THE AGE LADDER FROM LOW- TO HIGH-REDSHIFT POPULATIONS

ALVIO RENZINI
Astronomy Department,
University of Bologna,
CP 596 I-40100 Bologna, Italy

ABSTRACT. The determination of the age of stellar populations in progressively more distant stellar systems requires successive calibrations which in some way remind the procedures followed in determining astronomical distances. A few exemplifications of this *Age Ladder* are schematically reviewed, such as the determination of the age of galactic globular clusters, of the youngest population in bulges and ellipticals, and of the most distant, high redshift radiogalaxies.

1. INTRODUCTION: GETTING AGES AND AGE ERRORS

The *Distance Ladder*, from nearby trig parallax stars to quasars, through e.g. galactic star clusters, Cepheids, supergiants, HII-regions, supernovae, rotational velocity and velocity dispersion of galaxies, first ranked cluster galaxies, and the Hubble law, is rather familiar to all of us. How errors propagate from primary, to secondary, to tertiary distance indicators is well known in principle (if not always in practice), and a wise empirical approach has been generally followed, as opposed to distance determination methods making extensive use of theoretical models that may introduce systematic errors which are difficult to quantify. But once we know the distance to galaxies, and thus their size and luminosity, in order to make astrophysics and cosmology, to understand how galaxies have formed and evolved, we need also to know the age (distribution) of their stellar populations. Ages, unfortunately, are more difficult to get than distances. The use of stellar evolution models is unavoidable, and this has far too often resulted in less rigorous measuring procedures, in which observational facts and theory are intermingled in far from optimal combinations, often losing track of how errors propagate and pile up, rather than choose those operational sequences that minimize the final error in age. I believe that the same rigor commonly requested to methods of galactic and extragalactic distance determinations should also be asked to dating methods, and we should speak of an *Age Ladder*, as that particular sequence of operations that starting from the derivation of precision ages for galactic open and globular clusters (GC), goes on making use of such ages to calibrate so-for-saying *secondary* and then *tertiary* age indicators. In this brief review I will restrict to just a few concrete examples that allow to exemplify a method which can be of more general validity.

2. THE TURNOFF LUMINOSITY AND THE AGE OF GLOBULAR CLUSTERS

The output of stellar model calculations are evolutionary sequences which can be used to construct the theoretical *isochrones*. By comparing isochrones to cluster color-magnitude diagrams (CMD) one can derive cluster ages, and this comparison can be made following several different procedures. However, not only the result, but also its uncertainty (i.e. the age error) together with the possibility itself of quantifying such uncertainty may depend on the particular procedure. In this respect, the

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best procedure remains the classical one (e.g. Sandage 1970) in which the single detail of the isochrones that is used is the time-dependence of the luminosity of the main sequence turnoff:

$$\mathrm{Log}\, t_9 \simeq -0.41 + 0.37\, M_V^{TO} - 0.43\, Y - 0.13\, [\mathrm{Fe/H}], \tag{1}$$

where t_9 is the cluster age in Gyr units, M_V^{TO} the absolute visual magnitude of the main sequence turnoff (TO), Y the helium abundance, and [Fe/H] the metallicity in standard notations*. In turn, $M_V^{TO} = V^{TO}-$mod, where V^{TO} (the TO apparent magnitude) is the directly *observable* quantity, and mod is the cluster distance modulus. Equation (1) then allows to estimate the relative importance of the uncetainty in each of the four input quantities (namely: V^{TO}, mod, Y, and [Fe/H]) in determining the total uncertainty of the age determination.

Clearly the *great villain* is the error in the distance of the clusters. Here I have estimated that current distances are typically affected by a 1/4 magnitude error in the modulus – $\sigma(\mathrm{mod}) \simeq 0.^{m}25$ – which immediately translates into a $\sim$ 22% error in the derived cluster age ($\sim$ 3 Gyr for an age of 15 Gyr). All other IQs convey substantially smaller errors. The high photometric accuracy of CCDs now allows to determine a cluster's V^{TO} with an accuracy perhaps better than $0^{m}.1$, which translates into a $\sim$ 9% error in age. The helium abundance is very well known, from either the R method, primordial nucleosynthesis, or empirical determinations of the *pregalactic* abundance, which all indicate $Y = 0.23 - 0.24$. Anyway, a ± 0.02 uncertainty in Y gives a negligible 2% error in age. I assume the metal content of the best studied clusters to be uncertain by perhaps 0.3 dex, which translates into a $\sim$ 9% uncertainty in age. There may be a problem with the *composition* of metallicity (e.g. [O/Fe], see the contributions of Barbuy and Carney), but there is no room to discuss it here. Clearly the *great villain* is the error in the distance of the clusters, and the question *"How good are globular cluster ages?"* immediately becomes: *"How good are globular cluster distances?"*

All in all, just three main standard candles have been used (or are being considered) for the determination of GC accurate distances, namely: RR Lyraes, subdwarfs, and white dwarfs. Pros and cons of each method are discussed elsewhere in some detail (Renzini 1991), suffice here a few schematic considerations.

RR Lyrae are rather bright, but rare objects, and even the nearest one is too distant for a trigonometric parallax to be obtained with an interesting accuracy. The calibration of this standard candle then relies on indirect physical or astrophysical methods, each making extensive use of pulsation, evolution, and stellar atmosphere models in some combination. Unfortunately, the more theory is used, the less we are able to quantify errors. Ultimately, even if we reach self-consistent distances (and then ages), we are left with the doubt that the models may have been slightly inaccurate in some way, thus introducing an unknown bias. A linear relation is usually assumed between the absolute magnitude of RR Lyraes and their metallicity: $M_V^{RR} = a[\mathrm{Fe/H}] + b$,, and the question becomes *"how can we directly measure the slope (a) and the zero point (b)?"*. A calibration of the zero point with $0^{m}.1$ accuracy would need trig paralllaxes with $\sim$ 0.1 m.a.s. accuracy for a suitable sample of RR Lyrae stars, something we will not reach soon. The perspective looks more favorable for the slope a, which also plays an important role in the dating process. As Carney has reviewed at this meeting, some of the indirect methods give $a \simeq 0.35$, which would imply virtually coeval globular clusters in our Galaxy, while other methods suggest $a \simeq 0.2$ which may imply a large trend of the cluster age with metallicity if not compensated by a suitable trend in [O/Fe]. Therefore, disentangling among much different scenarios for the formation of the Galaxy ultimately relies on measuring the slope with better than $\sim 0^{m}.05$ accuracy per dex in [Fe/H]. Obtaining CMDs for the globular clusters in M31 would provide a direct determination of the slope, as such clusters are virtually all at the same diatance, and span the full range of metallicity from $[\mathrm{Fe/H}] \simeq -2$ to ~ 0. HST was supposed to provide the necessary data, but now we will have to wait for the recovery of its full capabilities.

* This relation has been derived by Buonanno *et al.* (1989) from the isochrones of VandenBerg and Bell (1985).

Subdwarfs distances are currently based on five such stars with reasonably well known trig parallax (see van Altena *et al.* 1988). The average error in the modulus of the five calibrators is $<\sigma(\text{mod})>= 0.^{\text{m}}15$, which is rather good. However, the MS location is rather sensitive to [Fe/H], and distance determinations by this method make use of the metallicity of the subdwarfs and of the cluster, which are both subject to errors. Unfortunately the five subdwarfs span a very narrow range in [Fe/H], and to extend the calibration to other metallicities one has to rely on theoretical ZAMS models (or on the main sequence of the Hyades cluster) and proceed to interpolations and extrapolations. When paying attention to the propagation of the errors, one finds that $\sigma(\text{mod}) \simeq \sigma([\text{Fe/H}])$, and therefore the relative error in age is nearly equal to the error in [Fe/H]: i.e. a 0.3 dex error in the cluster [Fe/H] propagates into an error of ~30% in age. All in all, while future efforts (and specially the Hipparcos mission) can improve the trig parallax determinations for very many subdwarfs, the problem with the uncertainty in metallicity may remain.

The basic idea of using white dwarf as standard candles is very simple: to fit the WD cooling sequence of a globular cluster to either the appropriate theoretical or empirical WD cooling sequence (Renzini 1991). The procedure is analogous to the classical main sequence fitting to the local subdwarfs, but with some non-trivial advantages: the method does not involve metallicity determinations which inevitably bring along their uncertainties, and there is no mixing length calibration involved. In fact, WDs have virtually metal free atmospheres, coming either in the DA or non-DA varieties (nearly pure hydrogen or pure helium, respectively), and their radius is insensitive to the mixing length. In the most straightforward case of the fitting to an empirical cooling sequence one has only to apply a small correction taking into account the mass difference between the cluster WDs and the local calibrators. Moreover, WDs are locally much more abundant than subdwarfs, and therefore an accurate trig parallax can be obtained for a much larger sample of calibrators. All in all – with HST working at nominal performance – the distance modulus of a cluster should be obtained with an accuracy better than $\sim 0.^{\text{m}}1$, which translates into a better then 10% accuracy in age. Unfortunately, WDs are very faint, and the useful ones are fainter than $V \simeq 24$ even in the closest globular cluster. The spherical aberration syndrome affecting HST makes presently impossible to reach such faint oblects in the crowded field of the clusters, and again, we may have to wait for the deployment of WFPC II, unless adaptive optics on large ground based telescopes comes first. But I believe that, ultimately, white dwarfs will prove the best standard candles, able to improve the calibration of all other calibrators (RR Lyraes, subdwarfs, etc.).

3. THE TURNOFF COLOR AND THE AGE OF STARS IN ELLIPTICALS

The turnoff luminosity is not the only time-dependent feature of theoretical isochrones. Turnoff colors (or the whole shape of the isochrone from the MS to the base of the RGB) also depend on age, and in principle may be used to estimate cluster ages. Actually, this has been a very popular approach to GC dating, a technique also known as *isochrone fitting*. Having thoretical isochrones in one hand, and observed cluster loci in the other, it may be difficult to resist the temptation of overlapping isochrones to data points, pick up the single isochrone which *best fits* the data, and read out its age. Yet, this temptation should be resisted. Unlike luminosities, the temperatures (colors) of theoretical isochrones are in fact seriously affected by our current, still very rough way of parameterizing the efficiency of the convective energy transfer, which is perhaps the most rudimentary of all ingredients entering in the construction of stellar models. Thus, the *shape* of the isochrones, as well as the location of the lower MS, both depend on the so-called mixing-length parameter $\alpha = \ell/H_{\text{P}}$, which is not a priori known, and which requires an independent calibration. Indeed, when using uncalibrated theoretical isochrones to get GC distances and ages we would build up distance and age ladders on the most uncertain aspect of theoretical models: certainly not a cleaver way of proceeding. Moreover, part of this limitation survives even after the calibration has been applied, as the available calibrators are very few: basically, just the sun and the five best subdwarfs already encountered in the previous section (see VandenBerg 1990). In other words, we cannot be sure that the value of α required to fit e.g. the solar radius, also gives the correct radius (i.e. temperature, color) for turnoff stars in a GC.

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Should we conclude that fitting isochrones to cluster CMDs is a futile exercise? In spite of the remarkably good match often reached, my answer is *"yes"*, if what matters is the determination of accurate GC ages. Indeed, if we have a direct access to TO luminosities, and we have to rely on subdwarfs for the calibration, why bring in all the uncertainties introduced by the theory of convection, that we cannot even quantify? However, for another purpose, matching isochrones to cluster loci is an indispensable step along the age ladder. Having determined in our best way GC distances and ages following the methods described in §2, then the calibration of α via accurate isochrone fitting provides us with a new tool, a clock that we can use for dating stellar systems that we cannot resolve into individual stars, but for which we have access only to the integrated light. This is the population synthesis technique, often applied to dating the stellar populations of elliptical galaxies, in which the correct sequence of operations is: 1) obtain GC distances using suitable standard candles, 2) get their ages, 3) calibrate α via isochrone fitting, 4) construct synthetic spectral energy distributions (SED) using calibrated isochrones, 5) estimate the age of ellipticals by comparing synthetic and observed SEDs. Far too often some of these operations have been omitted, or used in a different order, e.g. getting GC ages via isochrone fitting, or estimating the age of ellipticals using synthetic SEDs based on uncalibrated isochrones.

It is worth emphasizing that the calibration of the mixing-length parameter α is not such a trivial exercise, after all. Suffice to compare the results of different calibrations, such as Fig. 1 in Demarque *et al.* (1988), or Fig. 3 in VandenBerg (1990), which show that even if two calibrations coincide near the MS, they can dramatically differ in the TO region, to the extent that for fixed TO color the inferred age may differ by a factor of two. Therefore, before venturing into cosmological applications of population synthesis it is better to make sure that isochrones provide a nice match from the ZAMS all the way to the tip of the RGB, such as in the case e.g. of the calibrated isochrones of Straniero and Chieffi (1991).

Convection is not the only concern when using the TO color as an age indicator. As frequently emphasized, metallicity is another serious problem (see e.g. O'Connell 1980; Renzini 1986). Suppose we have been able to determine with extreme precision the TO color of a population, from either its CMD or its SED: to get the age we then need an independent estimate of the metallicity of the population. With little algebra, from Straniero and Chieffi (1991) one gets:

$$\left(\frac{\partial \mathrm{Log}\, t}{\partial [\mathrm{Fe/H}]}\right)_{(B-V)^{\mathrm{TO}}} = -0.88 - 0.35[\mathrm{Fe/H}], \tag{2}$$

i.e. $d\mathrm{Log}\, t \simeq d[\mathrm{Fe/H}]$ for near solar metallicity, and a 0.3 dex error in the estimated metallicity translates into a factor of two error in age. A comparison with eq. (1) immediately reveals how much more vulnerable to metallicity errors are ages estimated from TO colors, compared to ages derived from the TO luminosity. In the case of colors the error is up to ten times larger than in the latter case. This argument not only provides further strength to the case against the use of the isochrone fitting method to date GCs, but it reveals also one of the most disturbing intrinsic weaknesses of the population synthesis approach, a drawback that simply cannot be eliminated: the interesting quantity to determine is age, but the SED of a population is more sensitive to metallicity (and to its detailed distribution) than it is to age.

The case of the nearby dwarf elliptical M32 provides the best illustration of the problem. It does not matter here that M32 may not be a typical elliptical, that its evolutionary history may have been much different from that of giant ellipticals. What matters is that we have excellent spectroscopic data for it, photometry of individual stars is now possible, and a long sought test of the population synthesis results is becoming feasible. Over the years the population synthesis method has indicated for M32 1) near solar metallicity ($[\mathrm{Fe/H}] \simeq 0$), 2) no metallicity dispersion, and thus 3) the presence of a dominating intermediate age (~ 5 Gyr old) component (see O'Connell 1986, and references therein). However, thanks to the superior imaging capabilities of the CFH telescope on Mauna Kea, Freedman (1989) has been able to resolve M32 into individual RGB and AGB stars and found $< [\mathrm{Fe/H}] > \simeq -0.5$, or slightly larger (see also her contribution to these proceedings), and a 1σ metallicity dispersion of ~ 0.5 dex. Given the reduction by a factor of 3 in average metallicity, I would maintain that adopting the *observed* metallicity distribution, and applying the *same* population synthesis methods which gave 5 Gyr, thanks to eq. (2) one should now obtain ~ 15 Gyr, i.e. no need for an intermediate age component. After this discussion two questions remain. Why O'Connell's 5 Gyr, [Fe/H] = 0 synthetic spectrum fits so well M32, while we know from Freedman that there is a large metallicity dispersion, and the average is below solar? Why Bica *et al.* (1990) cluster-melange method prefers a small metallicity dispersion and a large spread in age instead of the contrary? Answering these questions will help to understand first how our population synthesis tools work, and then how to most effectively apply them to other galaxies.

Finally, I would like to call attention to another relevant aspect. To my understanding, all evolutionary population synthesis models constructed so far have assumed a RGB made of K-type giants. Yet, the upper RGB of near-solar metallicity GCs in the Galactic Bulge is instead populated by M-type giants (Ortolani *et al.* 1990, see also this volume). In super metal rich (SMR) populations – such as those dominating in giant ellipticals – an even larger fraction of the RGB and AGB is likely to be populated by M giants. This means that perhaps $\sim 30\%$ or more of the total bolometric light may come from M giants, thus considerably decreasing the RGB and AGB contributions in the visual, compared to models in which the giant branch is supposed to be made of K giants. Curiously enough, turning K giants into M giants will make the synthetic $B-V$ *bluer* (and the $V-K$ redder), thus increasing the estimated age by an amount which remains to be properly evaluated.

4. THE AGB TIP AND THE AGE OF YOUNGEST STARS IN BULGES

Besides the TO region, other parts of the CMD are sensitive to age. For example, the color and morphology of the horizontal branch (HB) and the luminosity extension of the AGB are both sensitive to age, and can be used in age estimates. Unfortunately, both depend also on the amount of mass loss during the red giant phases, which is not known with the necessary precision from either observations or theory. This implies that the HB morphology and the AGB extension cannot be used for *absolute* age determinations, but relative age estimates are possible once mass loss is properly calibrated using populations of independently known age. Again, we find here another methodological analogy with the distance ladder: the dermination of the age of GCs via eq. (1) allows a calibration of mass loss once we demand that theoretical HBs and AGBs fit those observed in Galactic GCs. Having done so, it becomes possible to use the HB and/or the AGB of other clusters or populations to infer their age difference with respect to the calibrating clusters. The HB as a clock is discussed by others at this meeting (e.g. Y.-W. Lee and Suntzeff in connection with *Second Parameter* effects, while for a possible use on moderate redshift galaxies see Greggio and Renzini, 1990). In this section I will then concentrate on the AGB.

In Galactic GCs the AGB barely extends above the RGB tip, which implies that mass loss has to prevent the stellar core from growing beyond $0.54 - 0.55M_{\odot}$ (Renzini and Fusi Pecci 1988). Several clusters of the Magellanic Clouds contain instead AGB stars which are much brighter than the RGB tip, and since Mould and Aaronson (1979) this is currently interpreted as an age effect. Fig. 1 shows theoretical RGB and AGB luminosity functions (LF) appropriate to stars less massive than $\sim 2M_{\odot}$. In these AGB models the core is allowed to grow up to $\sim 0.6M_{\odot}$, and therefore the composite RGB+AGB luminosity function is one appropriate to an intermediate age (few Gyr) stellar population. Note that the LF drops by a factor of ~ 4 at the tip of the RGB, as the RGB evolution is suddenly terminated by the onset of the core helium flash. As age is increased, the main effect is merely a decrease of the luminosity of the AGB tip, until in a very old (~ 15 Gyr) population the AGB tip gets very close or even fainter than the RGB tip whose luminosity is virtually independent of age. One can conclude that in a composite population consisting of an old plus an intemediate age component the LF must exhibit a drop by *more* than a factor of 4 at the luminosity corresponding to the tip of the RGB. Moreover, only having identified this feature in the LF one can draw the bottomline above which stars can certainly be ascribed to the AGB. More quantitatively, the expected RGB drop in the LF is given by $\sim 4(L_{\rm OLD} + L_{\rm IA})/L_{\rm IA}$, where $L_{\rm OLD}$ and $L_{\rm IA}$ are

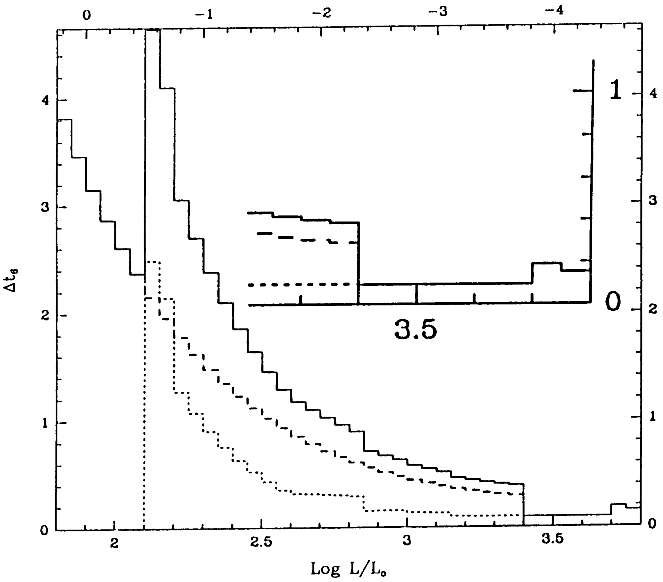

Fig. 1. The RGB (dashed line), AGB (dotted line), and cumulative (RGB+AGB, solid line) luminosity function for a population older than ∼ 1 Gyr, with $(Z, Y) = (10^{-3}, 0.3)$. The vertical scale is in 10^6 yr per luminosity bin $\Delta \mathrm{Log}\, L = 0.05$, while the upper scale gives the bolometric magnitude. The inlet shows a magnification of the top end of the LF, showing the drop associated to the RGB tip which for $Z = 10^{-3}$ is at $\mathrm{Log}\, L/L_\odot = 3.4$ ($M_{\rm bol} \simeq -3.8$). The RGB tip get ~ 0.25 mag brighter every dex increase in metallicity. The RGB and AGB models are respectively from Sweigart and Gross (1978) and Gingold (1974).

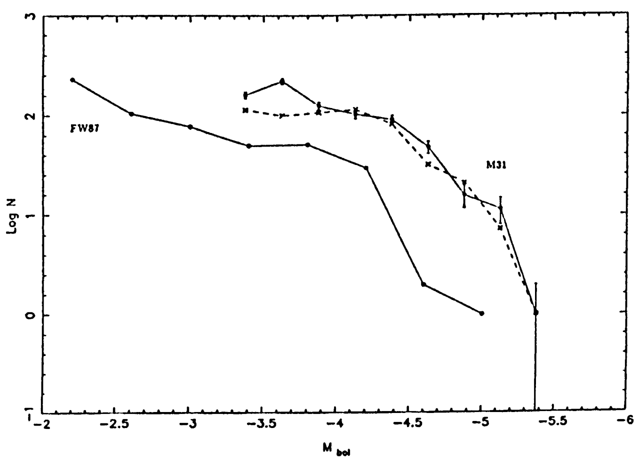

Fig. 2. The luminosity functions of the bulge of M31 (Mould and Rich 1991), and of Baade's window in the Galactic Bulge (Frogel and Whitford 1987, labelled FW). Note the absence in the LF of M31 of any feature resembling the RGB drop in Fig. 1. In the Galactic Bulge the only drop is at $M_{\rm bol} \simeq -4.2$, suggesting this feature being the RGB drop itself.

respectively the total luminosity of the old and of the intermediate age components.

Fig. 2 shows the LF of the Galactic Bulge from Frogel and Whitford (1987), compared to the one that Mould and Rich (1991) have obtained for the bulge of M31. From the lack of a drop off at $M_{\rm bol} \simeq -4.2$ Mould and Rich infer (among other possibilities) that the bulge of M31 may contain an intermediate-age component that according to Frogel and Whitford is not present in the Galactic Bulge. Yet, the LF of M31 does not show any sign of the expected RGB tip drop off, which in a SMR population should be found around $M_{\rm bol} \simeq -4$: something must have washed it out. I have no explanation for this, but certainly we need to uderstand why there is no RGB drop off before using the LF in Fig. 2 to make age inferences. Yet, this is still not enough. We need also to know what is the AGB tip luminosity in a SMR, 15 Gyr old population, so as to set the bottomline above which stars can be ascribed to an intermediate age component. Unfortunately, the composition dependence of the luminosity of the AGB tip has not been sufficiently explored, and recent calculations (e.g. Lattanzio 1991) don't extend beyond solar. Bulges instead contain SMR stars (see Rich, this volume), and for a moderate galactic helium enrichment ($\Delta Y/\Delta Z \simeq 3$) such stars would also be strongly enriched in helium (i.e. $Y \gg Y_{\odot}$, note that bulge stars would have really extreme composition if $\Delta Y/\Delta Z \simeq 6$, as advocated by Pagel at this meeting). In conclusion, while waiting for more extended evolutionary calculations we are left with the unanswered question: *"Are large AGB luminosities in bulges a result of young ages or of large $Z + Y$?"* To answer we need a suitable calibration of both giant branch tips as a function of composition, in particular in the SMR regime.

Even after the identification of the RGB tip and of the old age AGB tip, an additional complication remain. Blue stragglers (BS) are now found abundantly in globular clusters, and almost certainly BSs are the result of the merging of two main sequence stars in a close binary. Following merging BS evolve as single star with mass up to two times the turnoff mass of the parent population, and therefore when on the AGB they climb to much higher luminosities than single stars do. It follows that a BS progeny could be mistaken for an intermediate age component. Scaling from the frequency of BSs in M3, Renzini and Greggio (1990) estimate the number of BS progeny stars in a population to be $N_{\rm j}^{\rm BSP} \simeq 6 \times 10^{-13} L_{\rm T} t_{\rm j}$, where $L_{\rm T}$ is the total luminosity of the population, and $t_{\rm j}$ is the duration of the j$^{\rm th}$ evolutionary phase. From this relation, adopting $10^{10} L_{\odot}$ for the luminosity of the Bulge, and 2×10^6 yr for the duration of the TP-AGB we obtain $N_{\rm TP-AGB}^{\rm BSP} \simeq 6 \times 10^{-13} \times 10^{10} \times 2 \times 10^6 = 12,000$ stars in the whole Bulge, a number that can be appropriately scaled to the actual luminosity sampled by a CCD frame or IR array. This is not a completely negligible contribution, and the presence of an intermediate age component in bulges (and in M32, see Freedman's contribution) would be confirmed only if the actual number of AGB stars brighter than the (still poorly known) AGB limit of an old SMR populations were to exceed this estimate, i.e. ~ 1 star every $10^6 L_{\odot}$. Conversely, in a few Gyr single age population the expected frequency of TP-AGB stars is about 30 times larger: the actual counts will decide.

5. THE AGB, THE RGB, AND THE AGE OF HIGH-z RADIOGALAXIES

Dating stellar populations in nearby elliptical galaxies being a rather cumbersome affair (see §2), the existence of *features* in the time evolution of the SED would certainly help to pinpoint ages more precisely. The rapid development at fairly precise ages of the AGB and later of the RGB (the so-called phase transitions) represent two such features (Renzini and Buzzoni 1983, 1986), and soon it has been attempted to use them in dating high redshift galaxies (e.g. Wyse 1985). Here I would like to report on recent progress along these lines.

In drawing their Fig. 5 showing the relative contribution to the integrated light of a population, Renzini and Buzzoni made use of the best stellar models available in the early '80s, without any attempt at fudging with parameters, but making clear that a calibration of mixing length, mass loss, and overshooting was necessary before applying their theoretical predictions to galaxies. Also, the natural calibrators were indicated in the GCs of the Magellanic Clouds, which span the interesting range in age, and contrary to open clusters are populous enough for the purpose. As pointed out by RB (and later emphasized by many other investigators) their Fig. 5 gives a gross overestimate of the AGB contribution for clusters younger than a few 10^8 yr, such as NGC 1866 that *uncalibrated* theory predicted to have some 10 bright AGB stars, while it has none. Indeed, the problem of the missing bright AGB stars in Magellanic fields and clusters has been around for over 10 years (see e.g. Frogel *et al.* 1990, and references therein), and a number of possible solutions have been proposed. I should say that I have never been satisfied with any of such solutions of the *"AGB mystery"*, including those that I have entertained myself. Now, finally, the real solution may have been found. Blöcker and Schönberner (1991) have recently computed TP-AGB models for a $7 M_{\odot}$ star, with a mixing-length parameter $\alpha = 2$. As expected (see Renzini and Voli 1981), their models experience strong *envelope burning* process, but most surprizingly their luminosity evolution is much faster than predicted by the famous Paczyński's core mass-luminosity relation. The models spend between $M_{\rm bol} = -6$ and -7 a time which is nearly 10 times shorter than was the case for models assuming Paczyński's relation, such as those of Renzini and Voli. These calculations suggest that, climbing quickly to very high luminosity, the more massive AGB stars which experience envelope burning will rapidly run into severe mass loss, thus leaving the AGB and evolving towards their final white dwarf configuration. The final result is a drastic reduction of the TP-AGB lifetime and of the amount of fuel which is burned during the TP-AGB, i.e. of the TP-AGB contribution to the total light of a population. Both results now offer what appears to be an excellent opportunity to finally explain the AGB luminosity function of MC clusters and fields (see Mould's contribution), while the high lithium abundance in bright AGB stars provides independent evidence for the envelope burning process being indeed active for $M_{\rm bol} \lesssim -6$ (Smith and Lambert 1989). It is worth emphasizing that Blöcker and Schönberner's important results undermines two generally entertained dogmas: the universality of the core mass-luminosity relation, and the insensitivity of the stellar luminosity to the mixing-length parameter. *A posteriori*, it looks quite natural that both are invalid in presence of envelope burning, and – after all – it is somewhat reassuring for theory that the solution to the AGB mystery can evenually be found in just properly calibrating the envelope mixing-length parameter α!

Fig. 3 is an update of the old RB's Fig. 5. The input models are the same, apart from the use of Sweigart *et al.* (1989) models to nicely delineate the RGB phase transition at $t \simeq 5 \times 10^8$ yr*. Furthermore, a sharp switch on of envelope burning is assumed in stars more massive than $3M_\odot$ (the value appropriate for $\alpha = 2$ and $Z = 0.01$, see Renzini and Voli 1981), thus reducing by a factor of 10 the fuel consumption during the TP-AGB. Admittedly, this is still no more than an educated guess. Note that what was meant by AGB phase transition now splits into two evolutionary bifurcations: the first one still at $M \simeq 9M_\odot$ corresponds to the appearance of stars with degenerate C-O cores (i.e. the AGB transition in the *old* sense), the second one at $M = M_{\rm HBB}$ (in the notations of Renzini and Voli 1981) corresponds to the rather sharp separation between stars with and without envelope burning. The mass (age) at this separation is a function of α, and decreases from $M_{\rm HBB} \simeq 9M_\odot$ for $\alpha \lesssim 1$, to $\sim 3M_\odot$ for $\alpha = 2$, as assumed in drawing Fig. 3. Thus, for $\alpha = 2$ the first transition at $t \simeq 2 \times 10^7$ yr is not associated with the appearance of a major AGB contribution to the total light (see Fig. 3), while such a contribution is substantially delayed to $t \simeq 2 \times 10^8$ yr, and gets close to the RGB phase transition which rapidly brings the RGB contribution over 20%. In this way, the AGB+RGB contribution rapidly climbs from a fairly small fraction for $t \lesssim 2 \times 10^8$ yr, to over 50% at $t \simeq 10^9$ yr. It is particularly attractive to ascribe to this combined AGB+RGB development the very rapid jump of $V - K$ from ~ 1 to ~ 3.5 exhibited by SWB type 4 clusters of the Magellanic Clouds (see Fig. 3 in Renzini 1991). No such a major effect is predicted on the optical colors, since AGB and RGB stars radiate mostly in the near-IR.

What are the perspectives of detecting this ~ 2.5 mag color jump in young ($\lesssim 10^9$ yr), high redshift galaxies? Could the *"Red Bump"* in the SED of some 1 Jansky radiogalaxy be ascribed to the AGB or RGB phase transitions? Lilly (1988) has derived an age of $\gtrsim 10^9$ yr for one radiogalaxy at $z = 3.4$, and such an old age at such a large redshift would push the epoch of galaxy formation at $z > 10$, at odd with the standard CMD scenario. Lilly's determination relies on models which assume a 1 Gyr duration of the star formation process (Bruzual 1983), and in which the red giant component is not adequately treated. Either of these two aspects may have affected the result. More recently Chambers and Charlot (1990) have re-analyzed the problem, concluding that the galaxy with the highest redshift in their sample ($z = 3.8$) has an age of only $\sim 3.3 \times 10^8$ yr, thus somehow releaving the pressure on CDM. What is new in Chambers and Charlot's approach compared to that of Lilly? First they have assumed a shorter duration for the star formation process (i.e. $\sim 10^8$ rather than 10^9 yr), second, they have attempted a more sophisticated approach to the population synthesis, in particular improving the treatment of the red giant evolutionary phases, and assuming

* Sweigart *et al.* models have been implemented with a few HB and Early-AGB sequences kindly provided by Chieffi and Straniero.

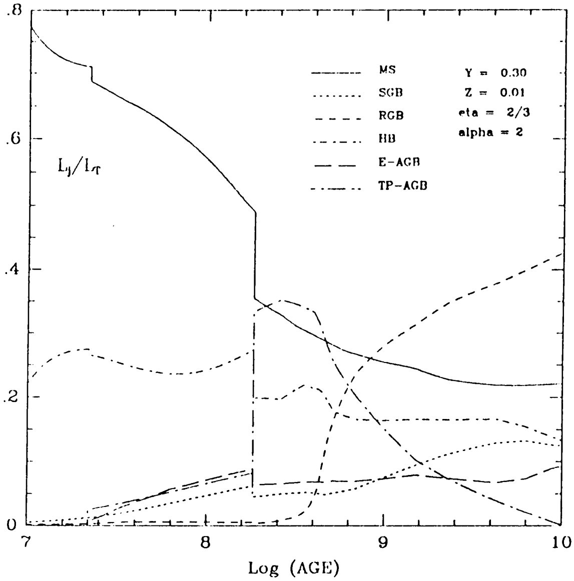

Fig. 3. The time evolution of the relative contributions L_j/L_T of stars in the various evolutionary stages to the integrated bolometric light of an evolving stellar population. Composition, mass loss, and mixing-length parameters (η and α) are indicated. No overshooting is assumed. The contributions of the Early- and Thermally Pulsing-AGB phases are plotted separately.

an important AGB contribution to appear at $t = 3 \times 10^8$ yr. At first sight I was striken by the derived age being only 10% older than the assumed AGB phase transition, is this transition the origin of the *red bump*? Probably not. A more meditated analysis of the result reveals in fact that Lilly's *red bump* isn't really very red, corresponding to a rest frame wavelength of only $\sim$ 5000 Å, where late type (M or C) AGB stars should contribute little flux. For this reason I am now inclined to believe that the younger age follows from the fact that Chambers and Charlot have assumed a 10 times shorter e-folding time for star formation (compared to Lilly). If this is the true reason, then it implies that the observed SED only constrains the bulk of star formation to have been completed some 10^8 yr ago, but tells very little on the age of the galaxy, i.e. on the beginning of the star formation, and therefore on the epoch of galaxy formation. It follows that other ways have to be envisaged to date high redshift galaxies, and the combined AGB+RGB phase transitions – with the associated 2.5 mag jump in $V-K$ – may help in this respect. Still, at $z = 4$ the restframe $V-K$ would be observed as $K - 10\mu$, and it may take a long time before we will be able to detect high redshift radiogalaxies at 10μ.

6. CONCLUSIONS

Having discussed four dating methods along the age ladder, I would like to conclude with a few schematic sentences:

- Current GC dating methods give ages of 13–15±3 Gyr. If $H_o = 100$, then we need $\Lambda \neq 0$, with $\Lambda + \Omega = 1$ to keep the inflationary scenario. For this among other reasons $\Lambda \neq 0$ is not regarded now as ugly as it used to be years ago (see e.g. Weinberg 1989; Peebles 1991).
- The question whether ellipticals contain an intermediate age component remains unsettled by population synthesis methods. Numerical experiments with synthetic populations would be very valuable in assessing the age vs. metallicity *resolution* of such methods.
- My prejudice is that bulges are (on average) older than halos (Renzini and Greggio 1990). To test this idea it would be interesting to compare the HB luminosity vs. HB color clocks, along the lines presented by Suntzeff and Y.-W. Lee at this meeting. Also, counts to luminosity ratios are needed to assess whether the brightest stars in bulges belong to an intermediate age AGB, or what.

• It may take some time before we are able to date very high-z radiogalaxies, but the mere existence of $z = 5$ quasars supports the notion of $z_{\rm GF} \gtrsim 10$ (see Turner 1991).

I am indebted to Laura Greggio and to Michele Guastamacchia for the plot of Fig. 1 and Fig. 3, respectively, and to Sandro Chieffi and Oscar Straniero for having expressly computed and provided several evolutionary sequences incorporated into Fig. 3.

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DISCUSSION

Y.-W. LEE: You mentioned the use of HST to observe M31 globular clusters to obtain the HB luminosity-[Fe/H] relationship directly. While I agree with you 100%, I like to remind you that there is another, much cheaper method already available. That is the use of RR Lyrae in ω Cen. Since there is a wide range in [Fe/H], and since all RR Lyrae in ω Cen are located at the same distance, we can use the apparent V mag vs. [Fe/H] relationship directly. The result (see Lee 1991, *Ap. J. (Letters)*, , June 1) supports the magnitude difference of 0.2 mag for 1 dex variation in [Fe/H], consistent with LDZ HB models.

RENZINI: You are right, ω Cen certaily helps, but its metallicity range (in total Z, not just Fe) may be rather modest and the precision with which one can derive the *slope* correspondingly reduced.

MATEO: I have two comments. (1) As you pointed out, some CGs have blue stragglers. As it turns out, some of these extend into the instability strip and are seen as pulsating dwarf Cepheids. These stars follow a P-L relation that can be trigonometrically calibrated by Hypparcos. So a 4th distance determination method for GCs may soon be available. (2) You may recall from my talk that I plotted the $(V-K)_o$–age relation using Persson *et al.* (1983) data and *new* age estimates for the clusters. I remind you that there was only one break in this relation at 10^9 yrs corresponding to the RGB phase transition; no feature is visible at the AGB transition which I assume will show up at about 10^8 yrs.

RENZINI: I think there are in LMC clusters in which the AGB is well developed, as testified by the presence of bright carbon stars, while the RGB is still lacking. One such case is NGC 2209, which unfortunately is not in the Persson *et al.* sample. Notice that only $\sim 2-3\times 10^8$ yr separate the development of the two branches (cf. Fig. 3), while SWB type 4 cluster are rather few and span a $\sim$ 3 to 4 times longer age interval. So we don't expect to find many clusters which are older than the AGB phase transition, but younger than the RGB one. Still, I agree with you that the AGB transition alone (at $t \simeq$ few 10^8 yr) cannot account for the whole jump in $V-K$, but it needs the assistence of the RGB transition.

MOULD: (1) You should be looking for the RGB tip discontinuity closer to $M_{\rm bol}=-3.6$, and at that luminosity the M31 bulge counts are very incomplete. (2) There are more than 10^5 stars brighter than $M_{\rm bol}=-4$ in a $10^{10}L_{V\odot}$ M31 bulge scaled to our field.

RENZINI: If the RGB tip is as faint as $M_{\rm bol}=-3.6$, then why at this luminosity the LF of Baade's window is so perfectly smooth? Frogel and Whitford 's counts are supposedly complete down to much fainter luminosities. The only clear drop is at $M_{\rm bol}=-4.2$, and thus I suspect that this is the RGB tip in Baade's window. Otherwise we have a serious problem with either the counts or the RGB theory. (2) OK, I'm looking forward to the detailed counts, and to their dependence on the assumed bottomline.

FROGEL: (1) The luminosity function for M31 that R. Davies, D. Terndrup and myself have determined has an even slower fall off above $M_{\rm bol}=-4.5$ than that of Rich and Mould. (2) The steep fall off in the bulge luminosity function undoubtely refers to the termination of AGB evolution, not of RGB evolution. I have no idea why the RGB tip is not seen in our Baades's window data. (3) It is really essential that models be made to predict terminations of AGB evolution as a function of mass and [Fe/H]. It is a mistake to keep using LMC clusters for galaxy models as the AGB stars in them have [Fe/H] significantly lower than the mean for E galaxies and spiral bulges.

RENZINI: (1) No comment. (2) I am not so certain. If the drop is due to the AGB then your Baade's window luminosity function should be dramatically incomplete already at the RGB tip, which has to be around $M_{\rm bol}=-4.0$. Again, it makes more sense to me to look at the $M_{\rm bol}=-4.2$ drop as due to the RGB tip. (3) I agree.

FREEDMAN: Jay Frogel has argued that in the Galactic Bulge the tip of the AGB occurs at $M_{\rm bol}=-4.2$ due to high metallicity. However the tip of the RGB has not been identified in the

Bulge luminosity function. As Alvio has pointed out, the tip of the RGB should be easy to detect, certainly in the case of the Galactic Bulge. The lack of a sharp cutoff in the Bulge LF at $M_{\rm bol} \gtrsim -3.7$ may be suggesting either that the slope of the RGB tip luminosity vs. metallicity relation is steeper at high metallicities (than measured for Globular clusters) or that the LF of the Bulge based on the Blanco M giant surveys is incomplete. In any case, it is still interesting that the M31 LF shows no sharp drop at $M_{\rm bol} = -4.2$ unlike the Galactic Bulge LF.

RENZINI: Yes, a range in metallicity will smooth out the RGB drop in the LF, but a sharp ridge line should still be recognizable in a well populated CMD.

PAGEL: (1) I do not claim $\Delta Y/\Delta Z = 6$ at high metallicity, only at low metallicity, while agreeing that the resulting trend is contrary to naïve theoretical expectations. (2) I believe that Th/Eu ratios in the few halo field stars for which they are available are an argument against an age as low as 10 Gyr; 15 Gyr seems more reasonable.

RENZINI: (1) Indeed, with a straight $\Delta Y/\Delta Z = 6$, in SMR stars with $Z = 5Z_\odot = 0.1$ hydrogen should be reduced to a trace element! I'm looking forward to see a non-naïve theory giving $\Delta Y/\Delta Z = 6$ at low metallicity, (2) 15 Gyr is also my preferred value.