermore, let us suppose that the catastrophic event occurs independently of the underlying financial market variables. Therefore, there will be four states in the model which we may identify as follows.

$$\begin{aligned}
\{\text{interest rate goes up, catastrophe occurs}\} &\equiv \{u,+\} \\
\{\text{interest rate goes up, no catastrophe occurs}\} &\equiv \{u,-\} \\
\{\text{interest rate goes down, catastrophe occurs}\} &\equiv \{d,+\} \\
\{\text{interest rate goes down, no catastrophe occurs}\} &\equiv \{d,-\}
\end{aligned} \qquad (6)$$

The reader will note that the symbol $\{u,+\}$ is shorthand for "interest rates go up" and "catastrophe occurs" and so forth. This information structure is represented on a single-period tree with four branches such as is shown in Figure 14.

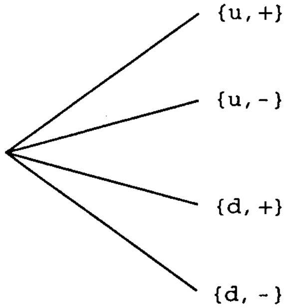

FIGURE 14: Revised states of the world with the introduction of catastrophe risk.

The values at time 1 of the one-period bond and the two-period bond are not linked to the occurrence or nonoccurrence of the catastrophic event and therefore do not depend on the catastrophic risk variable. We may represent the prices of the one-period and two-period bond in the extended model as shown in Figure 15.

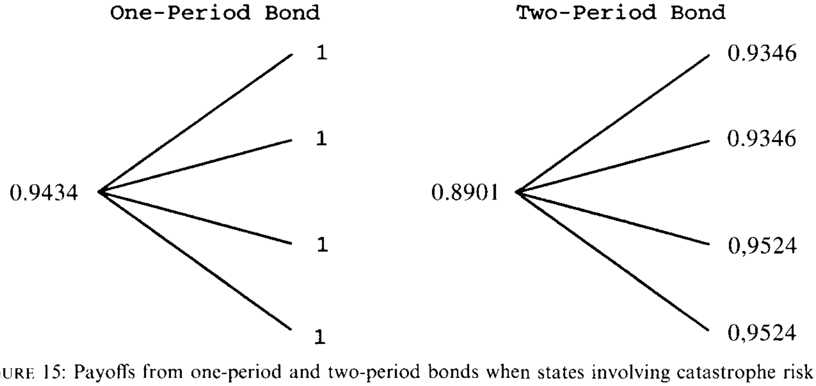

FIGURE 15: Payoffs from one-period and two-period bonds when states involving catastrophe risk are introduced into the model.

In contrast to equation (2), the state contingent payoffs at time 1 of a portfolio of the one-period and two-period bonds is now given by the following matrix equation.

$$\begin{bmatrix} 1 & 0.9346 \\ 1 & 0.9346 \\ 1 & 0.9524 \\ 1 & 0.9524 \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \tag{7}$$

The cost of this portfolio is still given by $0.9434n_1 + 0.8901n_2$. The most general vector of cash flows at time 1 in this model is of the following form:

$$\begin{bmatrix} c^{u,+} \\ c^{u,-} \\ c^{d,+} \\ c^{d,-} \end{bmatrix} \tag{8}$$

On reviewing equation (7) we see that the span of the assets available for trading in the model [*i.e.* the one-period and two-period bonds] are not sufficient to span all cash flows of the form (8). Consequently, we cannot derive a pricing relation such as (5) that is valid for all cash flow vectors of the form (8). The best we can do is to obtain bounds on the price of a general cash flow vector so that its price is consistent with the absence of arbitrage. This can be done using state price vectors and these calculations and some examples may be found in [4, Chapter 5].

Pricing in Incomplete Markets. We will offer only the briefest of indications on how one can price insurance risk securitizations when working in incomplete markets. Details may be found in [4, Chapter 4] and references cited therein.

The benchmark financial economics technique used to price uncertain cash flow streams in an incomplete markets setting is the *representative agent*. The representative agent technique consists of an assumed representative utility function and an aggregate consumption process. Let us suppose that we are in a T-period economy in which agents can make choices and consume each period. The agent makes choices about his future consumption, represented by the stochastic process $\{c(k) : k = 0, 1, ..., T\}$. The aggregate consumption process may be thought of as the total consumption available in the economy (for all agents) at each point in time and in each state of the world. Let us denote the aggregate consumption stochastic process by $\{C^*(k) | k = 0, 1, ..., T\}$. Only the first choice is known with certainty at time $k = 0$. The other choices at future times are random and depend on the random state prevailing when each time point is reached. In "simple" applications it is customarily assumed that the representative agent's utility is time-additive and separable as well as differentiable. Time-additive and separable means that there are utility functions $u_0, u_1, \ldots, u_T$ such that the agent's expected utility for a generic consumption process $\{c(k)|k = 0, 1, \ldots, T\}$ is given by

$$\mathrm{E}\left[\sum_{k=0}^{T} u_k(c(k))\right]. \tag{9}$$

It follows from the theory of the representative agent that the price, which we will denote $V(c)$, of a generic future cash flow process $\{c(k)|k = 1, \ldots, T\}$ at time 0 is given by the expectation

$$V(c) = \mathrm{E}\left[\sum_{k=1}^{T} \frac{u_k'(C^*(k))}{u_0'(C^*(0))} c(k)\right]. \tag{10}$$

If the aggregate consumption process in (10) is known (or can be determined for the model that is being used) then this equation gives a linear pricing relation for all uncertain consumption (or cash flow streams) for the model. This is very much like the risk-neutral expectation that occurs in complete markets valuation but here the model has been "closed" with an explicit assumption on utility. Evidently, different choices of utility functions will generally result in different pricing relations. Note that the aggregate consumption process plays a role in the pricing relation. In many implementations of this pricing relation the aggregate consumption process is assumed to evolve according to an exogenous process.

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Samuel H. Cox and Hal W. Pedersen
Actuarial Science Program,
Department of Risk Management and Insurance,
Georgia State University,
Atlanta, Georgia, 30302 USA
E-mail address, Samuel H. Cox: samcox@gsu.edu
E-mail address, Hal W. Pedersen: inshwp@panther.gsu.edu

Joseph R. Fairchild
Zürich Centre Group,
One Chase Manhattan Plaza, 39th Floor,
New York, NY 10005
E-mail address: joe.fairchild@zurich.com