OCEANIC TIDAL FRICTION: PRINCIPLES AND NEW RESULTS

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The main problems of the hydrodynamical integrations are analyzed. New results are presented for the effect of an extreme ice age.

1. PRINCIPLES

It should be recognized that the angular momentum transfer between Earth and Moon via the hydrosphere has essentially two constituents:

a) The interaction between the Moon and water of the oceans, mediated by the bodily tidal forces. In order to compute the time-averaged torque around the axis of rotation of the Earth, we need only the east-west tangential component, F, of the space density of the force; since F is purely periodical, its time average vanishes. Consequently, we need the tidal elevation part, ζ, of the water depth alone. Then the (time-averaged) net torque, L, on the water turns out to be

$$L_1 = \int_{\substack{\text{surface of} \\ \text{the oceans}}} \int_{\substack{\text{tidal} \\ \text{period}}} \rho[(R \cos \phi)\ \zeta F] dt\ dq, \qquad (1)$$

(R = radius of the Earth, ϕ = latitude, ρ = density of the water, dq and dt are the surface and time differentials). Note that the essential part is the product, ζF.

b) The interaction between the water and the solid Earth mediated by the surface forces of bottom friction. The most realistic representation for the absolute value of the surface density of these forces is thought to be given by the empirical law

$$K^* = r\rho w^2, \qquad (2)$$

where w is the velocity of the water relative to the ground, and r a dimensionless constant (in our computations r = 0.003). With u and v being the east-west and north-south components of w, we can write the east-west component of K*

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$$K = K^* u/w = r\rho uw = r\rho u \ (u^2 + v^2)^{\frac{1}{2}}. \quad (3)$$

In this case the net torque acting on the water (and, with reversed sign, on the solid Earth) is

$$L_2 = \int_{\substack{\text{surface of}\\ \text{the ocean}\\ \text{bottom}}} \int_{\substack{\text{tidal}\\ \text{period}}} [(R \cos \phi)\ K] dt\ dq. \quad (4)$$

The essential "hard core" here is the product uw.

Since the water is neither a sink nor a source of angular momentum, we have

$$L_1 + L_2 = 0. \quad (5)$$

Consequently, the determination of either L_1 or L_2 suffices computationally, but for a complete understanding (and for a test of the computations) the independent determination of both L_1 and L_2 is very desirable. This would seem conceivable using oceanographic observations, but for the time being these are too sparse. At present, we have to rely on theoretical models of oceanic tides. It seems doubtful whether such models can provide independent estimates of L_1 and L_2 because, in general, we expect that only those which include both kinds of interactions will give correct results. This "all or nothing" point of view is, however - and luckily! - not supported by the computational experience. Models without any bottom friction or those which use Laplace's tidal equation and hence use precise harmonic motions lead to $L_2 = 0$ but an astonishingly realistic $L_1 \neq 0$. Since these models violate equation (5) they cannot be stationary in a strict sense. Our models contain the friction according to equation (2) and are therefore, in principle, able to represent both kinds of interactions. In practice, L_1 quickly converged to a stationary value after a few tidal cycles of iterations in which the tidal movements themselves reached a degree of stationarity which is sufficient for usual oceanographic purposes. On the contrary, L_2 needed many more iterations and an unprecedented accuracy to reach even the right order of magnitude.

We can still not give a complete explanation, but two heuristic points are undoubtedly near to the cause: First, the ratio between the terms of the bottom friction and of the pressure gradient in the hydrodynamical equations depends strongly on the depth in such a way that the friction is comparable with the latter only for depths up to several tens of meters (Sündermann and Brosche, 1978). Therefore the bottom friction can manifest itself sufficiently quickly only in the case of shallow seas. Second, the time behavior of the tidal elevation, ζ, and of the tidal velocities, u and v, is mainly harmonic. That is, within the coefficients of a Fourier expansion, the two coefficients belonging to the tidal frequency dominate over all others. Because of the orthogonality of Fourier series, we have Parseval's equation for the time average of a product xy:

$$\overline{xy} = X_0Y_0 + \tfrac{1}{2}\sum_{\nu=1}^{\infty} (X_\nu Y_\nu + X'_\nu Y'_\nu), \tag{6}$$

where X_0, Y_0 are the constants and X_ν, X'_ν, Y_ν, Y'_ν, the coefficients of the sine and cosine terms. Looking into equation (1), we recognize that the time average, $\overline{\zeta F}$, can be reduced to $X_1Y_1 + X'_1Y'_1$ because the force is, by definition, precisely harmonic. Since only the main term in ζ is involved, we get the desired result from first order quantities. In contrast, the Fourier expansion of w contains only even terms ($\nu = 0$, 2, 4, ...) if u and v are purely harmonic (w being, then, the radius in the elliptic path of the velocity vector). The presence of second order terms other than $\nu = 1$ in u and v leads to the occurrence of odd terms of second order in w (and also to small changes of the even terms). Then the time average of uw in equations (3) and (4) becomes a series where all the terms contain a second order factor alternately arising from u or w! In summing up, the numerical solution of the hydrodynamical equations has to be correct in the first order quantities to get a meaningful result from equation (1), but the second order accuracy is necessary for obtaining such a result from equation (4). The numerical results presented in the following section for global ocean models are therefore based on applications of equation (1).

2. NEW RESULTS

So far, all our results refer to the M_2 tide and to a schematic allowance for the elasticity of the Earth by using a reduction factor 0.69 for the effective tidal constant. The resulting torque for our model of the present oceans is in good accordance with astronomical values. As a contrast, we have treated models of the Pangea situation of the continents. We obtained smaller values than for the present configuration, which is desirable in order to prevent an apocalyptic Gerstenkorn event (Brosche and Sündermann, 1977; Sündermann and Brosche, 1978).

A plausible reason for considerable variations of the torque acting in the more recent geological past is the alternation between ice ages and intervening periods. In the case of an ice age more sea water is tied up in the polar ice caps than today and the sea level is consequently lower. While the relative topography is practically the same as at present, it is sufficient as a first approximation to lower the sea level in this case by 100 m for the purpose of representing an extreme ice age. The corresponding tides are very similar to those of the present situation; the latitude distribution is somewhat less concentrated towards an equatorial belt than that of the present tides. The main result is that the average torque is only a few percent greater than the present value:

$$L_1 = -\ 5.2 \times 10^{23} \text{ dyn cm.}$$

Thus we can conclude that at least for more recent geological epochs the variation of the glaciation of the Earth does not seem to complicate the reconstruction of the Earth's rotation. This need not be true for every epoch, because, as we learned from the Permian models, the opening or blocking of crucial passages is of importance for our aims.

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