In Chapter 1 univalent functions are considered and various classical inequalities are obtained. The class $\mathfrak{S}$ of univalent functions $f(z)$ with Taylor expansions

$$f(z) = z + a_2 z^2 + a_3 z^3 + \ldots \qquad (\,|\,z\,| < 1)$$

is studied, and it is shown that $|\,a_2\,| \leqslant 2$, not only for $f(z) \in \mathfrak{S}$, but also for a rather wider class of functions. It is also shown that the Bieberbach conjecture $|\,a_n\,| \leqslant n$ holds for certain subclasses of $\mathfrak{S}$ such as Rogosinski's typically real functions.

In Chapter 2 the Cartwright-Spencer theorem on areally mean p-valent functions is proved. A regular function f is said to be areally mean p-valent in $|\,z\,| < 1$ if the average number of roots of the equation $f(z) = w$ is not greater than p, as w ranges over any disc $|\,w\,| < R$, and the theorem states that

$$\sup_{|z|=r} |\,f(z)\,| < A(p)\mu_p(1-r)^{-2p} \qquad (0 < r < 1),$$

where μ_p is max $|\,a_q\,|$ for $1 \leqslant q \leqslant p$. This is used to obtain upper bounds for $|\,a_n\,|$ in Chapter 3.

Chapter 4 is devoted to Steiner and circular (Pólya) symmetrisation and their applications to functions regular in the unit circle.

In Chapter 5 circumferentially mean p-valent functions, which form a subclass of the class of areally mean p-valent functions, are studied. Most of the theorems are due to the author himself, and they include the theorem which states, in the particular case when $p = 1$, that, for a fixed function $f \in \mathfrak{S}$, $|\,a_n\,| \leqslant n$ for all $n > n_0(f)$; thus, for a fixed function, Bieberbach's conjecture holds for all sufficiently large n.

The final chapter gives an account of the deep and difficult theory of K. Löwner, from which it is deduced, in particuar, that $|\,a_3\,| \leqslant 3$.

The author's style is succinct and clear and the book is beautifully printed. Most of the material contained in it has not appeared in book form before and some of it is quite new. The tract forms a most valuable addition to the library of any mathematician interested in the theory of functions of a complex variable.

R. A. RANKIN

GRENANDER, U., AND SZEGÖ, G., *Toeplitz Forms and their Applications* (University of California Press, 1958), 246 pp., $6.00.

It should perhaps be remarked at the outset, for those to whom Toeplitz matrices mean the matrices of regular summability methods, that this book is concerned with quadratic forms and specialisations of them. The finite Hermitian forms $T_n = \Sigma\, c_{\nu-\mu} u_\mu \bar{u}_\nu (\mu, \nu = 0, 1, \ldots, n)$, $c_{-\nu} = \bar{c}_\nu$, are called the Toeplitz forms associated with a given function when the c_n are either the coefficients in a power series expansion of a harmonic function, or the complex Fourier coefficients of a real-valued function Lebesgue-integrable over $[-\pi, \pi]$, or the Fourier-Stieltjes coefficients of a distribution function. Toeplitz's original L-forms were associated with Laurent series, and various writers have established connections between these L-forms and the work of Carathéodory on the Fourier coefficients of a harmonic function. Szegö has done much work on Toeplitz forms associated with a Lebesgue-integrable function, in particular on the distribution of their eigenvalues, and some of this work is incorporated into the text. In the place of the Toeplitz matrix $(c_{\nu,\mu}) = (c_{\nu-\mu})$ there is an analogue for functions, using a kernel $K(s, t) = K(s-t)$.

The preliminary chapter provides, with the minimum of proofs, a useful source of reference material for the later chapters. Chapters 2 and 3 deal with the algebraic and limit properties of orthogonal polynomials, and chapters 4, 5, 6 with the trigonometric moment problem, eigenvalues of Toeplitz forms, and generalisations and analogues. Chapter 7 consists of further generalisations to the case of Toeplitz operators associated with a kernel $K(s, t)$, defined with reference to two measure spaces and a function $\phi(x, s)$ measurable with respect to their product measure. In the later part of the chapter some previous results are re-examined in the light of these generalisations—for instance, in § 7.6, there is an ingenious alternative proof of a theorem of § 5.2. If a certain limiting property holds, the class of Toeplitz matrices corresponding to a class of real-valued functions is said to be trace-complete ; the first part of chapter 8 concerns the properties of such classes of matrices. §§ 8.4, 8.5 concern Toeplitz matrices associated with certain special orthogonal systems, while the rest of the chapter examines two special kernels $K(s, t)$.

The first chapter (9) of the Applications section of the book makes use of the properties of the eigenvalues of Toeplitz forms to deduce several results on analytic functions regular within the unit circle. Some of the results in the final chapters (10, 11), which deal with the applications to probability theory and statistics, are recent, and there is a link between Toeplitz forms and the work of Kolmogorov and Wiener on stationary processes. Some of the work of Grenander on stochastic processes is incorporated into these chapters.

This is a stimulating book, designed primarily for the research worker who is anxious to have available, in a compact form, some of the recent work on the subject, together with a survey of older results. Its value is enhanced by the bibliography and by the Appendix which provides (with additional references) an annotated commentary on the text. An unusual amount of material is compressed into the 228 pages, mainly by the omission of tedious detail. Despite this, the printing is uncramped and the book remains, both in appearance and content, a pleasure to read.

DENNIS C. RUSSELL

CHURCHILL, R. V., *Operational Mathematics*, 2nd ed. (McGraw-Hill Book Co., New York, 1958), 306 pp., 41s.

The general arrangement of the book is unaltered from the first edition published in 1944 under the title *Modern Operational Mathematics in Engineering*. There is, however, an extensive revision of detail, the concept of the delta-function is introduced, the chapter on elementary applications now contains sections on electric circuits and servomechanisms, and many more examples for the reader to work are included. The bibliography extends to twenty-two titles including references to tables of Laplace and Fourier transforms.

Perhaps the most obvious change is in the title, implying that the book is now of interest to a wider class of reader than was suggested by the title of the first edition. The change is justified for this is a sound mathematical treatment of the theory of the Laplace transform and its application to problems of physical origin. Great care is taken to state clearly and precisely the conditions under which results are established. These conditions are usually simple and practical, the author having resisted the temptation to enshroud the theory in mathematical sophistication. The treatment should satisfy all with a reasonable outlook whether they incline to pure or applied mathematics.

Recent developments, applying operator methods to partial differential equations by the use of integral transforms with kernels of various types, are reflected in the chapters entitled " Sturm-Liouville Systems " and " Fourier Transforms ". The first of these has been radically revised providing a much more satisfying account of second order linear differential systems ; an addition of particular interest in the other chapter is the account of how the kernel appropriate to a given differential system may be determined.