

# The systematically varying stellar IMF

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**Abstract.** Some ultra-compact dwarf galaxies have large dynamical mass to light ( $M/L$ ) ratios and also appear to contain an overabundance of LMXB sources, and some Milky Way globular clusters have a low concentration and appear to have a deficit of low-mass stars. These observations can be explained if the stellar IMF becomes increasingly top-heavy with decreasing metallicity and increasing gas density of the forming object. The thus constrained stellar IMF then accounts for the observed trend of metallicity and  $M/L$  ratio found amongst M31 globular star clusters. It also accounts for the overall shift of the observationally deduced galaxy-wide IMF from top-light to top-heavy with increasing star formation rate amongst galaxies. If the IMF varies similarly to deduced here, then extremely young very massive star-burst clusters observed at a high redshift would appear quasar-like (Jerabkova *et al.* 2017).

**Keywords.** stars: luminosity function, mass function; galaxies: stellar content; galaxies: star clusters; galaxies: starburst; galaxies: high-redshift; quasars: general

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## 1. Introduction

The stellar IMF is valid for a simple stellar population as emerges, on a time-scale of about one Myr, from a molecular cloud core (i.e. on its dynamical collapse time scale) in which forms an embedded cluster on a scale of a pc or less containing dozens to many millions of stars. The galaxy-wide IMF, gwIMF, in contrast is the composite IMF resulting from the addition of the individual IMFs forming throughout the system.

The above is summarised in this contribution, with the observationally constrained gwIMF being used as a boundary condition on the independently deduced variation of the IMF. Definitions:

- The *stellar IMF* (IMF),  $\xi(m) = dN/dm$ , where  $dN$  is the infinitesimal number of stars born together (in one embedded cluster) with individual masses in the interval  $m$  to  $m + dm$ . The *canonical IMF* has the shape  $\xi(m) \propto m^{-\alpha_1}$ , with  $\alpha_2 \approx 2.3$ ,  $0.5 M_\odot < m < m_{\max}$  being the Salpeter power-law index and  $\alpha_1 \approx 1.3$ ,  $m < 0.5 M_\odot$ . Here,  $m_{\max}$  is the mass of the most massive star forming in an embedded cluster with a total stellar mass of  $M_{\text{ecl}}$  (Weidner *et al.* 2013). Since stars typically form as binary systems (Goodwin & Kroupa 2005), the molecular cloud filament density variations which fragment to stars (André *et al.* 2010) are related to the initial system mass function (eq. 4-57 in Kroupa *et al.* 2013).

- The *galaxy-wide IMF* (gwIMF) is the sum of all IMFs over a whole galaxy (Yan *et al.* 2017; Jerabkova *et al.* 2018; Hopkins 2018). The calculation of the gwIMF is performed, in the integrated galaxy IMF (IGIMF) theory, by integrating over all embedded clusters formed in the galaxy over a period of about 10 Myr. The 10 Myr time-scale (see Schulz *et al.* 2015; Yan *et al.* 2017; Jerabkova *et al.* 2018 for discussions) is given by the time-scale for local collapse of the interstellar medium (ISM), the thickness of the gas disk being

about 100 pc and the velocity dispersion of the warm ISM being about 10 pc/Myr. In general, the shapes,  $\text{gwIMF} \neq \text{IMF}$ , because the  $m_{\text{max}} - M_{\text{ecl}}$  relation (Kirk & Myers 2011, 2012; Ramírez Alegría *et al.* 2016; Stephens *et al.* 2017; Oh & Kroupa 2018) implies low-mass embedded clusters to not have massive stars. Within the IGIMF theory, the gwIMF is only comparable to the IMF in a galaxy with a star-formation rate (SFR)  $\text{SFR} \approx 1 M_{\odot}/\text{yr}$  occurring at about solar metallicity (Yan *et al.* 2017; Jerabkova *et al.* 2018; Zonoozi *et al.* 2019). The IGIMF varies systematically with the star formation rate and metallicity of the galaxy because the IMF varies on the molecular-cloud-core scale. This variation of the IMF and of the gwIMF carries implications for extragalactic astrophysics and for cosmology.

## 2. The IMF from solar-neighbourhood star counts

A vast amount of effort has been conducted to constrain the stellar IMF from the nearby stars, starting with the Salpeter-1955 foundation through to the seminal contributions by Miller & Scalo in 1979 and Scalo in 1986. That the internal structure of stars changes around  $0.3 M_{\odot}$  (less massive stars being fully convective) led to the realisation (Kroupa *et al.* 1990; Kroupa & Tout 1997) that the stellar luminosity function ( $\Psi(M_X) = dN/dM_X$ , where  $M_X$  is the absolute stellar magnitude in the photometric  $X$ -band, e.g.  $X = V$ ) has a pronounced maximum at  $M_V \approx 11.5$ ,  $M_I \approx 8.5$ . Together with corrections of the star-counts for unresolved binary stars, biases in photometric- and trigonometric parallaxes as well as the age, metallicity and spatial distribution of low-mass stars of different masses established the canonical IMF for  $m \lesssim 1 M_{\odot}$  (Kroupa *et al.* 1993). This form, corrected for the first time for unresolved binary systems, is virtually identical to the confirmation (subject to the caveat that corrections for unresolved binary stars need to be applied properly) by Chabrier in 2003 and Bochanski, Hawley, Covey *et al.* in 2010 who however, used a log-normal form for  $\xi(m)$  for  $m < 1 M_{\odot}$ . For  $m \gtrsim 1 M_{\odot}$ , significant corrections for short stellar life times and for the different spatial distribution of the massive stars compared to low-mass stars need to be taken into account. The Milky-Way (MW) gwIMF has  $\alpha_3 \approx 2.7$  (Kroupa *et al.* 1993, based on Scalo's 1986 analysis), while more recent determinations yield  $\alpha_3 \approx 2$  (Mor *et al.* 2019; Zonoozi *et al.* 2019).

Important is to realise that this solar-neighbourhood IMF is in fact the MW gwIMF, sampled from the stars within the solar neighbourhood. The true form of the MW gwIMF remains disputable, because the stellar ensembles used to constrain it differ: stars with  $m \lesssim 0.5 M_{\odot}$  come from a region around the Sun spanning only a few pc; stars with  $m \approx 1 M_{\odot}$  extend the ensemble to about 20 pc while more massive stars are taken from regions of the Galactic disk extending to kpc distances. Thus, differences in the velocity distribution function (and thus vertical scale height) and the time variation of the past local SFR have an impact on the form of the local gwIMF deduced from the star counts (Elmegreen & Scalo 2006).

## 3. The IMF from embedded, open and globular star clusters and from ultra-compact dwarf galaxies

Direct star counts in nearby embedded and young clusters have been taken to imply an invariant IMF which is consistent with the canonical form (Kroupa 2002; Bastian *et al.* 2010; Offner *et al.* 2014). But perhaps the most dramatic example of a non-canonical stellar population is the observationally deduced IMF near the Galactic centre where stars appear to have formed in the past few Myr with  $\alpha_3 \approx 0.45$  and very few if any detected low-mass stars (Bartko *et al.* 2010). This suggests a strong variation of the shape of the IMF with the physical conditions of the star forming gas, whereby shear may be playing a significant role in this particular case. The dynamical mass-to-light ratio,  $M/L$ , of some ultra-compact dwarf galaxies (UCDs, ‘‘Hilker objects’’, Hilker *et al.* 1999) can be

represented by an initial stellar population with a top-heavy IMF which depends on the birth density of the UCD (Dabringhausen *et al.* 2009). Some UCDs have an oversurplus of low-mass X-ray binaries, which can be explained by the same dependency of the IMF (Dabringhausen *et al.* 2012). A very young UCD with a top-heavy IMF expands to become a present-day UCD (Dabringhausen *et al.* 2010). Some globular clusters (GCs) have a deficit of low-mass stars and at the same time a low concentration (De Marchi *et al.* 2007). While inconsistent with energy-equipartition-driven cluster evolution, this trend can be explained by an IMF which becomes increasingly top-heavy with decreasing metallicity and increasing density (Marks *et al.* 2012). A non-trivial outcome of this work is that the UCD and GC calculations lead to the same IMF variation. This is non-trivial because the data sets are completely different involving different physical processes, yet yield a consistent result. The GCs in Andromeda show a  $M/L$  trend with metallicity which is explainable by this same IMF variation (Zonoozi *et al.* 2016; Haggi *et al.* 2017). Local star-burst clusters are efficient ejectors of their massive stars (Oh *et al.* 2015; Oh & Kroupa 2016). When statistically adding them back into an observed very young cluster, such as R136 in the Large Magellanic Cloud, the IMF of R136 is implied to have been top-heavy (Banerjee & Kroupa 2012). Direct star-counts have confirmed this (Schneider *et al.* 2018). Direct star-counts also support the IMF of low-metallicity very young clusters to be top-heavy (Kalari *et al.* 2018).

In summary, the evidence for a systematic IMF variation has become very significant. While the detailed form of the variation remains unclear to some degree, the particular formulation available (Marks *et al.* 2012) appears to largely embrace this variation. As a note for completeness, MW star counts for different populations with different metallicity have been noted to suggest the IMF to be bottom heavy at super-solar metallicity and bottom-light at low-metallicity (see Marks *et al.* 2012 and references therein).

#### 4. Consistency with theoretical expectations

Until about 2008 the lack of evidence for a variable IMF was disconcerting (see discussion in Kroupa *et al.* 2013). The evidence which started forthcoming thereafter (Sec. 3) became possible through improved resolved stellar population data and improved theoretical understanding of the stellar and dynamical evolution of these populations.

The theoretical expectation for a variable IMF has been rigorous and time-lasting, because two main broad but entirely independent arguments lead to the same expectation of an IMF which should shift to top-heavy with decreasing metallicity and increasing density. The one argument rests on the Jeans mass instability in a molecular cloud (Larson 1998) while the other argument rests on self-regulation of the accretion flow onto a forming star (Adams & Fatuzzo 1996; Adams & Laughlin 1996). Metallicity plays a decisive role in so far as it regulates the cooling through line-emission of the collapsing gas cloud and the coupling of the stellar photons to the accretion flow, as is indeed supported observationally (De Marchi *et al.* 2017). Auxiliary arguments pertain to the coagulation of proto-stars at high densities of the embedded-cluster forming cloud core (Dib *et al.* 2007) and heating of the cloud core through supernova type II generated cosmic rays in star-burst regions (Papadopoulos 2010).

The recent developments in IMF studies (Sec. 3 and this section) have thus nicely shown broad convergence of empirical and theoretical results.

#### 5. Consistency with galaxy-wide stellar populations

The IMF variation suggested above ought to be evident in a corresponding variation of the gwIMF, since, trivially, the young population in a galaxy is composed of the sum over all its embedded clusters.

The G-dwarf problem (too few low-metallicity G-dwarfs found locally) has been used to argue that the gwIMF must have been top-heavy in past cosmological times (e.g. [Davé 2008](#)). The emission of H $\alpha$  light is a direct tracer of the young ionising massive-stellar content of a galaxy with stellar life-times up to about 50 Myr, while the brightness in the UV spectral bands assesses the intermediate-mass stellar population with stellar life-times of up to about 300 Myr ([Pflamm-Altenburg et al. 2009](#); [Jerabkova et al. 2018](#)). Broad-band optical colours also allow an assessment of the intermediate to low-mass stellar population. A combination of these tracers thus allows, under appropriate consideration of photon leakage and dust obscuration and scattering, information to be gleaned on the shape of the gwIMF. Surveys of local-volume star-forming dwarf galaxies have shown these to have a decreasing H $\alpha$ /UV flux ratio with decreasing SFR (implying an increasingly top-light gwIMF with decreasing SFR, [Lee et al. 2009](#)). Star-counts in the  $\approx 4$  Mpc distant dwarf galaxy DDO 154 with a  $SFR \approx 10^{-3} M_{\odot}/\text{yr}$  have uncovered a deficit of massive stars ([Watts et al. 2018](#)), consistent with the above survey. Surveys of disk galaxies with  $SFR \gtrsim 1 M_{\odot}/\text{yr}$  imply an increasingly top-heavy gwIMF with increasing SFR ([Gunawardhana et al. 2011](#)). Independently-performed surveys of star-forming galaxies come to the same conclusion ([Hoversten, & Glazebrook 2008](#); [Meurer et al. 2009](#)). This observationally inferred general trend of the gwIMF becoming increasingly top-heavy with increasing SFR is consistent with the early deduction based on  $\alpha$ - and Fe-element abundances in elliptical galaxies which formed with SFRs as high as a few  $10^3 M_{\odot}/\text{yr}$  ([Matteucci 1994](#)). A potentially powerful tool to assess the shape of the gwIMF in high-redshift star-bursting galaxies is the  $^{13}\text{C}/^{18}\text{O}$  isotope abundance ratio in the cold molecular gas. Probing it via the rotational transitions of the  $^{13}\text{CO}$  and  $\text{C}^{18}\text{O}$  isotopologues provides a sensitive measure of the slope of the gwIMF at high stellar masses. Using this method leads to the result that the gwIMF becomes increasingly top heavy with increasing SFR ([Zhang et al. 2018](#)).

This documented variation of the gwIMF from the least-massive star-forming dwarf galaxies to the most massive now dormant elliptical galaxies is well described by the IGIMF theory ([Yan et al. 2017](#); [Jerabkova et al. 2018](#)).

## 6. Final comments

The evidence for the IMF and, by implication, also for the gwIMF to vary appears to be rather robust. Various independent lines of argument, also based on independent data, point to the IMF being more top-heavy at low metallicity, high density, and thus to have been top-heavy in the early Universe. This is well consistent with theoretical expectations and with galaxy-wide stellar populations. An explicit mathematical formulation of this dependency is available ([Marks et al. 2012](#); [Jerabkova et al. 2018](#)). Observations at high redshift and thus in extreme environments will allow this formulation to be updated, but any changes must always retain consistency with the observational constraints in the MW. The current prediction, based on an extrapolation to extreme conditions, is that cosmologically early star-burst clusters (proto-UCDs) would appear quasar-like ([Jerabkova et al. 2017](#)). It may thus be that some of the very high-red shift quasars may not be accreting super massive black holes, but may rather be a prior stage, namely a-few-Myr old hyper-star-burst clusters.

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