18
Solutions

Chapter 1

1.1 $100\,\text{keV} \ll m_e c^2 \Rightarrow$ classical (non-relativistic) treatment acceptable

$$E_{\text{kin}} = \frac{1}{2} m_e v^2 \quad \Rightarrow \quad v = \sqrt{\frac{2E_{\text{kin}}}{m_e}} = 1.9 \cdot 10^8\,\text{m/s} \ ,$$

$$\text{range } s = \frac{1}{2}\,at^2 \ , \quad v = at \ ,$$

$$\Rightarrow \quad t = \frac{2s}{v} = 2.1 \cdot 10^{-12}\,\text{s} = 2.1\,\text{ps} \ .$$

1.2 $m_\mu c^2 \ll 1\,\text{TeV}$, therefore in this approximation $m_\mu \approx 0$;

$$R = \int_E^0 \frac{\mathrm{d}E}{\mathrm{d}E/\mathrm{d}x} = \int_0^E \frac{\mathrm{d}E}{a+bE} = \frac{1}{b}\ln\left(1 + \frac{b}{a}\,E\right) \ ,$$

$$R(1\,\text{TeV}) = 2.64 \cdot 10^5\,\text{g/cm}^2$$

$$\widehat{=}\ 881\,\text{m rock } (\varrho_{\text{rock}} = 3\,\text{g/cm}^3 \text{ assumed}) \ .$$

1.3

$$\frac{\sigma(E)}{E} = \frac{\sqrt{F} \cdot \sqrt{n}}{n} = \frac{\sqrt{F}}{\sqrt{n}} \ ;$$

n is the number of produced electron–hole pairs,

$$n = \frac{E}{W} \ .$$

https://doi.org/10.1017/9781009401531.022 Published online by Cambridge University Press

$W = 3.65\,\mathrm{eV}$ is the average energy required for the production of an electron–hole pair in silicon:

$$\frac{\sigma(E)}{E} = \frac{\sqrt{F \cdot W}}{\sqrt{E}} = 8.5 \cdot 10^{-4} = 0.085\% \ .$$

1.4

$$R = \int_{E_{\mathrm{kin}}}^{0} \frac{\mathrm{d}E_{\mathrm{kin}}}{\mathrm{d}E_{\mathrm{kin}}/\mathrm{d}x} = \int_{0}^{E_{\mathrm{kin}}} \frac{E_{\mathrm{kin}}\,\mathrm{d}E_{\mathrm{kin}}}{az^2\,\ln(bE_{\mathrm{kin}})}$$
$$\approx \frac{1}{az^2}\int_0^{E_{\mathrm{kin}}} \frac{E_{\mathrm{kin}}\,\mathrm{d}E_{\mathrm{kin}}}{(bE_{\mathrm{kin}})^{1/4}} \approx \frac{1}{a\sqrt[4]{b}\,z^2}\int_0^{E_{\mathrm{kin}}} E_{\mathrm{kin}}^{3/4}\,\mathrm{d}E_{\mathrm{kin}}$$
$$= \frac{4}{7a\sqrt[4]{b}\,z^2}\,E_{\mathrm{kin}}^{7/4} \propto E_{\mathrm{kin}}^{1.75} \ ;$$

experimentally, the exponent is found to vary depending on the energy range and the type of particle. For low-energy protons with energies between several MeV and 200 MeV it is found to be 1.8, and for α particles with energies between 4 MeV and 7 MeV, it is around 1.5 [1, 2].

1.5 Longitudinal- and transverse-component momentum conservation requires, see Fig. 18.1:

longitudinal component	$h\nu - h\nu' \cos\Theta_\gamma$	$= p\cos\Theta_e,$
transverse component	$h\nu' \sin\Theta_\gamma$	$= p\sin\Theta_e,$
($c = 1$ assumed):		

$$\cot\Theta_e = \frac{h\nu - h\nu'\cos\Theta_\gamma}{h\nu'\sin\Theta_\gamma} \ .$$

Because of

$$\frac{h\nu'}{h\nu} = \frac{1}{1+\varepsilon(1-\cos\Theta_\gamma)} \ :$$

$$\cot\Theta_e = \frac{1+\varepsilon(1-\cos\Theta_\gamma) - \cos\Theta_\gamma}{\sin\Theta_\gamma} = \frac{(1+\varepsilon)(1-\cos\Theta_\gamma)}{\sin\Theta_\gamma} \ .$$

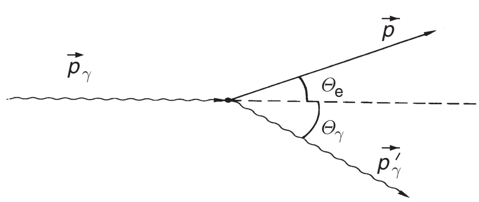

Fig. 18.1. Kinematics of Compton scattering.

https://doi.org/10.1017/9781009401531.022 Published online by Cambridge University Press

Because of

$$1 - \cos\Theta_\gamma = 2\,\sin^2\frac{\Theta_\gamma}{2}$$

one gets

$$\cot\Theta_e = (1+\varepsilon)\,\frac{2\,\sin^2\frac{\Theta_\gamma}{2}}{\sin\Theta_\gamma}\ .$$

With $\sin\Theta_\gamma = 2\,\sin(\Theta_\gamma/2)\cdot\cos(\Theta_\gamma/2)$ follows

$$\cot\Theta_e = (1+\varepsilon)\,\frac{\sin(\Theta_\gamma/2)}{\cos(\Theta_\gamma/2)} = (1+\varepsilon)\tan\frac{\Theta_\gamma}{2}\ .$$

This relation shows that the scattering angle of the electron can never exceed 90°.

1.6 $q_\mu + q_e = q'_\mu + q'_e \Rightarrow$

$$\begin{pmatrix} E_\mu \\ \vec{p}_\mu \end{pmatrix}\begin{pmatrix} m_e \\ \vec{0} \end{pmatrix} = \begin{pmatrix} E'_\mu \\ \vec{p}'_\mu \end{pmatrix}\begin{pmatrix} E'_e \\ \vec{p}'_e \end{pmatrix}\ , \quad m_eE_\mu = E'_\mu E'_e - \vec{p}'_\mu\cdot\vec{p}'_e\ .$$

Head-on collision gives maximum energy transfer $\Rightarrow \cos\Theta = 1$:

$$\begin{aligned}
m_eE_\mu &= E'_\mu E'_e - \sqrt{E'^2_\mu - m^2_\mu}\sqrt{E'^2_e - m^2_e} \\
&= E'_\mu E'_e - E'_\mu E'_e\sqrt{1-\left(\frac{m_\mu}{E'_\mu}\right)^2}\sqrt{1-\left(\frac{m_e}{E'_e}\right)^2} \\
&= E'_\mu E'_e\left\{1-\left[1-\frac{1}{2}\left(\frac{m_\mu}{E'_\mu}\right)^2+\cdots\right]\left[1-\frac{1}{2}\left(\frac{m_e}{E'_e}\right)^2+\cdots\right]\right\} \\
&= E'_\mu E'_e\left[\frac{1}{2}\left(\frac{m_\mu}{E'_\mu}\right)^2+\frac{1}{2}\left(\frac{m_e}{E'_e}\right)^2+\cdots\right]\ ,
\end{aligned}$$

$$2m_eE_\mu \approx \frac{E'_e}{E'_\mu}\,m^2_\mu + \frac{E'_\mu}{E'_e}\,m^2_e \ \Rightarrow\ 2m_eE_\mu E'_e E'_\mu = E'^2_e m^2_\mu + E'^2_\mu m^2_e\ ,$$

$$m^2_e E'^2_\mu \ll m^2_\mu E'^2_e \ \Rightarrow\ 2m_eE_\mu E'_e E'_\mu \approx E'^2_e m^2_\mu\ ,$$

https://doi.org/10.1017/9781009401531.022 Published online by Cambridge University Press

energy conservation: $E'_\mu + E'_e = E_\mu + m_e, m_e \ll E_\mu$;

$$2m_e E_\mu (E_\mu - E'_e) = m_\mu^2 E'_e = 2m_e E_\mu^2 - 2m_e E_\mu E'_e \ ,$$
$$E'_e = \frac{2m_e E_\mu^2}{m_\mu^2 + 2m_e E_\mu} = \frac{E_\mu^2}{E_\mu + \frac{m_\mu^2}{2m_e}} = \frac{E_\mu^2}{E_\mu + 11\,\mathrm{GeV}} \ ,$$
therefore $E'_e = 90.1\,\mathrm{GeV}$.

1.7 Argon: $Z = 18,\ A = 40,\ \varrho = 1.782 \cdot 10^{-3}\,\mathrm{g/cm^3}$,

$$\phi(E)\,\mathrm{d}E = 1.235 \cdot 10^{-4}\,\mathrm{GeV}\ \frac{\mathrm{d}E}{\beta^2 E^2} = \alpha\ \frac{\mathrm{d}E}{\beta^2 E^2} \ .$$

For a 10 GeV muon $\beta \approx 1$,

$$P(> E_0) = \int_{E_0}^{E_{\max}} \phi(E)\,\mathrm{d}E = \alpha \int_{E_0}^{E_{\max}} \frac{\mathrm{d}E}{E^2} = \alpha \left(\frac{1}{E_0} - \frac{1}{E_{\max}} \right) \ ,$$
$$E_{\max} = \frac{E_\mu^2}{E_\mu + 11\,\mathrm{GeV}} = 4.76\,\mathrm{GeV} \ ,$$
$$P(> E_0) = 1.235 \cdot 10^{-4} \left(\frac{1}{10} - \frac{1}{4760} \right) = 1.235 \cdot 10^{-5} \approx 0.0012\% \ .$$

1.8 The sea-level muon spectrum can be approximated by

$$N(E)\,\mathrm{d}E \propto E^{-\alpha}\,\mathrm{d}E \ , \quad \text{where } \alpha \approx 2 \ ,$$

$$\frac{\mathrm{d}E}{\mathrm{d}x} = \text{constant } (= a) \ \Rightarrow\ E = a \cdot h \ (h - \text{depth}),$$

$$I(h) = \text{const } h^{-\alpha} \ ,$$
$$\left| \frac{\Delta I}{I} \right| = \frac{\alpha h^{-\alpha-1} \Delta h}{h^{-\alpha}} = \alpha\ \frac{\Delta h}{h} = 2 \cdot \frac{1}{100} = 2\% \ .$$

Chapter 2

2.1

$$\rho(\mathrm{Al}) = 2.7\,\mathrm{g/cm^3} \to \mu = (0.189 \pm 0.027)\,\mathrm{cm^{-1}} \ ,$$
$$I(x) = I_0 \exp(-\mu \cdot x) \to x = 1/\mu \cdot \ln(I_0/I) \ .$$

https://doi.org/10.1017/9781009401531.022 Published online by Cambridge University Press

Statistical error of the count rates:

$$\sqrt{I_0}/I_0 = 1/\sqrt{I_0} \approx 4.2\% \ , \quad \sqrt{I}/I = 1/\sqrt{I} \approx 5.0\% \ .$$

The fractional error of I_0/I is

$$\sqrt{(4.2\%)^2 + (5.0\%)^2} \approx 6.5\% \ .$$

Hence $I_0/I = 1.440_{\pm 6.5\%}$.

Since $x \propto \ln(I_0/I) = \ln r \to \mathrm{d}x \propto \mathrm{d}r/r$, so that the absolute error in $\ln r$ is equal to the fractional error in I_0/I.

Therefore, $\ln(I_0/I) = \ln 1.44 \pm 0.065 \approx 0.365 \pm 0.065 \approx 0.37_{\pm 18\%}$.

The fractional error in μ was 14.3%, so the fractional error in x is

$$\sqrt{(18\%)^2 + (14.3\%)^2} \approx 23\% \ .$$

Therefore

$$x = 1/\mu \cdot \ln(I_0/I) = 1.93\,\mathrm{cm}_{\pm 23\%} = (1.93 \pm 0.45)\,\mathrm{cm} \ .$$

2.2

$$P(n,\mu) = \frac{\mu^n \cdot \mathrm{e}^{-\mu}}{n!} \ , \quad n = 0,1,2,3,\ldots \ \to$$
$$P(5,10) = \frac{10^5 \cdot \mathrm{e}^{-10}}{5!} \approx 0.0378 \ ,$$
$$P(2,1) = \frac{1^2 \cdot \mathrm{e}^{-1}}{2!} \approx 0.184 \ , \quad P(0,10) = \frac{10^0 \cdot \mathrm{e}^{-10}}{0!} \approx 4.5 \cdot 10^{-5} \ .$$

2.3 The true dead-time-corrected rate at $d_1 = 10\,\mathrm{cm}$ is

$$R_1^* = \frac{R_1}{1 - \tau R_1} \ .$$

Because of the inverse square law ($\propto 1/r^2$) the true rate at $d_2 = 30\,\mathrm{cm}$ is

$$R_2^* = \left(\frac{d_1}{d_2}\right)^2 R_1^* \ ;$$

and because of $R_2^* = R_2/(1 - \tau R_2)$ one gets

$$\left(\frac{d_1}{d_2}\right)^2 \frac{R_1}{1 - \tau R_1} = \frac{R_2}{1 - \tau R_2} \ .$$

https://doi.org/10.1017/9781009401531.022 Published online by Cambridge University Press

Solving for τ yields

$$\tau = \frac{\left(\frac{d_2}{d_1}\right)^2 R_2 - R_1}{\left[\left(\frac{d_2}{d_1}\right)^2 - 1\right] R_1\, R_2} = 10\,\mu\mathrm{s} \ .$$

Chapter 3

3.1

$$\begin{aligned} \text{dose} &= \frac{\text{absorbed energy}}{\text{mass unit}} = \frac{\text{activity} \cdot \text{energy per Bq} \cdot \text{time}}{\text{mass}} \\ &= \frac{10^9\,\mathrm{Bq} \cdot 1.5 \cdot 10^6\,\mathrm{eV} \cdot 1.602 \cdot 10^{-19}\,\mathrm{J/eV} \cdot 86\,400\,\mathrm{s}}{10\,\mathrm{kg}} \\ &= 2.08\,\mathrm{J/kg} = 2.08\,\mathrm{Gy} \ . \end{aligned}$$

Here, also a common unit for the energy, like eV (electron volt), in addition to Joule, is used: $1\,\mathrm{eV} = 1.602 \cdot 10^{-19}\,\mathrm{J}$.

3.2 The decrease of the activity in the researcher's body has two components. The total decay rate λ_{eff} is

$$\lambda_{\mathrm{eff}} = \lambda_{\mathrm{phys}} + \lambda_{\mathrm{bio}} \ .$$

Because of $\lambda = \frac{1}{\tau} = \frac{\ln 2}{T_{1/2}}$ one gets

$$T_{1/2}^{\mathrm{eff}} = \frac{T_{\mathrm{phys}}\, T_{\mathrm{bio}}}{T_{\mathrm{phys}} + T_{\mathrm{bio}}} = 79.4\,\mathrm{d} \ .$$

Using $\dot{D} = \dot{D}_0\, \mathrm{e}^{-\lambda\, t}$ and $\dot{D}/\dot{D}_0 = 0.1$ one has[†]

$$t = \frac{1}{\lambda} \ln\left(\frac{\dot{D}_0}{\dot{D}}\right) = \frac{T_{1/2}^{\mathrm{eff}}}{\ln 2} \ln\left(\frac{\dot{D}_0}{\dot{D}}\right) = 263.8\,\mathrm{d} \ .$$

† The notation $\dot{D}_0$ describes the dose rate at $t = 0$. $\dot{D}_0$ does not represent the time derivative of the constant dose D_0 (which would be zero, of course).

https://doi.org/10.1017/9781009401531.022 Published online by Cambridge University Press

A mathematically more demanding calculation allows to work out the dose that the researcher has received in this time span:

$$D_{\text{total}} = \int_0^{263.8\,\text{d}} \dot{D}_0\, \mathrm{e}^{-\lambda\, t}\, \mathrm{d}t = \dot{D}_0 \left(-\frac{1}{\lambda}\right)\, \mathrm{e}^{-\lambda\, t}\Big|_0^{263.8\,\text{d}}$$
$$= \frac{\dot{D}_0}{\lambda}\left(1 - \mathrm{e}^{-\lambda\cdot 263.8\,\text{d}}\right) \ .$$

With

$$\lambda = \frac{1}{\tau} = \frac{\ln 2}{T_{1/2}^{\text{eff}}} = 8.7\cdot 10^{-3}\,\text{d}^{-1}$$

one obtains ($1\,\mu\text{Sv/h} = 24\,\mu\text{Sv/d}$)

$$D_{\text{total}} = \frac{24\,\mu\text{Sv/d}}{\lambda}(1 - 0.1) = 2.47\,\text{mSv} \ .$$

The 50-year dose equivalent commitment $D_{50} = \int_0^{50\,\text{a}} \dot{D}(t)\,\mathrm{d}t$ is worked out to be

$$D_{50} = \int_0^{50\,\text{a}} \dot{D}_0\, \mathrm{e}^{-\lambda\, t}\mathrm{d}t = \frac{\dot{D}_0}{\lambda}\left(1 - \mathrm{e}^{-\lambda\cdot 50\,\text{a}}\right) \approx \frac{\dot{D}_0}{\lambda} = 2.75\,\text{mSv} \ .$$

3.3 The recorded charge ΔQ is related to the voltage drop ΔU by the capacitor equation

$$\Delta Q = C\,\Delta U = 7\cdot 10^{-12}\,\text{F}\cdot 30\,\text{V} = 210\cdot 10^{-12}\,\text{C} \ .$$

The mass of the air in the ionisation chamber is

$$m = \varrho_{\text{L}}\, V = 3.225\cdot 10^{-3}\,\text{g} \ .$$

This leads to an ion dose of

$$I = \frac{\Delta Q}{m} = 6.5\cdot 10^{-8}\,\text{C/g} = 6.5\cdot 10^{-5}\,\text{C/kg} \ .$$

Because of $1\,\text{R} = 2.58\cdot 10^{-4}\,\text{C/kg}$, this corresponds to a dose of 0.25 Röntgen or, respectively, because of $1\,\text{R} = 8.8\,\text{mGy}$,

$$D = 2.2\,\text{mGy} \ .$$

3.4 The total activity is worked out to be

$$A_{\text{total}} = 100\,\text{Bq/m}^3\cdot 4000\,\text{m}^3 = 4\cdot 10^5\,\text{Bq} \ .$$

https://doi.org/10.1017/9781009401531.022 Published online by Cambridge University Press

This leads to the original activity concentration in the containment area of

$$A_0 = \frac{4 \cdot 10^5\,\mathrm{Bq}}{500\,\mathrm{m}^3} = 800\,\mathrm{Bq/m}^3 \ .$$

3.5 For the activity one has

$$A = \lambda\, N = \frac{1}{\tau}\, N = \frac{\ln 2}{T_{1/2}}\, N \ ,$$

corresponding to

$$N = \frac{A\, T_{1/2}}{\ln 2} = 1.9 \cdot 10^{12} \text{ cobalt nuclei}$$

and $m = N\, m_{\mathrm{Co}} = 0.2\,\mathrm{ng}$. Such a small amount of cobalt can hardly be detected with chemical techniques.

3.6 The radiation power is worked out to be

$$S = 10^{17}\,\mathrm{Bq} \cdot 10\,\mathrm{MeV} = 10^{24}\,\mathrm{eV/s} = 160\,\mathrm{kJ/s} \ ;$$

the temperature increase is calculated to be

$$\Delta T = \frac{\text{energy deposit}}{m\, c} = \frac{160\,\mathrm{kJ/s} \cdot 86\,400\,\mathrm{s/d} \cdot 1\,\mathrm{d}}{120\,000\,\mathrm{kg} \cdot 0.452\,\mathrm{kJ/(kg\,K)}} = 255\,\mathrm{K} \ .$$

This temperature rise of 255 °C eventually leads to a temperature of 275 °C.

3.7 X rays are attenuated according to

$$I = I_0\, \mathrm{e}^{-\mu\, x} \ \Rightarrow \ \mathrm{e}^{\mu\, x} = \frac{I_0}{I} \ .$$

This leads to

$$x = \frac{1}{\mu}\, \ln\left(\frac{I_0}{I}\right) = 30.7\,\mathrm{g/cm}^2 \ ,$$

and accordingly

$$x^* = \frac{x}{\varrho_{\mathrm{Al}}} = 11.4\,\mathrm{cm} \ .$$

3.8 With modern X-ray tubes the patient gets an effective whole-body dose on the order of 0.1 mSv. For a holiday spent at an altitude of 3000 m at average geographic latitudes the dose rate by cosmic rays amounts to about 0.1 µSv/h corresponding to 67 µSv in a period

https://doi.org/10.1017/9781009401531.022 Published online by Cambridge University Press

of 4 weeks [3]. If, in addition, the radiation load due to terrestrial radiation is also taken into account (about 40 μSv in 4 weeks), one arrives at a total dose which is very similar to the radiation dose received by an X ray of the chest. It has to be mentioned, however, that older X-ray tubes can lead to higher doses, and that the period over which the dose is applied is much shorter for an X ray, so that the dose rate in this case is much higher compared to the exposure at mountain altitudes.

3.9 The effective half-life for ^{137}Cs in the human body is

$$T_{1/2}^{\text{eff}} = \frac{T_{1/2}^{\text{phys}}\, T_{1/2}^{\text{bio}}}{T_{1/2}^{\text{phys}} + T_{1/2}^{\text{bio}}} = 109.9\,\text{d} \ .$$

The remaining amount of ^{137}Cs after three years can be worked out by two different methods:

a) the period of three years corresponds to $\frac{3\cdot 365}{109.9} = 9.9636$ half-lives:

$$\text{activity}(3\,\text{a}) = 4 \cdot 10^6 \cdot 2^{-9.9636} = 4006\,\text{Bq} \ ;$$

b) on the other hand, one can consider the evolution of the activity,

$$\text{activity}(3\,\text{a}) = 4 \cdot 10^6 \cdot \text{e}^{-3\,\text{a}\cdot \ln 2/T_{1/2}^{\text{eff}}} = 4006\,\text{Bq} \ .$$

3.10 The specific dose constants for β and γ radiation of ^{60}Co are

$$\mathit{\Gamma}_\beta = 2.62 \cdot 10^{-11}\,\text{Sv}\,\text{m}^2/\text{Bq}\,\text{h} \ , \quad \mathit{\Gamma}_\gamma = 3.41 \cdot 10^{-13}\,\text{Sv}\,\text{m}^2/\text{Bq}\,\text{h} \ .$$

For the radiation exposure of the hands the β dose dominates. Assuming an average distance of 10 cm and an actual handling time of the source with the hands of 60 s, this would lead to a partial-body dose of

$$H_\beta = \mathit{\Gamma}_\beta \frac{A}{r^2}\,\Delta t = 2.62 \cdot 10^{-11} \cdot \frac{3.7 \cdot 10^{11}}{0.1^2} \cdot \frac{1}{60}\,\text{Sv} = 16.1\,\text{Sv} \ .$$

The whole-body dose, on the other hand, is related to the γ radiation of ^{60}Co. For an average distance of 0.5 m and an exposure time of 5 minutes the whole-body dose is worked out to be

$$H_\gamma = \mathit{\Gamma}_\gamma \frac{A}{r^2}\,\Delta t = 42\,\text{mSv} \ .$$

https://doi.org/10.1017/9781009401531.022 Published online by Cambridge University Press

Actually, a similar accident has happened to an experienced team of technicians in Saintes, France, in 1981. The technicians should have under no circumstances handled the strong source with their hands! Because of the large radiation exposure to the hands and the corresponding substantial radiation hazard, the hands of two technicians had to be amputated. For a third technician the amputation of three fingers was unavoidable.

3.11 After the first decontamination the remaining activity is $N(1-\varepsilon)$, where N is the original surface contamination. After the third procedure one has $N(1-\varepsilon)^3$. Therefore, one gets

$$N = \frac{512\,\mathrm{Bq/cm^2}}{(1-\varepsilon)^3} = 64\,000\,\mathrm{Bq/cm^2} \ .$$

The third decontamination reduced the surface contamination by

$$N(1-\varepsilon)^2\,\varepsilon = 2048\,\mathrm{Bq/cm^2} \ .$$

The number of decontaminations to reduce the level to $1\,\mathrm{Bq/cm^2}$ can be worked out along very similar lines ($N_\mathrm{n} = N/(\mathrm{Bq/cm^2})$):

$$N(1-\varepsilon)^n = 1\,\mathrm{Bq/cm^2} \ \rightarrow \quad (1-\varepsilon)^n = \frac{1}{N_\mathrm{n}} \ \rightarrow$$

$$n \cdot \ln(1-\varepsilon) = \ln\left(\frac{1}{N_\mathrm{n}}\right) = -\ln N_\mathrm{n} \ \rightarrow \quad n = \frac{-\ln N_\mathrm{n}}{\ln(1-\varepsilon)} = 6.9 \ ,$$

i.e. ≈ seven times.

Chapter 4

4.1

$$\begin{aligned} s = E^2_\mathrm{CMS} &= (q_1+q_2)^2 \\ &= (E_1+E_2)^2 - (\vec{p}_1+\vec{p}_2)^2 \\ &= E_1^2 - p_1^2 + E_2^2 - p_2^2 + 2E_1E_2 - 2\vec{p}_1\cdot\vec{p}_2 \\ &= 2m^2 + 2E_1E_2(1-\beta_1\beta_2\cos\Theta) \end{aligned}$$

because $p = \gamma m_0 \beta = E\beta \quad (c = 1 \text{ assumed})$.

In cosmic rays $\beta_1 \approx 1$ and $\beta_2 = 0$, since the target is at rest ($E_2 = m$); also $2E_1 m \gg 2m^2$:

$$s \approx 2mE_1 \ .$$

https://doi.org/10.1017/9781009401531.022 Published online by Cambridge University Press

Under these conditions, one gets

$$E_{\text{lab}} = E_1 = \frac{s}{2m} = \frac{(14\,000\,\text{GeV})^2}{2 \cdot 0.938\,\text{GeV}} = 1.045 \cdot 10^8\,\text{GeV} \approx 10^{17}\,\text{eV} \ .$$

4.2 Centrifugal force $F = \frac{mv^2}{R} = evB_{\text{St}}$:

$$B_{\text{St}} = \frac{m}{e} \cdot \frac{v}{R} \ . \tag{18.1}$$

$$(4.13) \ \Rightarrow \ \frac{\text{d}}{\text{d}t}(mv) = e|\vec{E}| = \frac{eR}{2}\,\frac{\text{d}B}{\text{d}t} \ \Rightarrow \ mv = \frac{eR}{2}\,B \ . \tag{18.2}$$

Compare Eqs. (18.1) and (18.2):

$$B_{\text{St}} = \frac{1}{2}\,B \ ,$$

which is called the *Wideroe condition*.

4.3

$$m(\text{Fe}) = \varrho \cdot 300\,\text{cm} \cdot 0.3\,\text{cm} \cdot 1\,\text{mm} = 68.4\,\text{g} \ ,$$

$$\Delta T = \frac{\Delta E}{m(\text{Fe}) \cdot c} = \frac{2 \cdot 10^{13} \cdot 7 \cdot 10^3\,\text{GeV} \cdot 1.6 \cdot 10^{-10}\,\text{J/GeV} \cdot 3 \cdot 10^{-3}}{0.56\,\text{J/(g}\cdot\text{K)} \cdot 68.4\,\text{g}}$$
$$= 1754\,\text{K}$$

$\Rightarrow$ the section hit by the proton beam will melt.

4.4 Effective bending radius

$$\rho = \frac{27\,\text{km} \cdot 2/3}{2\pi} = 2866\,\text{m} \ ,$$
$$\frac{mv^2}{\rho} = evB \Rightarrow p = eB\rho \ ,$$
$$pc = eB\rho c \ ,$$
$$10^9 \ pc\,[\text{GeV}] = 3 \cdot 10^8 \ B\,[\text{T}] \cdot \rho\,[\text{m}] \ ,$$
$$pc\,[\text{GeV}] = 0.3 \ B\,[\text{T}] \cdot \rho\,[\text{m}] \ ,$$
$$pc^{\max}(\text{LEP}) = 116\,\text{GeV} \ ,$$
$$pc^{\max}(\text{LHC}) = 8.598\,\text{TeV} \ .$$

4.5 Magnetic potential $V = -g \cdot x \cdot y$
with g – quadrupole field strength or gradient of the quadrupole;

$$\vec{B} = -\,\text{grad}\,V = (gy, gx) \ ;$$

https://doi.org/10.1017/9781009401531.022 Published online by Cambridge University Press

the surface of the magnet must be an equipotential surface $\Rightarrow$

$$V = -g \cdot x \cdot y = \text{const} \quad \Rightarrow \quad x \cdot y = \text{const} \quad \Rightarrow \quad \text{hyperbolas} \ .$$

Chapter 5

5.1

$$R_{\text{true}} = \frac{R_{\text{measured}}}{1 - \tau_{\text{D}} \cdot R_{\text{measured}}} = 2\,\text{kHz} \ . \tag{18.3}$$

5.2 For vertical incidence

$$\Delta E = \frac{\text{d}E}{\text{d}x} \cdot d \ , \tag{18.4}$$

for inclined incidence $\Delta E(\Theta) = \Delta E / \cos\Theta$;
measured energy for vertical incidence: $E_1 = E_0 - \Delta E$,
measured energy for inclined incidence: $E_2 = E_0 - \Delta E / \cos\Theta$,

$$E_1 - E_2 = \Delta E \left(\frac{1}{\cos\Theta} - 1 \right) \ ;$$

plot $E_1 - E_2$ versus $\left(\frac{1}{\cos\Theta} - 1\right) \Rightarrow$ gives a straight line with a slope ΔE. With the known $\text{d}E/\text{d}x$ (from tables) for semiconductors Eq. (18.4) can be solved for d.

5.3

$$q = \begin{pmatrix} E \\ \vec{p} \end{pmatrix} \ , \quad q' = \begin{pmatrix} E' \\ \vec{p}\,' \end{pmatrix} \ , \quad q_\gamma = \begin{pmatrix} h\nu \\ \vec{p}_\gamma \end{pmatrix}$$

are the four-momentum vectors of the incident particle, the particle after Cherenkov emission, and the emitted Cherenkov photon;

$$\begin{aligned} q' &= q - q_\gamma \ , \\ E'^2 - p'^2 = (q - q_\gamma)^2 &= \begin{pmatrix} E - h\nu \\ \vec{p} - \vec{p}_\gamma \end{pmatrix}^2 \\ &= E^2 - 2h\nu E + h^2\nu^2 - (p^2 + p_\gamma^2 - 2\vec{p} \cdot \vec{p}_\gamma) \ . \end{aligned}$$

Since $E^2 = m^2 + p^2$ and $\vec{p}_\gamma = \hbar\vec{k}$:

$$0 = -m^2 + m^2 + p^2 - 2h\nu E + h^2\nu^2 - p^2 + 2p\hbar k \cos\Theta - \hbar^2 k^2 \ ,$$

https://doi.org/10.1017/9781009401531.022 Published online by Cambridge University Press

$2p\hbar k\,\cos\Theta = 2h\nu E - h^2\nu^2 + \hbar^2 k^2,$

$$\cos\Theta = \frac{2\pi\nu E}{pk} + \frac{\hbar k}{2p} - \frac{2\pi h\nu^2}{2pk}\ ;$$

because of $\frac{c}{n} = \nu\cdot\lambda = \frac{2\pi\nu}{k}$ one has $(c = 1)$

$$\cos\Theta = \frac{E}{np} + \frac{\hbar k}{2p} - \frac{\hbar k}{2pn^2}$$

with $E = \gamma m_0, \gamma = \dfrac{1}{\sqrt{1-\beta^2}}$, and $p = \gamma m_0\beta$ one gets

$$\cos\Theta = \frac{1}{n\beta} + \frac{\hbar k}{2p}\left(1 - \frac{1}{n^2}\right)\ .$$

Normally $\hbar k/2p \ll 1$, so that the usually used expression for the Cherenkov angle is quite justified.

5.4 Let us assume that the light flash with the total amount of light, I_0, occurs at the centre of the sphere. In a first step the light intensity qI_0, where $q = S_{\rm p}/S_{\rm tot}$ arrives at the photomultiplier. The majority of the light $((1-q)I_0)$ misses the PM tube and hits the reflecting surface. Then, let us select a small pad S_1 anywhere on the sphere, at a distance r from the photomultiplier and calculate how much light reflected by this pad reaches the photomultiplier after just one reflection (see Fig. 5.46). Denoting the total amount of light reflected from S_1 as ΔJ_0 we find

$$\Delta I_1^{\rm PM} = \frac{\Delta J_0}{\pi}\cos\chi\,\Delta\Omega = \frac{\Delta J_0}{\pi}\cos\chi\,\frac{S_{\rm p}\,\cos\chi}{(2R\,\cos\chi)^2} = \Delta J_0\,q\ .$$

Since $\Delta I_1^{\rm PM}$ has no angular dependence, this value can be simply integrated over the sphere which gives the total amount of light collected by the photomultiplier after the first reflection:

$$I_1^{\rm PM} = I_0 q + I_0(1-q)(1-\mu)q\ .$$

The iteration of this argument leads to an expression for the total amount of light collected by the photomultiplier after an infinite number of reflections:

$$I_{\rm tot}^{\rm PM} = I_0 q + I_0(1-q)(1-\mu)q + I_0(1-q)^2(1-\mu)^2 q + \cdots$$
$$= I_0 q\frac{1}{1-(1-\mu)(1-q)}\ . \qquad (18.5)$$

https://doi.org/10.1017/9781009401531.022 Published online by Cambridge University Press

Then the light collection efficiency, $\eta = I_{\text{tot}}^{\text{PM}}/I_0$, is

$$\eta = \frac{q}{\mu + q - \mu q} \approx \frac{q}{\mu + q} \ . \tag{18.6}$$

Similar considerations for non-focussing Cherenkov counters were presented already a long time ago by M. Mando [4].

Chapter 6

6.1 If a small-diameter tube is submerged in a liquid, the liquid level will rise in the tube because the saturation vapour pressure of the concave liquid surface in the tube is smaller than the corresponding pressure over the planar liquid surface (capillary forces). An equilibrium condition is obtained for an elevation h of

$$2\pi r\sigma = \pi \varrho r^2 hg \ , \tag{18.7}$$

where

r – radius of the capillary vessel,

σ – surface tension,

ϱ – density of the liquid,

g – acceleration due to gravity.

For convex droplets the barometric scale formula

$$p_r = p_\infty \exp\left(\frac{Mgh}{RT}\right)$$

with

M – molar mass,

R – gas constant,

T – temperature

can be combined with (18.7) to give

$$\ln(p_r/p_\infty) = \frac{M}{RT}\frac{2\sigma}{\varrho r} \ .$$

https://doi.org/10.1017/9781009401531.022 Published online by Cambridge University Press

With numbers:

$$
\begin{aligned}
M &= 18\,\mathrm{g/mol} \text{ for water} && (46\,\mathrm{g/mol} \text{ for } C_2H_5OH),\\
\sigma &= 72.8\,\mathrm{dyn/cm} \text{ for water} && (22.3\,\mathrm{dyn/cm} \text{ for } C_2H_5OH),\\
\varrho &= 1\,\mathrm{g/cm^3} && (0.79\,\mathrm{g/cm^3} \text{ for } C_2H_5OH),\\
R &= 8.31\,\mathrm{J/mol\,K},\\
T &= 20\,^\circ\mathrm{C},\\
p_r/p_\infty &= 1.001,\\
\to r &= 1.08\cdot 10^{-6}\,\mathrm{m} && (1.07\cdot 10^{-6}\,\mathrm{m}),
\end{aligned}
$$

i.e., droplets of diameter $\approx 2\,\mu\mathrm{m}$ will form.

If the droplets are electrically charged, the mutual repulsive action will somewhat reduce the surface tension.

6.2 Increase of electron number

$$\mathrm{d}n_e = (\alpha - \beta) n_e \,\mathrm{d}x \;;$$

$\alpha =$ first Townsend coefficient,

$\beta =$ attachment coefficient,

$$
\begin{aligned}
n_e &= n_0\,\mathrm{e}^{(\alpha-\beta)d} \;,\\
\mathrm{d}n_{\mathrm{ion}} &= \beta n_e \,\mathrm{d}x \;,\\
\mathrm{d}n_{\mathrm{ion}} &= \beta n_0\,\mathrm{e}^{(\alpha-\beta)x}\,\mathrm{d}x \;,\\
n_{\mathrm{ion}} &= \beta n_0 \int_0^d \mathrm{e}^{(\alpha-\beta)x}\,\mathrm{d}x\\
&= \frac{n_0\beta}{\alpha-\beta}\left[\mathrm{e}^{(\alpha-\beta)d} - 1\right] \;,\\
\frac{n_e + n_{\mathrm{ion}}}{n_0} &= \frac{n_0\,\mathrm{e}^{(\alpha-\beta)d} + \frac{n_0\beta}{\alpha-\beta}\left[\mathrm{e}^{(\alpha-\beta)d}-1\right]}{n_0}\\
&= \frac{1}{\alpha-\beta}\left\{(\alpha-\beta)\,\mathrm{e}^{(\alpha-\beta)d} + \beta\left[\mathrm{e}^{(\alpha-\beta)d} - 1\right]\right\}\\
&= \frac{1}{\alpha-\beta}\left(\alpha\,\mathrm{e}^{(\alpha-\beta)d} - \beta\right)\\
&= \frac{1}{18}\left(20\,\mathrm{e}^{18} - 2\right) = 7.3\cdot 10^7 \;.
\end{aligned}
$$

6.3

$$
\begin{aligned}
\sqrt{\langle\theta^2\rangle} &= \frac{13.6\,\mathrm{MeV}}{\beta c p}\sqrt{\frac{x}{X_0}}\,[1 + 0.038\ln(x/X_0)] \;,\\
\beta c p &= 12.86\,\mathrm{MeV} \;.
\end{aligned}
$$

https://doi.org/10.1017/9781009401531.022 Published online by Cambridge University Press

For electrons of this energy $\beta \approx 1 \Rightarrow p = 12.86\,\mathrm{MeV}/c$. More precisely, one has to solve the equation

$$\begin{aligned}
\beta c \gamma m_0 \beta c &= 12.86\,\mathrm{MeV} \ , \\
\frac{\beta^2}{\sqrt{1-\beta^2}} &= \frac{12.86\,\mathrm{MeV}}{m_0 c^2} = 25.16 = \alpha \ , \\
\beta^2 = \sqrt{1-\beta^2} \cdot \alpha \quad &\Rightarrow \quad \beta^4 = \alpha^2 - \alpha^2 \beta^2 \ , \\
\beta^4 + \alpha^2 \beta^2 - \alpha^2 &= 0 \ , \\
\beta^2 &= -\frac{\alpha^2}{2} + \sqrt{\frac{\alpha^4}{4} + \alpha^2} = 0.998\,42 \ , \\
\gamma &= 25.16 \ , \\
p &= 12.87\,\mathrm{MeV}/c \ .
\end{aligned}$$

Chapter 7

7.1

$$\Delta t = \frac{T_1 + T_3}{2} - T_2 \ ; \tag{18.8}$$

resolution on Δt:

$$\sigma^2(\Delta t) = \left(\frac{\sigma_1}{2}\right)^2 + \left(\frac{\sigma_3}{2}\right)^2 + \sigma_2^2 = \frac{3}{2} \cdot \sigma_t^2 \ ; \tag{18.9}$$

for one wire one has

$$\sigma_t = \sqrt{\frac{2}{3}}\,\sigma(\Delta t) = 5\,\mathrm{ns} \ \rightarrow \ \sigma_x = v \cdot \sigma_t = 250\,\mu\mathrm{m} \ . \tag{18.10}$$

Correspondingly, the spatial resolution on the vertex is (Fig. 18.2)

$$\sin\frac{\alpha}{2} = \frac{\sigma_x}{\sigma_z} \ \rightarrow \ \sigma_z = \frac{\sigma_x}{\sin\frac{\alpha}{2}} = 500\,\mu\mathrm{m} \ . \tag{18.11}$$

7.2 What matters is the transverse packing fraction. A simple geometrical argument (Fig. 18.3) leads to the maximum area that can be covered.

https://doi.org/10.1017/9781009401531.022 Published online by Cambridge University Press

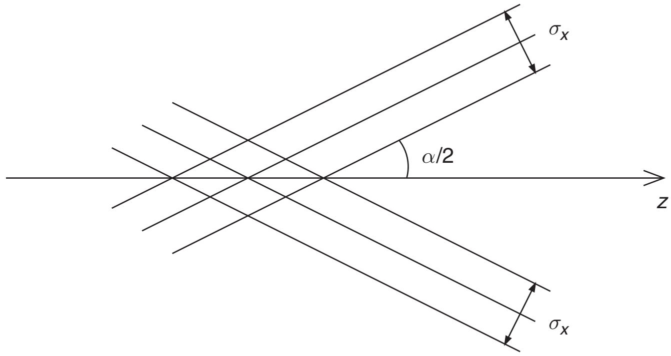

Fig. 18.2. Illustration of the vertex resolution σ_z as derived from the track resolution σ_x.

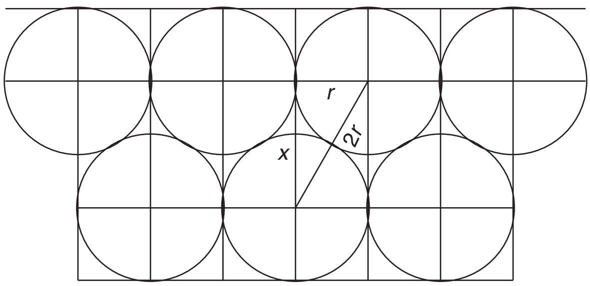

Fig. 18.3. Determination of the maximum packing fraction for a scintillating fibre tracker.

From

$$r^2 + x^2 = (2r)^2 \tag{18.12}$$

one gets $x = \sqrt{3} \cdot r$ and finds the fraction

$$\frac{\pi r^2/2}{r \cdot \sqrt{3} r} = \pi/(2\sqrt{3}) \approx 90.7\% \ , \tag{18.13}$$

$$A_{\text{fibre}} = \pi \cdot 0.5^2\,\text{cm}^2 = 0.785\,\text{mm}^2 \quad \rightarrow \quad N = \frac{A \cdot \pi/(2\sqrt{3})}{A_{\text{fibre}}} = 46\,211 \ . \tag{18.14}$$

7.3

$$\frac{mv^2}{\rho} = evB \ , \tag{18.15}$$

$$\rho = \frac{mv}{eB} = \frac{9.1 \cdot 10^{-31}\,\text{kg} \cdot 0.1 \cdot 10^6\,\text{m/s}}{1.6 \cdot 10^{-19}\,\text{A s} \cdot B} \leq 10^{-5}\,\text{m} \ , \tag{18.16}$$

$$\rightarrow \quad B \geq 0.057\,\text{T} = 570\,\text{Gauss} \ . \tag{18.17}$$

https://doi.org/10.1017/9781009401531.022 Published online by Cambridge University Press

7.4

$$Q = C \cdot U \ . \tag{18.18}$$

The liberated charge is

$$q = \frac{60\,\text{keV}}{26\,\text{eV}} \cdot q_e = 3.70 \cdot 10^{-16}\,\text{A s} \ , \tag{18.19}$$

and the required gain is obtained to be

$$G = \frac{C \cdot U}{q} = \frac{180 \cdot 10^{-12} \cdot 10^{-2}}{3.70 \cdot 10^{-16}} = 4865 \ . \tag{18.20}$$

For the energy resolution one gets

$$\frac{\sigma}{E} = \frac{\sqrt{N \cdot F}}{N} = \frac{\sqrt{F}}{\sqrt{N}} = \frac{\sqrt{F \cdot W}}{\sqrt{E}} = 8.58 \cdot 10^{-3} \ , \tag{18.21}$$

i.e. $(60 \pm 0.5)\,\text{keV}$.

7.5 The horizontal force (tension) F_h does not change along the wire, whereas the vertical one F_v is position-dependent, more precisely, the vertical force at the left boundary is diminished by the weight of the wire to the left of position x:

$$F_\text{v}(x) = F_\text{v} - \int_{x'=x_1}^{x} \varrho g \sqrt{1 + y'^2(x)}\,\text{d}x \ , \quad \text{d}m = \varrho\,\text{d}s \ ,$$
$$\text{d}s = \sqrt{\text{d}x^2 + \text{d}y^2} = \sqrt{1 + y'^2(x)}\,\text{d}x \ .$$

From the above assumptions the slope $y'(x)$ reads

$$y'(x) = -\frac{F_\text{v}(x)}{F_\text{h}} = -\frac{F_\text{v}}{F_\text{h}} + \frac{\varrho g}{F_\text{h}} \int_{x'=x_1}^{x} \sqrt{1 + y'^2(x')}\,\text{d}x' \ ,$$

with $L(x) = \int_{x'=x_1}^{x} \sqrt{1 + y'^2(x')}\,\text{d}x'$ being the length of the wire measured from the left boundary. Differentiating this equation

https://doi.org/10.1017/9781009401531.022 Published online by Cambridge University Press

leads to a differential equation for y' that can be directly integrated by separation of variables,

$$y''(x) = \frac{\varrho g}{F_\mathrm{h}}\sqrt{1+y'^2(x)} \ , \qquad \frac{\frac{\mathrm{d}}{\mathrm{d}x}y'(x)}{\sqrt{1+y'^2(x)}} = \frac{\varrho g}{F_\mathrm{h}} \ .$$

Its solution is

$$\operatorname{arsinh} y'(x) = \frac{\varrho g}{F_\mathrm{h}}x + c \ , \quad y'(x) = \sinh\left(\frac{\varrho g}{F_\mathrm{h}}x + c\right) \ ,$$

and a subsequent integration straightforwardly leads to the curve

$$y(x) = \frac{F_\mathrm{h}}{\varrho g}\cosh\left(\frac{\varrho g}{F_\mathrm{h}}x + c\right) + y_0 \ ,$$

where the integration constant c and the horizontal force F_h are to be determined from the geometry and the total length L of the wire. This solution for the form of the wire shows that it is a catenary rather than a parabola. In a symmetric environment and/or for an appropriate choice of the coordinate system the constants can be chosen to be $c = 0$ and $y_0 = -\frac{F_\mathrm{h}}{\varrho g}$. This also guarantees $y(x = 0) = 0$. For the further calculation we set the horizontal tension to $T := F_\mathrm{h}$.

The sag of the wire will be small compared to its length. Therefore, the cosh can be expanded into a series

$$\cosh\left(\frac{\varrho g x}{T}\right) = 1 + \frac{1}{2}\left(\frac{\varrho g x}{T}\right)^2 + \cdots$$

giving

$$y(x) = \text{sag} = -\frac{T}{\varrho g} + \frac{T}{\varrho g}\left[1 + \frac{1}{2}\left(\frac{\varrho g x}{T}\right)^2 + \cdots\right] \ ,$$

$$x = \frac{\ell}{2} \Rightarrow \ y\left(\frac{\ell}{2}\right) = \frac{1}{2}\frac{\varrho g}{T}\left(\frac{\ell}{2}\right)^2 = \frac{\varrho g \ell^2}{8T} \ ,$$

$$\varrho = \frac{\mathrm{d}m}{\mathrm{d}s} = \pi r_\mathrm{i}^2 \varrho^*$$

mass per unit length, with ϱ^* = density of the wire material,

$$y\left(\frac{\ell}{2}\right) = \frac{1}{8}\pi r_\mathrm{i}^2 \cdot \varrho^* \cdot \frac{g}{T}\ell^2 \ .$$

For a tension of 50 g, corresponding to $T = m_T \cdot g = 0.49\,\mathrm{kg\,m/s^2}$, $\ell = 1\,\mathrm{m}$, $\varrho^*(\text{tungsten}) = 19.3\,\mathrm{g/cm^3} = 19.3 \cdot 10^3\,\mathrm{kg/m^3}$, and $r_\mathrm{i} = 15\,\mu\mathrm{m}$ one gets a sag of $34\,\mu\mathrm{m}$.

https://doi.org/10.1017/9781009401531.022 Published online by Cambridge University Press

Chapter 8

8.1 If ε_1 and ε_2 are the energies of the two photons and ψ the opening angle between them, the two-gamma invariant mass squared is:

$$m_{\gamma\gamma}^2 = (\varepsilon_1 + \varepsilon_2)^2 - (\vec{p_1} + \vec{p_2})^2 = 4\varepsilon_1\varepsilon_2 \sin^2(\psi/2) \ .$$

Using the common error-propagation formula one gets for the relative m^2 uncertainty:

$$\frac{\delta(m^2)}{m^2} = \sqrt{\left[\frac{\delta(\varepsilon_1)}{\varepsilon_1}\right]^2 + \left[\frac{\delta(\varepsilon_2)}{\varepsilon_2}\right]^2 + \cot^2\frac{\psi}{2}\,\delta_\psi^2} \ ,$$

where $\delta(\varepsilon_i)$ and δ_ψ are the energy and angular resolution, respectively. The angular distribution is peaked near ψ_{min} ($\sin(\psi_{\text{min}}/2) = m_\eta/E_0$), so that one can take as an estimation a value of $\psi_{\text{min}} = 31.8°$. Since

$$\frac{\delta(m^2)}{m^2} = \frac{m_1^2 - m_2^2}{m^2} = \frac{(m_1 + m_2)(m_1 - m_2)}{m^2} = 2\frac{\delta m}{m}$$

or, just by differentiating,

$$\frac{\delta(m^2)}{m^2} = 2m\frac{\delta(m)}{m^2} = 2\frac{\delta m}{m} \ ,$$

one gets

$$\frac{\delta m}{m} = \frac{1}{2}\sqrt{2 \cdot (0.05)^2 + \cot^2(15.9°)(0.05)^2} \approx 9.5\% \ .$$

One can see that in this case the angular accuracy dominates the mass resolution.

8.2 The photon interaction length in matter is $\lambda = (9/7)\,X_0$. Then the probability that the photon passes the aluminium layer without interaction is

$$W_{\text{n}} = \exp\left(-\frac{L}{\lambda}\right) = \exp\left(-\frac{7}{18}\right) = 0.68 \ .$$

In this case the calorimeter response function remains unchanged, namely, it is close to a Gaussian distribution $g(E, E_0)$, where E is the measured energy and E_0 is the incident photon energy.

https://doi.org/10.1017/9781009401531.022 Published online by Cambridge University Press

If the photon produces an e^+e^- pair in the aluminium, at a distance x from the calorimeter, the electron and positron lose part of their energy:

$$\Delta E = 2\varepsilon_{\mathrm{MIP}} x \ ,$$

where $\varepsilon_{\mathrm{MIP}} = (\mathrm{d}E/\mathrm{d}x)_{\mathrm{MIP}}$ is the specific ionisation loss. For Al one has $\varepsilon_{\mathrm{MIP}} = 1.62\,\mathrm{MeV/(g/cm^2)}$ and $X_0 = 24\,\mathrm{g/cm^2}$ resulting in ΔE to vary from 0 MeV to 39 MeV. As one can see, e.g. for 100 MeV, the measured energy spectrum will consist of a narrow peak ($g(E, E_0)$) comprising 68% of the events, and a wide spectrum ranging from $0.6\,E_0$ to the full energy E_0 containing the other 32%. For a 1 GeV photon the events with pair production cannot be resolved from the main peak and just result in increasing the width of it.

To estimate the resulting rms one can use a simplified form of the probability density function (PDF):

$$\varphi(E) = pf_1(E) + (1-p)g(E, E_0) \ ,$$

where p is the photon conversion probability in Al and $f_1(E)$ is just an uniform distribution between $E_{\min} = E_0 - \Delta E_{\max}$ and E_0. The modified rms can be calculated as

$$\sigma^2_{\mathrm{res}} = p\sigma_1^2 + (1-p)\sigma_0^2 + p(1-p)(E_1 - E_0)^2 \ ,$$

where $\sigma_1, E_1, \sigma_0, E_0$ are the rms and average values for $f_1(E)$ and $g(E, E_0)$, respectively. For $f_1(E)$ one has $E_1^{\min} = E_0 - 2\varepsilon_{\mathrm{MIP}} L = E_0 - 39\,\mathrm{MeV}$ and $\sigma_1 = 2 \cdot \varepsilon_{\mathrm{MIP}} L/\sqrt{12} = \varepsilon_{\mathrm{MIP}} L/\sqrt{3} = 11\,\mathrm{MeV}$ (see Chap. 2, Eq. (2.6)). One has to consider that the energy loss ΔE varies uniformly between 0 MeV and 39 MeV with an average value of $E_1 = 19.5\,\mathrm{MeV}$, and this value has to be used in the formula for σ_{res}. With these numbers one gets $\sigma_{\mathrm{res}} \approx 11\,\mathrm{MeV}$ for the 100 MeV photon and $\sigma_{\mathrm{res}} \approx 17\,\mathrm{MeV}$ for the 1 GeV photon.

8.3 When the pion interacts at depth t, the energy deposited in the calorimeter is a sum of the pion ionisation losses before the interaction (E_{ion}) and the shower energy (E_{sh}) created by the π^0,

$$E_{\mathrm{C}} = E_{\mathrm{ion}} + E_{\mathrm{sh}} \ , \quad E_{\mathrm{ion}} = \frac{\mathrm{d}E}{\mathrm{d}X} t X_0 = E_{\mathrm{cr}} t \ ,$$
$$E_{\mathrm{sh}} = (E_0 - E_{\mathrm{ion}}) \int_0^{L-t} \left(\frac{\mathrm{d}E}{\mathrm{d}t}\right) \mathrm{d}t \ ,$$

https://doi.org/10.1017/9781009401531.022 Published online by Cambridge University Press

where E_{cr} is the critical energy. Formula (8.7) describes the electromagnetic shower development. For this estimation one can take

$$\frac{\mathrm{d}E}{\mathrm{d}t} = E_\gamma F(t) \ ,$$

where E_γ is the energy of both photons from the π^0 decay and t is the thickness measured in radiation lengths X_0. Let us assume that the resolution of the calorimeter is $\sigma_E/E = 2\%$ and the condition of the correct particle identification as a pion is

$$\Delta E(t_{\mathrm{c}}) = (E_e - E_{\mathrm{C}}) > 3\sigma_E \ ,$$

where E_e is the energy deposition for an electron in the calorimeter.

For an electron–positron shower of 200–500 MeV in the NaI absorber, the parameter a in Formula (8.7) can be roughly estimated as $a \approx 2$. Then Formula (8.7) simplifies,

$$\frac{1}{E_\gamma}\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{1}{4}\left(\frac{t}{2}\right)^2 \exp(-t/2) \ ,$$

and can be easily integrated for any t. To find t_{c} one has to tabulate the function $\Delta E(t_{\mathrm{c}})$ numerically. The calculated dependencies of E_{ion}, E_{sh} and E_{C} on t are shown in Fig. 18.4. Since $\sigma_E = 2\% \cdot E = 10\,\mathrm{MeV}$ for a 500 MeV shower, and $E_e - E_{\mathrm{C}} > 3\sigma_E$ is required, one has to ask for $E_{\mathrm{C}} < 470\,\mathrm{MeV}$. Reading this limit from the

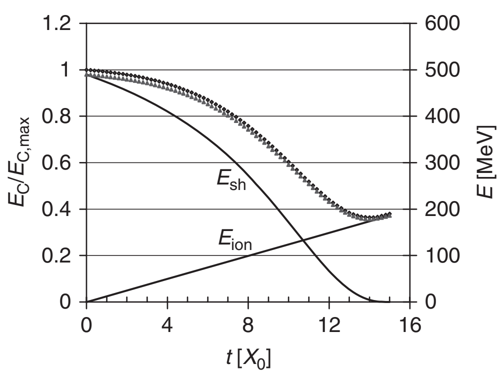

Fig. 18.4. The calculated dependencies of E_{ion}, E_{sh} and E_{C} (lower line with triangle symbols) on t. The upper line with diamond symbols shows the ratio of the energy deposition in the calorimeter to its maximum value (without leakage). Even when the charge exchange occurs in the very beginning of the calorimeter, some part of the energy leaks through the rear side.

https://doi.org/10.1017/9781009401531.022 Published online by Cambridge University Press

figure leads to $t_c \approx 4$ which corresponds to a thickness of $38\,\mathrm{g/cm}^2$. Working out the interaction probability W from the interaction length $\lambda_{\mathrm{int}} = 151\,\mathrm{g/cm}^2$ and with the knowledge of the charge-exchange probability of 0.5, one obtains the pion misidentification probability P to be

$$P_{\mathrm{M}} = 0.5\,W(t < t_{\mathrm{c}}) = 0.5\,[1 - \exp(-t_{\mathrm{c}}/\lambda_{\mathrm{int}})] \approx 0.12 \ .$$

The probability of misidentification of the electron as pion is much lower.

Chapter 9

9.1 Convert momenta to total energy:

$$E = c\sqrt{p^2 + m_0^2 c^2} = \begin{cases} 3.0032\,\mathrm{GeV} & \text{for} \quad 3\,\mathrm{GeV}/c \\ 4.0024\,\mathrm{GeV} & \text{for} \quad 4\,\mathrm{GeV}/c \\ 5.0019\,\mathrm{GeV} & \text{for} \quad 5\,\mathrm{GeV}/c \end{cases} ,$$

$$m_0 = 139.57\,\mathrm{MeV}/c^2 \ ,$$

$$\gamma = \frac{E}{m_0 c^2} = \begin{cases} 21.518 & \text{for} \quad 3\,\mathrm{GeV}/c \\ 28.677 & \text{for} \quad 4\,\mathrm{GeV}/c \\ 35.838 & \text{for} \quad 5\,\mathrm{GeV}/c \end{cases} ,$$

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = \begin{cases} 0.9989195 & \text{for} \quad 3\,\mathrm{GeV}/c \\ 0.9993918 & \text{for} \quad 4\,\mathrm{GeV}/c \\ 0.9996106 & \text{for} \quad 5\,\mathrm{GeV}/c \end{cases} ,$$

$$\cos\theta_{\mathrm{c}} = \frac{1}{n\beta} \quad \Rightarrow \quad \theta_{\mathrm{c}} = \arccos\left(\frac{1}{n\beta}\right) \ ;$$

	$3\,\mathrm{GeV}/c$	$4\,\mathrm{GeV}/c$	$5\,\mathrm{GeV}/c$
Lucite	47.8°	47.8°	47.8°
silica aerogel	12.40°–21.37°	12.52°–21.44°	12.58°–21.47°
Pyrex	47.08°	47.10°	47.11°
lead glass	58.57°	58.59°	58.60°

https://doi.org/10.1017/9781009401531.022 Published online by Cambridge University Press

9.2

$$
\begin{aligned}
m_K &= 493.677\,\text{MeV}/c^2 \;, \quad n_{\text{water}} = 1.33 \;, \\
E_K &= c\sqrt{p^2 + m_K^2 c^2} = 2.2547\,\text{GeV} \;; \\
\beta &= \sqrt{1 - \frac{1}{\gamma^2}} = 0.9757 \quad \Rightarrow \quad \theta_{\text{C}} = 39.59^\circ \;, \\
\frac{\text{d}E}{\text{d}L} &= \frac{\text{d}N}{\text{d}L} \cdot h\nu = \frac{\text{d}N}{\text{d}L}\frac{hc}{\lambda} = 2\pi\alpha z^2 hc \int_{\lambda_1}^{\lambda_2} \left(1 - \frac{1}{\beta^2 n^2}\right) \frac{\text{d}\lambda}{\lambda^3} \;;
\end{aligned}
$$

assume $n \neq f(\lambda)$, then

$$
\begin{aligned}
\frac{\text{d}E}{\text{d}L} &= 2\pi\alpha z^2 hc \left(1 - \frac{1}{\beta^2 n^2}\right) \frac{1}{2} \left(\frac{1}{\lambda_1^2} - \frac{1}{\lambda_2^2}\right) \\
&= \pi\alpha z^2 hc \left(1 - \frac{1}{\beta^2 n^2}\right) \left(\frac{1}{\lambda_1^2} - \frac{1}{\lambda_2^2}\right) \;; \\
h &= 2\pi\hbar = 41.36 \cdot 10^{-22}\,\text{MeV s} \;, \\
c &= 3 \cdot 10^{17}\,\text{nm/s} \;, \quad n = 1.33 \quad \text{for water} \;, \\
\lambda_1 &= 400\,\text{nm}, \; \lambda_2 = 700\,\text{nm} \\
&\to \frac{\text{d}E}{\text{d}L} = 0.49\,\text{keV/cm} \;.
\end{aligned}
$$

9.3

$$
E_p = c\sqrt{p^2 + m^2 \cdot c^2} = 5.087\,\text{GeV} \;.
$$

If water is considered as Cherenkov medium, one has

$$
\begin{aligned}
\beta &= \sqrt{1 - \frac{1}{\gamma^2}} = 0.9805 \;\Rightarrow\; \theta_{\text{C}} = 40.1^\circ \quad \text{in water} \;, \\
N &= 203.2 \quad \text{photons per cm} \;, \\
n &= 12 \text{ photoelectrons} = N \cdot x \cdot \eta_{\text{PM}} \cdot \eta_{\text{Geom}} \cdot \eta_{\text{Transfer}} \;, \\
x &= \frac{n}{N \cdot \eta_{\text{PM}} \cdot \eta_{\text{Geom}} \cdot \eta_{\text{Transfer}}} = 1.48\,\text{cm} \;.
\end{aligned}
$$

A counter of $\approx$ 1.5 cm thickness is required for the assumed collection/conversion efficiencies.

https://doi.org/10.1017/9781009401531.022 Published online by Cambridge University Press

9.4

$$n_{\text{Lucite}} = 1.49 \ ,$$

threshold energy for electrons

$$\beta > \frac{1}{n} = 0.67 \ \Rightarrow \ \gamma = \frac{1}{\sqrt{1-\beta^2}} = 1.35 \ \Rightarrow \ E = 689\,\text{keV} \ ,$$
$$\frac{\mathrm{d}^2 N}{\mathrm{d}x\,\mathrm{d}T} = \frac{1}{2} K \cdot z^2 \frac{Z}{A} \frac{1}{\beta^2} \frac{1}{T^2} \ ;$$

T – kinetic energy of the δ rays,

$$K = 4\pi N_{\text{A}} \ [\text{mol}^{-1}]/\text{g}\, r_e^2 m_e c^2 = 0.307\,\text{MeV}/(\text{g}/\text{cm}^2) \ ,$$
$$\frac{\mathrm{d}N}{\mathrm{d}T} = \frac{1}{2} \cdot 0.307 \, \frac{\text{MeV}}{\text{g}/\text{cm}^2} \cdot \frac{6}{12} \frac{1}{\beta^2} \frac{1}{T^2} x$$
$$= 0.171 \, \frac{\text{MeV}}{\text{g}/\text{cm}^2} \cdot \frac{1}{T^2} x \ \rightarrow \ N = x \cdot \int_T^\infty 0.171 \, \frac{\text{MeV}}{\text{g}/\text{cm}^2} \cdot \frac{1}{T'^2} \, \mathrm{d}T' \ ,$$
$$N = 0.171 \, \frac{\text{MeV}}{\text{g}/\text{cm}^2} \cdot x \frac{1}{T} \ ,$$

$T_{\text{threshold}} = 689\,\text{keV} - 511\,\text{keV} = 178\,\text{keV}$.
This gives $N = 9.6$ δ rays above threshold. These electrons are distributed according to a $1/T^2$ spectrum. The maximum transferable energy to electrons by $3\,\text{GeV}/c$ protons is

$$E_{\text{kin}}^{\text{max}} = \frac{E^2}{E + m_p^2/2m_e} = 3.56\,\text{MeV} \ .$$

However, the $1/T^2$ dependence of the δ rays is strongly modified close to the kinematic limit (the spectrum gets steeper). The 9.6 δ rays should be taken from a $1/T^2$ spectrum by a suitable Monte Carlo. Here we argue that the chance to find a δ ray with more than $1\,\text{MeV}$ is only

$$P = \left(\frac{178}{1000}\right)^2 \approx 3\% \ .$$

https://doi.org/10.1017/9781009401531.022 Published online by Cambridge University Press

Therefore, we average the energies over the range 178 keV to 1 MeV,

$$\langle T\rangle = \frac{\int_{178\,\text{keV}}^{1\,\text{MeV}} T \cdot \frac{1}{T^2}\,\mathrm{d}T}{\int_{178\,\text{keV}}^{1\,\text{MeV}} \frac{1}{T^2}\,\mathrm{d}T} = 372\,\text{keV} \ ,$$

$$\beta_{372\,\text{keV}} = \sqrt{1-\frac{1}{\gamma^2}} = 0.815 \ , \quad \gamma = 1.73 \ ,$$

$$\cos\Theta = \frac{1}{n\beta} = 0.82 \ \Rightarrow \ \Theta = 34.6^\circ \ ,$$

$$N_{\text{Photons}} = 9.6 \cdot 490 \cdot \sin^2\Theta \cdot 0.08 = 121 \text{ photons} \ ,$$

where $x = 0.08\,\text{cm}$ is the range of the δ rays of 372 keV (see Chap. 1).

$$n = N_{\text{Photons}} \cdot \eta_{\text{PM}} \cdot \eta_{\text{Geom}} \cdot \eta_{\text{Transfer}} = 0.97$$

if all efficiencies are assumed to be 20%.

$$\Rightarrow \ \epsilon = 1 - \mathrm{e}^{-n} = 62\%$$

is the efficiency for δ rays.

9.5 Imaging Air Cherenkov telescopes measure γ-ray cascades initiated in the atmosphere. Because of the large cross section of photons these cascades are initiated at large altitudes, where the refractive index is smaller than the value given at STP. Does the observed angle of 1° allow to determine the typical altitude where these showers develop?

Density variation in the atmosphere

$$\varrho = \varrho_0 \cdot \mathrm{e}^{-h/h_0} \ ,$$

where $h_0 = 7.9\,\text{km}$ for an isothermal atmosphere.

The index of refraction n varies with the permittivity ε like $n = \sqrt{\varepsilon}$. Since $\varepsilon - 1 \propto \varrho$, one has

$$n^2 = \varepsilon - 1 + 1 \propto \varrho + 1 \ \rightarrow \ \frac{\varrho(h)}{\varrho_0} = \frac{n^2(h)-1}{n_0^2-1} \ .$$

Because of $\Theta = 1^\circ \rightarrow n(h) = 1.000\,152$, if $\beta = 1$ is assumed.

$$\Rightarrow \ \frac{\varrho_0}{\varrho(h)} = 1.94 \ \Rightarrow \ h = h_0 \ln\frac{n_0^2-1}{n^2(h)-1} \approx 5235\,\text{m} \ .$$

https://doi.org/10.1017/9781009401531.022 Published online by Cambridge University Press

9.6 Since

$$\frac{\mathrm{d}E}{\mathrm{d}x} = a\frac{mz^2}{E_{\mathrm{kin}}} \cdot \ln\left(b\frac{E_{\mathrm{kin}}}{m}\right)$$

a measurement of

$$\frac{\mathrm{d}E}{\mathrm{d}x} \cdot E_{\mathrm{kin}}$$

identifies $m \cdot z^2$, since the logarithmic term is usually comparable for non-relativistic particles, and the Lorentz factor is always close to unity. Therefore a measurement of $(\mathrm{d}E/\mathrm{d}x)E_{\mathrm{kin}}$ provides a technique for particle identification.

Let us first assume that muons and pions of 10 MeV kinetic energy can be treated non-relativistically, and that we can approximate the Bethe–Bloch formula in the following way:

$$\frac{\mathrm{d}E}{\mathrm{d}x} = K \cdot z^2 \frac{Z}{A}\frac{1}{\beta^2} \cdot \ln\left(\frac{2m_e c^2 \beta^2 \gamma^2}{I}\right)$$

with $K = 0.307\,\mathrm{MeV/(g/cm^2)}$ and $\beta^2 = (2 \cdot E_{\mathrm{kin}})/(m \cdot c^2)$ in the classical approximation. (The correction terms characterising the saturation effect (Fermi plateau) should be rather small in this kinematic domain.)

For singly charged particles one has

$$\begin{aligned}\frac{\mathrm{d}E}{\mathrm{d}x} &= K\frac{Z}{A}\frac{mc^2}{2E_{\mathrm{kin}}} \cdot \ln\left(\frac{2m_e c^2}{I}\frac{2E_{\mathrm{kin}}}{mc^2}\gamma^2\right)\\ &= 0.076\,75\,\frac{\mathrm{MeV}}{\mathrm{g/cm^2}}\,\frac{mc^2}{E_{\mathrm{kin}}} \cdot \ln\left(14\,600 \cdot \frac{E_{\mathrm{kin}}}{mc^2}\gamma^2\right)\end{aligned}$$

leading to $6.027\,\mathrm{MeV/(g/cm^2)}$ for muons and $7.593\,\mathrm{MeV/(g/cm^2)}$ for pions. Since $\Delta x = 300\,\mu\mathrm{m} \cdot 2.33\,\mathrm{g/cm^3} = 6.99 \cdot 10^{-2}\,\mathrm{g/cm^2}$, one gets $\Delta E(\text{muons}) = 0.421\,\mathrm{MeV}$ and $\Delta E(\text{pions}) = 0.531\,\mathrm{MeV}$.

For muons one would obtain $\Delta E \cdot E_{\mathrm{kin}} = 4.21\,\mathrm{MeV}^2$ and for pions, correspondingly, $\Delta E \cdot E_{\mathrm{kin}} = 5.31\,\mathrm{MeV}^2$.

Neither of these results agrees with the measurement. Redoing the calculation and dropping the assumption that muons and pions can be treated in a non-relativistic fashion, one gets for a consideration of the non-approximated Bethe–Bloch formula and a full relativistic treatment for muons, $\Delta E \cdot E_{\mathrm{kin}} = 4.6\,\mathrm{MeV}^2$, and for pions, correspondingly, $\Delta E \cdot E_{\mathrm{kin}} = 5.7\,\mathrm{MeV}^2$. The difference to the earlier result mainly comes from the correct relativistic treatment. Therefore the above measurement identified a pion.

For the separation of the beryllium isotopes it is justified to use the non-relativistic approach

$$\Delta E = 0.076\,75\,\frac{\mathrm{MeV}}{\mathrm{g/cm^2}}\,z^2\,\frac{mc^2}{E_{\mathrm{kin}}}\cdot\ln\left(14\,600\cdot\frac{E_{\mathrm{kin}}}{mc^2}\gamma^2\right)\cdot\Delta x\ .$$

This leads to $\Delta E\cdot E_{\mathrm{kin}} = 3056\,\mathrm{MeV}^2$ for $^7\mathrm{Be}$ and $\Delta E\cdot E_{\mathrm{kin}} = 3744\,\mathrm{MeV}^2$ for $^9\mathrm{Be}$.

The full non-approximated consideration gives results which differ only by about 1%.

Therefore the measured isotope is $^9\mathrm{Be}$, and $^8\mathrm{Be}$ did not show up because it is highly unstable and disintegrates immediately into two α particles.

Chapter 10

10.1 The neutrino flux ϕ_ν is given by the number of fusion processes $4p \to {}^4\mathrm{He} + 2e^+ + 2\nu_e$ times 2 neutrinos per reaction chain:

$$\begin{aligned}\phi_\nu &= \frac{\text{solar constant}}{\text{energy gain per reaction chain}}\cdot 2\\ &\approx \frac{1400\,\mathrm{W/m^2}}{26.1\,\mathrm{MeV}\cdot 1.6\cdot 10^{-13}\,\mathrm{J/MeV}}\cdot 2 \approx 6.7\cdot 10^{10}\,\mathrm{cm^{-2}\,s^{-1}}\ .\end{aligned}$$

10.2

$$(q_{\nu_\alpha} + q_{e^-})^2 = (m_\alpha + m_{\nu_e})^2\ ,\quad \alpha = \mu,\tau\ ;$$

assuming m_{ν_α} to be small ($\ll m_e, m_\mu, m_\tau$) one gets

$$\begin{aligned}&2E_{\nu_\alpha}m_e + m_e^2 = m_\alpha^2 \quad\Rightarrow\quad E_{\nu_\alpha} = \frac{m_\alpha^2 - m_\mathrm{e}^2}{2m_e} \quad\Rightarrow\\ &\alpha = \mu : E_{\nu_\mu} = 10.92\,\mathrm{GeV}\ ,\quad \alpha = \tau : E_{\nu_\tau} = 3.09\,\mathrm{TeV}\ ;\end{aligned}$$

since solar neutrinos cannot convert into such high-energy neutrinos, the proposed reactions cannot be induced.

10.3 The interaction rate is

$$R = \sigma_\mathrm{N} N_\mathrm{A}[\mathrm{mol}^{-1}]/\mathrm{g}\,d\,A\,\phi_\nu\ ,$$

where σ_N is the cross section per nucleon, $N_\mathrm{A} = 6.022\times 10^{23}\,\mathrm{mol}^{-1}$ is the Avogadro constant, d the area density of the target, A the target area and ϕ_ν the solar neutrino flux. With $d \approx 15\,\mathrm{g\,cm^{-2}}$,

https://doi.org/10.1017/9781009401531.022 Published online by Cambridge University Press

$A = 180 \times 30\,\mathrm{cm}^2$, $\phi_\nu \approx 7 \cdot 10^{10}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$, and $\sigma_\mathrm{N} = 10^{-45}\,\mathrm{cm}^2$ one gets $R = 3.41 \cdot 10^{-6}\,\mathrm{s}^{-1} = 107\,\mathrm{a}^{-1}$. A typical energy of solar neutrinos is 100 keV, i.e., 50 keV are transferred to the electron. Consequently, the total annual energy transfer to the electrons is

$$\Delta E = 107 \cdot 50\,\mathrm{keV} = 5.35\,\mathrm{MeV} = 0.86 \cdot 10^{-12}\,\mathrm{J} \ .$$

With the numbers used so far the mass of the human is 81 kg. Therefore, the equivalent annual dose comes out to be

$$H_\nu = \frac{\Delta E}{m} w_R = 1.06 \cdot 10^{-14}\,\mathrm{Sv} \ ,$$

actually independent of the assumed human mass. The contribution of solar neutrinos to the normal natural dose rate is negligible, since

$$H = \frac{H_\nu}{H_0} = 5.3 \cdot 10^{-12} \ .$$

10.4 Four-momentum conservation yields

$$q_\pi^2 = (q_\mu + q_\nu)^2 = m_\pi^2 \ . \tag{18.22}$$

In the rest frame of the pion the muon and neutrino are emitted in opposite directions, $\vec{p}_\mu = -\vec{p}_{\nu_\mu}$,

$$\begin{pmatrix} E_\mu + E_\nu \\ \vec{p}_\mu + \vec{p}_{\nu_\mu} \end{pmatrix}^2 = (E_\mu + E_\nu)^2 = m_\pi^2 \ . \tag{18.23}$$

Neglecting a possible non-zero neutrino mass for this consideration, one has

$$E_\nu = p_{\nu_\mu}$$

with the result

$$E_\mu + p_\mu = m_\pi \ .$$

Rearranging this equation and squaring it gives

$$\begin{aligned} E_\mu^2 + m_\pi^2 - 2E_\mu m_\pi &= p_\mu^2 \ , \\ 2E_\mu m_\pi &= m_\pi^2 + m_\mu^2 \ , \\ E_\mu &= \frac{m_\pi^2 + m_\mu^2}{2m_\pi} \ . \end{aligned} \tag{18.24}$$

For $m_\mu = 105.658\,369\,\mathrm{MeV}$ and $m_{\pi^\pm} = 139.570\,18\,\mathrm{MeV}$ one gets $E_\mu^\mathrm{kin} = E_\mu - m_\mu = 4.09\,\mathrm{MeV}$. For the two-body decay of the kaon,

https://doi.org/10.1017/9781009401531.022 Published online by Cambridge University Press

$K^+ \to \mu^+ + \nu_\mu$, (18.24) gives $E_\mu^{\text{kin}} = E_\mu - m_\mu = 152.49\,\text{MeV}$ ($m_{K^\pm} = 493.677\,\text{MeV}$).

The neutrino energies are then just given by

$$E_\nu = m_\pi - E_\mu = 29.82\,\text{MeV}$$

for pion decay and

$$E_\nu = m_K - E_\mu = 235.53\,\text{MeV}$$

for kaon decay.

10.5 The expected difference of arrival times Δt of two neutrinos with velocities v_1 and v_2 emitted at the same time from the supernova is

$$\Delta t = \frac{r}{v_1} - \frac{r}{v_2} = \frac{r}{c}\left(\frac{1}{\beta_1} - \frac{1}{\beta_2}\right) = \frac{r}{c}\,\frac{\beta_2 - \beta_1}{\beta_1\,\beta_2} \;. \tag{18.25}$$

If the recorded electron neutrinos had a rest mass m_0, their energy would be

$$E = mc^2 = \gamma m_0 c^2 = \frac{m_0 c^2}{\sqrt{1-\beta^2}} \;, \tag{18.26}$$

and their velocity

$$\beta = \left(1 - \frac{m_0^2 c^4}{E^2}\right)^{1/2} \approx 1 - \frac{1}{2}\,\frac{m_0^2 c^4}{E^2} \;, \tag{18.27}$$

since one can safely assume that $m_0 c^2 \ll E$. This means that the neutrino velocities are very close to the velocity of light. Obviously, the arrival-time difference Δt depends on the velocity difference of the neutrinos. Using (18.25) and (18.27), one gets

$$\Delta t \approx \frac{r}{c}\,\frac{\frac{1}{2}\,\frac{m_0^2 c^4}{E_1^2} - \frac{1}{2}\,\frac{m_0^2 c^4}{E_2^2}}{\beta_1 \beta_2} \approx \frac{1}{2}\,m_0^2 c^4\,\frac{r}{c}\,\frac{E_2^2 - E_1^2}{E_1^2\,E_2^2} \;. \tag{18.28}$$

The experimentally measured arrival-time differences and individual neutrino energies allow in principle to work out the electron-neutrino rest mass

$$m_0 = \left(\frac{2\Delta t}{r\,c^3}\,\frac{E_1^2\,E_2^2}{E_2^2 - E_1^2}\right)^{1/2} \;. \tag{18.29}$$

10.6 The interaction cross section of high-energy neutrinos was measured at accelerators to be

$$\sigma(\nu_\mu N) = 6.7 \cdot 10^{-39}\, E_\nu\,[\mathrm{GeV}]\,\mathrm{cm}^2/\mathrm{nucleon}\ . \qquad (18.30)$$

For 100 TeV neutrinos one would arrive at a cross section of $6.7 \cdot 10^{-34}\,\mathrm{cm}^2$/nucleon. For a target thickness of 1 km an interaction probability W per neutrino of

$$W = N_\mathrm{A}[\mathrm{mol}^{-1}]/\mathrm{g}\,\sigma\, d\,\varrho = 4 \cdot 10^{-5} \qquad (18.31)$$

is obtained ($d = 1\,\mathrm{km} = 10^5\,\mathrm{cm}$, $\varrho(\mathrm{ice}) \approx 1\,\mathrm{g/cm}^3$).

The total interaction rate R is obtained from the integral neutrino flux Φ_ν, the interaction probability W, the effective collection area $A_\mathrm{eff} = 1\,\mathrm{km}^2$ and a measurement time t. This leads to an event rate of

$$R = \Phi_\nu\, W\, A_\mathrm{eff} \qquad (18.32)$$

corresponding to 250 events per year. If a target volume of $1\,\mathrm{km}^3$ is fully instrumented, the effective collection area will be even larger.

Chapter 11

11.1

$$\frac{\mathrm{d}E}{\mathrm{d}x} = a + bE$$

is a good approximation for the energy loss, where a represents the ionisation loss and b stands for the losses due to pair production, bremsstrahlung and photonuclear interactions. For 1 TeV muons one finds [5]

$$a \approx 2.5\,\mathrm{MeV/(g/cm^2)}\ ,$$
$$b \approx 7.5 \cdot 10^{-6}\,(\mathrm{g/cm^2})^{-1}\ .$$

For 3 m of iron ($\varrho \cdot x = 2280\,\mathrm{g/cm}^2$) one gets

$$\Delta E = 3\,\mathrm{m}\,\frac{\mathrm{d}E}{\mathrm{d}x} = 22.8\,\mathrm{GeV}\ .$$

Because of energy-loss fluctuations, one gets a radiative tail in the momentum distributions of an originally monoenergetic muon beam as sketched in Fig. 18.5 [5].

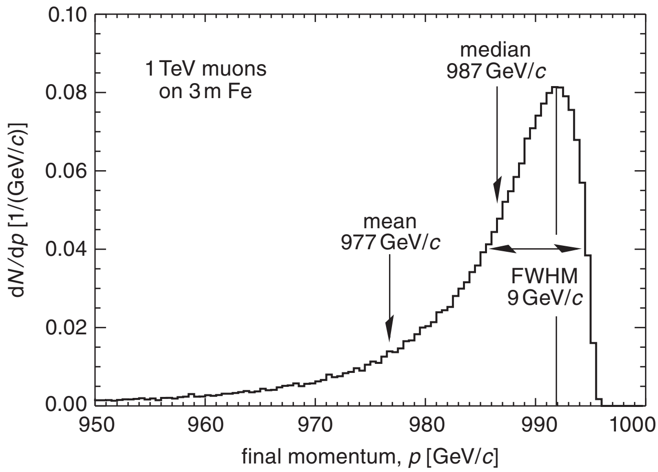

Fig. 18.5. The momentum distribution of 1 TeV/c muons after traversing 3 m of iron [6].

11.2 The production probability can be determined along the lines of Eq. (1.25) and the references given in that context. For argon ($Z = 18,\ A = 36,\ \varrho = 1.782 \cdot 10^{-3}\,\mathrm{g/cm^3}$) the column density is

$$d = 0.5346\,\mathrm{g/cm^2} \ .$$

Bending radii from 5 cm to 20 cm correspond to momenta

$$p\,[\mathrm{GeV}/c] = 0.3\,B\,[\mathrm{T}] \cdot R\,[\mathrm{m}]$$

of 30 MeV/c to 120 MeV/c. The δ-electron differential energy spectrum for high-momentum muons can be approximated by

$$\phi(\varepsilon)\,\mathrm{d}\varepsilon = 2Cm_ec^2\frac{\mathrm{d}\varepsilon}{\varepsilon^2} \ ,$$

where ε is the energy of the δ electron and m_e the electron rest mass [7, 8]. With $C = 0.150\,\frac{Z}{A}\,\mathrm{g^{-1}\,cm^2}$ one gets

$$P = \int_{30\,\mathrm{MeV}}^{120\,\mathrm{MeV}} \phi(\varepsilon)\,\mathrm{d}\varepsilon = 0.150 \cdot \frac{Z}{A}\left(\frac{1}{30} - \frac{1}{120}\right)\frac{\mathrm{cm^2}}{\mathrm{g}}$$
$$= 1.875 \cdot 10^{-3}\,\frac{\mathrm{cm^2}}{\mathrm{g}} \ ,$$
$$P \cdot d = 10^{-3} = 0.1\% \quad \text{per track} \ .$$

For 100 tracks one has a 10% probability that one of the charged particles will create a δ electron with the properties in question.

https://doi.org/10.1017/9781009401531.022 Published online by Cambridge University Press

11.3 (a)

$$\Theta_\rho = \Theta_\varphi \quad \text{for} \quad n = \frac{1}{2} \quad \Rightarrow \quad \Theta = \pi\sqrt{2} = 255.6^\circ \ ,$$
$$B(\rho) = B(\rho_0)\left(\frac{\rho_0}{\rho}\right)^{1/2} \ .$$

(b)

$$\frac{\mathrm{d}E}{\mathrm{d}x}(10\,\mathrm{keV}) = 27\,\frac{\mathrm{keV}}{\mathrm{cm}} \cdot \pi\sqrt{2} \cdot \rho_0 \cdot \frac{p}{p_{\mathrm{atm}}}$$
$$= 27 \cdot 10^3 \cdot \pi\sqrt{2} \cdot 50 \cdot \frac{10^{-3}}{760}\,\mathrm{eV} = 7.9\,\mathrm{eV} \ ,$$

i.e., just about one or perhaps even zero ionisation processes will occur.

11.4

$$B_y \cdot \ell \propto x \ , \quad B_x \cdot \ell \propto y \ ,$$
$$\ell = \text{const} \ \Rightarrow \ B_y = g \cdot x \ , \quad B_x = g \cdot y \ .$$

This leads to a magnetic potential of

$$V = -g \cdot x \cdot y$$

with

$$g = \frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y} \ ,$$

where g is called the gradient of the quadrupole;

$$-\,\mathrm{grad}\,V = -\left(\frac{\partial V}{\partial x}\vec{e}_x + \frac{\partial V}{\partial y}\vec{e}_y\right) = \underbrace{g \cdot y}_{B_x}\,\vec{e}_x + \underbrace{g \cdot x}_{B_y}\,\vec{e}_y \ .$$

Since the surface of the yoke must have constant potential, one has

$$V = V_0 = -g \cdot x \cdot y \ , \quad \text{i.e.} \quad x \propto \frac{1}{y} \ ,$$

which means that the surface of the yoke must be hyperbolic.

https://doi.org/10.1017/9781009401531.022 Published online by Cambridge University Press

Chapter 12

12.1

$$\tau(T^*) = \frac{1}{12}\tau(T) \ , \quad \tau_0\,\mathrm{e}^{E_\mathrm{a}/kT^*} = \frac{1}{12}\tau_0\,\mathrm{e}^{E_\mathrm{a}/kT} \ ;$$

solve for $\frac{kT^*}{kT} \rightarrow$

$$\frac{kT^*}{kT} = \frac{1}{1-\frac{kT}{E_\mathrm{a}}\ln 12} = 1.18 \ .$$

$\rightarrow$ The temperature has to be increased by 18%.

12.2

$$\frac{\Delta U^{-*}}{\Delta U^{-}} = \frac{-\frac{N\,e}{C\ln[r_\mathrm{a}/(1.1\,r_\mathrm{i})]}\ln[r_0/(1.1\,r_\mathrm{i})]}{-\frac{N\,e}{C\ln(r_\mathrm{a}/r_\mathrm{i})}\ln(r_0/r_\mathrm{i})} = \frac{1-\frac{\ln 1.1}{\ln(r_0/r_\mathrm{i})}}{1-\frac{\ln 1.1}{\ln(r_\mathrm{a}/r_\mathrm{i})}} \approx 0.88 \ .$$

$\rightarrow$ The gain is decreased by 12%.

Chapter 13

13.1 Assume Poisson statistics:

$$\text{efficiency} = 50\% \ \Rightarrow \ \mathrm{e}^{-m} = 0.5 \ \Rightarrow \ m = 0.6931 \ ,$$

$$N = \frac{m}{\eta_\mathrm{PM}\cdot\eta_\mathrm{Geom}\cdot\eta_\mathrm{Transfer}} = 43.32 \ ,$$

$$\frac{\mathrm{d}N}{\mathrm{d}x} = 490\,\sin^2\theta_\mathrm{C}\,\mathrm{cm}^{-1}\cdot 150\,\mathrm{cm} = 43.32 \ ,$$

$$\sin^2\theta_\mathrm{C} = 5.89\cdot 10^{-4} \ ,$$

$$\theta_\mathrm{C} = 1.39^\circ \ ;$$

$$\cos\theta_\mathrm{C} = \frac{1}{n\beta} \Rightarrow \beta = \frac{1}{n\,\cos\theta_\mathrm{C}} \ .$$

Index of refraction of CO_2 at 3 atm:

$$n = 1.00123 \Rightarrow \beta = 0.99907$$

$$\Rightarrow \gamma = 23.14$$

$$\Rightarrow E_\pi = 3.23\,\mathrm{GeV} \ .$$

https://doi.org/10.1017/9781009401531.022 Published online by Cambridge University Press

13.2 By grouping the photons into pairs one can work out the invariant mass of the different $\gamma\gamma$ combinations,

$$m^2 = (q_{\gamma_i} + q_{\gamma_j})^2 = 2 \cdot E_{\gamma_i} \cdot E_{\gamma_j}(1 - \cos\theta) \ .$$

One finds for $m(\gamma_1, \gamma_2) = 135\,\text{MeV}$ and for $m(\gamma_3, \gamma_4) = 548\,\text{MeV}$, i.e., the four photons came from a π^0 and an η.

13.3

$$\Delta t = \frac{L \cdot c}{2 \cdot p^2}(m_2^2 - m_1^2) = \frac{L \cdot c}{2 \cdot p^2}(m_2 - m_1)(m_2 + m_1) \ ;$$

if $m_1 \approx m_2 \rightarrow$

$$\Delta t = \frac{L \cdot c}{2 \cdot p^2} \cdot 2m \cdot \Delta m \ ;$$

since

$$p^2 = \gamma^2 \cdot m^2 \cdot \beta^2 \cdot c^2$$

one gets

$$\Delta t = \frac{L \cdot c}{\gamma^2 \cdot \beta^2 \cdot c^2} \cdot \frac{\Delta m}{m} \ ;$$

$\rightarrow$

$$\frac{\Delta m}{m} = \gamma^2 \cdot \frac{c^2 \cdot \beta^2}{L \cdot c} \cdot \Delta t \ .$$

For $\beta \approx 1$ one has

$$\frac{\Delta m}{m} = \gamma^2 \cdot \frac{c}{L} \cdot \Delta t = \gamma^2 \frac{\Delta t}{t} \ . \qquad (18.33)$$

For a momentum of $1\,\text{GeV}/c$ the flight-time difference for muons and pions is

$$\Delta t = \frac{L}{c} \cdot \left(\frac{1}{\beta_1} - \frac{1}{\beta_2}\right) \ .$$

From $\gamma\beta mc^2 = 1\,\text{GeV}$ one gets $\gamma_\mu \cdot \beta_\mu = 9.46$, $\gamma_\pi \cdot \beta_\pi = 7.16$ corresponding to $\beta_\mu = 0.989$, $\gamma_\mu = 9.57$ and $\beta_\pi = 0.981$, $\gamma_\pi = 7.30$. With these values the flight-time difference becomes $\Delta t = 27.5\,\text{ps}$. The absolute flight times for pions and muons are not very different (this is the problem!), namely $t_\mu = 3.37\,\text{ns}$ and $t_\pi = 3.40\,\text{ns}$, resulting in

$$\frac{\Delta t}{t} \approx 8.12 \cdot 10^{-3} \ .$$

This excellent value is, however, spoiled by the factor γ^2 in Eq. (18.33), leading to a relatively poor mass resolution.

https://doi.org/10.1017/9781009401531.022 Published online by Cambridge University Press

13.4 The quantity E_{CM}^2 is equal to the kinematical invariant $s = (p_+ + p_-)^2$, where p_+ and p_- are the positron and electron four-momenta, respectively. Then this value can be expressed as

$$s = (p_+ + p_-)^2 = 2m_e^2 + 2(E_+E_- - \vec{p}_+\vec{p}_-) \ .$$

Neglecting the electron mass and the angle between the beams, 22 mrad, one gets

$$E_{\mathrm{CM}} = 2\sqrt{E_+E_-} = 10.58\,\mathrm{GeV} \ .$$

Considering the finite crossing angle of 22 mrad results in a decrease of the centre-of-mass energy of 200 keV only!

13.5 Since a particle energy loss is recovered at every revolution when it passes the RF cavities, let us calculate first the probability of the emission of a bremsstrahlung photon carrying away more than 1% of the particle's energy. The number of photons which are emitted along the path ΔX in the energy interval $[\varepsilon, \varepsilon + \mathrm{d}\varepsilon]$ to first approximation is (see Rossi's book [7])

$$\mathrm{d}n = \frac{\Delta X}{X_0} \frac{\mathrm{d}\varepsilon}{\varepsilon} \ .$$

Integration of this expression from ε_0 to the beam energy, E_0, gives the required probability

$$w_1 = \frac{\Delta X}{X_0} \ln \frac{E_0}{\varepsilon_0} \ .$$

The density of the residual gas (assuming air, having the density of $1.3 \cdot 10^{-3}\,\mathrm{g/cm^3}$ at 100 kPa) is $1.3 \cdot 10^{-15}\,\mathrm{g/cm^3}$ which results in $w_1 \approx 0.5 \cdot 10^{-10}$, which means that after an average of $1/w_1 \approx 2 \cdot 10^{10}$ revolutions a bremsstrahlung process with an energy transfer of more than 1% of the beam energy occurs. This corresponds to a beam lifetime of $t_\mathrm{b} \approx 2 \cdot 10^5\,\mathrm{s}$. In a real experiment with intensively colliding beams, the beam lifetime is much shorter and it is determined by other effects, such as beam–beam interactions, the Touschek effect,‡ nuclear interactions of the electrons with the residual gas, interactions with the ambient blackbody photons of room temperature and so on.

‡ An effect observed in electron–positron storage rings in which the maximum particle concentration in the counterrotating electron bunches is limited at low energies by the loss of electrons in Møller scattering [9].

13.6 The differential cross section for this process is expressed as (see, for example, [5] (2006), p. 325)

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}(1 + \cos^2\theta) \ .$$

Integration of this formula over the mentioned solid angle and converting the 'natural units' of the cross-section formula into numerical values by using $\hbar c = 0.1973$ GeV fm results in

$$\sigma_{\rm det} = \frac{\pi\alpha^2}{s}\left(z_0 + \frac{z_0^3}{3}\right) = \frac{65.1\,\text{nb}}{s\,[\text{GeV}^2]}\left(z_0 + \frac{z_0^3}{3}\right) = \frac{70.5\,\text{nb}}{s\,[\text{GeV}^2]} \ ,$$

where $z_0 = \cos\theta_0$. Thus, one gets $\sigma_{\rm det} = 0.63$ nb at $E_{\rm CM} = 10.58$ GeV corresponding to a muon event rate of 6.3 Hz.

Chapter 14

14.1 When the overall resolution of a system is determined by the convolution of multiple Gaussian distributions, the individual resolutions add in quadrature:

$$\Delta t = \sqrt{\Delta t_1^2 + \Delta t_2^2} = \sqrt{100^2 + 50^2}\,\text{ps} = 112\,\text{ps} \ .$$

14.2 (a)

$$Q_{\rm n} = \sqrt{Q_{\rm ni}^2 + Q_{\rm nv}^2} = \sqrt{120^2 + 160^2}\,\text{eV} = 200\,\text{eV} \ .$$

(b)

$$Q_{\rm n} = \sqrt{Q_{\rm ni}^2 + Q_{\rm nv}^2} = \sqrt{10^2 + 160^2}\,\text{eV} = 160\,\text{eV} \ .$$

After cooling, the current noise contribution is not discernible.

14.3 (a) The two Gaussian peaks are adequately resolved at $\sigma_E = \Delta E/3$, so since the spacing between the two peaks is $\Delta E = (72.87 - 70.83)$ keV $= 2.04$ keV, the required resolution is $\sigma_E = 0.68$ keV or 1.6 keV FWHM. Note that in systems dominated by electronic noise it is more useful to specify absolute resolution rather than relative resolution, as the linewidth is essentially independent of energy.

(b) Since the individual resolutions add in quadrature, $\sigma_E^2 = \sigma_{\rm det}^2 + \sigma_{\rm n}^2$, the allowable electronic noise is $\sigma_{\rm n} = 660$ eV.

https://doi.org/10.1017/9781009401531.022 Published online by Cambridge University Press

14.4 (a) The noise current sources are the detector bias current, contributing $i_{\mathrm{nd}}^2 = 2eI_{\mathrm{d}}$, and the bias resistor with $i_{\mathrm{nb}}^2 = 4kT/R_{\mathrm{b}}$. The noise voltage sources are the series resistance and the amplifier, contributing $e_{\mathrm{nR}}^2 = 4kTR_{\mathrm{s}}$ and $e_{\mathrm{na}}^2 = 10^{-18}\,\mathrm{V}^2/\mathrm{Hz}$, respectively. The shape factors for a CR–RC shaper are $F_{\mathrm{i}} = F_{\mathrm{v}} = 0.924$. This results in an equivalent noise charge

$$
\begin{aligned}
Q_{\mathrm{n}}^2 &= i_{\mathrm{n}}^2 T_{\mathrm{s}} F_{\mathrm{i}} + C_{\mathrm{d}}^2 e_{\mathrm{n}}^2 \frac{F_{\mathrm{v}}}{T_{\mathrm{s}}} \ , \\
Q_{\mathrm{n}}^2 &= \left(2eI_{\mathrm{d}} + \frac{4kT}{R_{\mathrm{b}}}\right) \cdot T_{\mathrm{s}} \cdot F_{\mathrm{i}} + C_{\mathrm{d}}^2 \cdot (4kTR_{\mathrm{s}} + e_{\mathrm{na}}^2) \cdot \frac{F_{\mathrm{v}}}{T_{\mathrm{s}}} \ , \\
Q_{\mathrm{n}}^2 &= (3.2 \cdot 10^{-26} + 1.66 \cdot 10^{-27}) \cdot 10^{-6} \cdot 0.924\,\mathrm{C}^2 + \qquad (18.34) \\
&\quad + 10^{-20} \cdot (1.66 \cdot 10^{-19} + 10^{-18}) \cdot \frac{0.924}{10^{-6}}\,\mathrm{C}^2 \ .
\end{aligned}
$$

The detector bias current contributes $1075\,e$, the bias current $245\,e$ the series resistance $246\,e$ and the amplifier $601\,e$. These add in quadrature to yield the total noise of $Q_{\mathrm{n}} = 1280\,e$ or 4.6 keV rms (10.8 keV FWHM).

(b) As calculated in (a) the current noise contribution is

$$Q_{\mathrm{ni}} = \sqrt{1075^2 + 245^2}\,e = 1103\,e$$

and the voltage noise contribution is

$$Q_{\mathrm{nv}} = \sqrt{246^2 + 601^2}\,e = 649\,e \ .$$

Minimum noise results when the current and voltage noise contributions are equal. From Eq. (14.18) this condition yields the optimum shaping time

$$T_{\mathrm{s,opt}} = C_{\mathrm{i}} \frac{e_{\mathrm{n}}}{i_{\mathrm{n}}} \sqrt{\frac{F_{\mathrm{v}}}{F_{\mathrm{i}}}} \ .$$

This yields $T_{\mathrm{s,opt}} = 589\,\mathrm{ns}$ and $Q_{\mathrm{n,min}} = 1196\,e$.

(c) Without the bias resistor, the noise is $1181\,e$. For the resistor to add 1% to the total, its noise may be 2% of $1181\,e$ or $24\,e$, so $R_{\mathrm{b}} > 34\,\mathrm{M}\Omega$.

14.5 (a) Equation (14.26) yields the timing jitter

$$\sigma_{\mathrm{t}} = \frac{\sigma_{\mathrm{n}}}{(\mathrm{d}V/\mathrm{d}t)_{V_{\mathrm{T}}}} \ .$$

https://doi.org/10.1017/9781009401531.022 Published online by Cambridge University Press

The noise level is $\sigma_{\rm n} = 10\,\mu\text{V}$ and the rate of change is

$$\frac{\mathrm{d}V}{\mathrm{d}t} \approx \frac{\Delta V}{t_{\rm r}} = \frac{10 \cdot 10^{-3}\,\text{V}}{10 \cdot 10^{-9}\,\text{s}} = 10^{6}\,\text{V/s} \ ,$$

yielding the timing jitter

$$\sigma_{\rm t} = \frac{10 \cdot 10^{-6}}{10^{6}}\,\text{s} = 10\,\text{ps} \ .$$

(b) For the 10 mV signal the threshold of 5 mV is at 50% of the rise time, so the comparator fires at $(5+1)$ ns, whereas for the 50 mV signal the threshold is at 10% of the rise time, so the comparator fires at $(1+1)$ ns. The time shift is 4 ns. Note that the time t_0 drops out, so it can be disregarded.

Chapter 15

15.1

$$N_{\rm acc} = \varepsilon_e N_e + \varepsilon_\pi N_\pi = \varepsilon_e N_e + \varepsilon_\pi (N_{\rm tot} - N_e)$$

Solving for N_e gives

$$N_e = \frac{N_{\rm acc} - \varepsilon_\pi N_{\rm tot}}{\varepsilon_e - \varepsilon_\pi} \ .$$

In case of $\varepsilon_e = \varepsilon_\pi$ there would obviously be no chance to determine N_e.

15.2

$$\begin{aligned}
E[t] &= \frac{1}{\tau}\int_0^\infty t\,\mathrm{e}^{-t/\tau}\,\mathrm{d}t = \tau \ , \\
\sigma^2[t] &= \frac{1}{\tau}\int_0^\infty (t-\tau)^2\,\mathrm{e}^{-t/\tau}\,\mathrm{d}t \\
&= \frac{1}{\tau}\left[\int_0^\infty t^2\,\mathrm{e}^{-t/\tau}\,\mathrm{d}t - \int_0^\infty 2t\tau\,\mathrm{e}^{-t/\tau}\,\mathrm{d}t + \tau^2\int_0^\infty \mathrm{e}^{-t/\tau}\,\mathrm{d}t\right] \\
&= \frac{1}{\tau}\left(2\tau^3 - 2\tau^3 + \tau^3\right) = \tau^2 \ .
\end{aligned}$$

15.3 Source rate

$$n_\nu = \frac{N_1}{t_1} - \frac{N_2}{t_2} = (n_\nu + n_\mu) - n_\mu \ .$$

https://doi.org/10.1017/9781009401531.022 Published online by Cambridge University Press

Standard deviation from error propagation:

$$\sigma_{n_\nu} = \left[\left(\frac{\sigma_{N_1}}{t_1}\right)^2 + \left(\frac{\sigma_{N_2}}{t_2}\right)^2\right]^{1/2} = \left(\frac{N_1}{t_1^2} + \frac{N_2}{t_2^2}\right)^{1/2}$$
$$= \left(\frac{n_\nu + n_\mu}{t_1} + \frac{n_\mu}{t_2}\right)^{1/2} \quad .$$

$t_1 + t_2 = T$ is fixed. Therefore, $\mathrm{d}T = \mathrm{d}t_1 + \mathrm{d}t_2 = 0$. Squaring and differentiating σ_{n_ν} with respect to the measurement times gives

$$2\sigma_{n_\nu}\,\mathrm{d}\sigma_{n_\nu} = -\frac{n_\nu + n_\mu}{t_1^2}\,\mathrm{d}t_1 - \frac{n_\mu}{t_2^2}\,\mathrm{d}t_2 \quad .$$

Setting

$$\mathrm{d}\sigma_{n_\nu} = 0$$

yields the optimum condition ($\mathrm{d}t_1 = -\mathrm{d}t_2$):

$$\frac{n_\nu + n_\mu}{t_1^2}\,\mathrm{d}t_2 - \frac{n_\mu}{t_2^2}\,\mathrm{d}t_2 = 0 \quad \Rightarrow \quad \frac{t_1}{t_2} = \sqrt{\frac{n_\nu + n_\mu}{n_\mu}} = \sqrt{\frac{n_\nu}{n_\mu} + 1} = 2 \quad .$$

15.4 $y = mE$, m – slope. The fitted linear relation is obtained from $y + Am = 0$ (C_y – error matrix) with [10]

$$A = -\begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} \quad ,$$

$$C_y = \begin{pmatrix} 0.3^2 & & & & & \\ & 0.3^2 & & & 0 & \\ & & 0.3^2 & & & \\ & & & 0.3^2 & & \\ & 0 & & & 0.3^2 & \\ & & & & & 0.3^2 \end{pmatrix} = 0.09\, I \quad ,$$

https://doi.org/10.1017/9781009401531.022 Published online by Cambridge University Press

$$m = -(A^{\mathrm{T}}A)^{-1}A^{\mathrm{T}}y = \left\{ \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} \right\}^{-1} \cdot$$

$$\begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \end{pmatrix} \begin{pmatrix} 0 \\ 0.8 \\ 1.6 \\ 2.5 \\ 2.8 \\ 4.0 \end{pmatrix} = \frac{1}{55} \cdot 42.7 \approx 0.7764 \ ,$$

$$\begin{aligned}(\Delta m)^2 &= (A^{\mathrm{T}}C_y^{-1}A)^{-1} \\ &= \left\{ \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \end{pmatrix} 0.09^{-1} \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} \right\}^{-1} \\ &= 0.09 \cdot \frac{1}{55} \approx 0.00164 \ , \\ \to m &= 0.7764 \pm 0.0405 \ .\end{aligned}$$

The data points – corrected for the offset – along with the best fit are shown in Fig. 18.6.

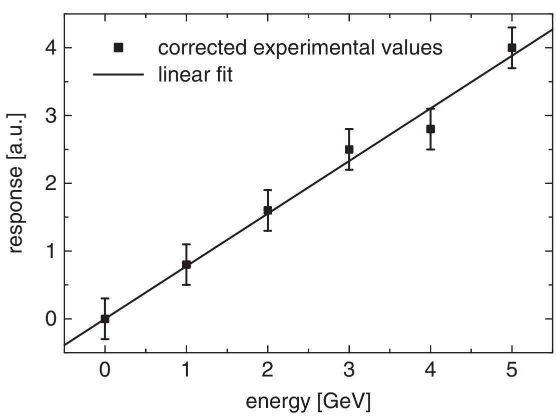

Fig. 18.6. Calibration data, corrected for the offset, along with the best fit calibration function.

https://doi.org/10.1017/9781009401531.022 Published online by Cambridge University Press

Chapter 16

16.1 $P = 10\,\mathrm{mW}$ laser power at the frequency ν; rate of photons $n = P/h\nu$, h – Planck's constant; momentum of the photon (after de Broglie) $p = h/\lambda = h\nu/c$; change of momentum upon reflection $2p = 2h\nu/c$; the force has two components: (a) reflected photons $F_1 = n \cdot 2p \cdot \epsilon = (P/h\nu)2(h\nu/c)\epsilon = 2(P/c)\epsilon$; (b) absorbed photons $F_2 = (P/h\nu)(h\nu/c)(1-\epsilon) = (P/c)(1-\epsilon)$; $F = F_1 + F_2 = \frac{P}{c} \cdot (\epsilon + 1) = 5 \cdot 10^{-11}\,\mathrm{N}$.

16.2 Number of ^{238}U nuclei: $N = N_0 \cdot \mathrm{e}^{-\lambda t}$, where $\lambda = \ln 2/T_{1/2}$; number of lead nuclei: $N_0(1 - \mathrm{e}^{-\lambda t})$. $r = N_0(1 - \mathrm{e}^{-\lambda t})/N_0\,\mathrm{e}^{-\lambda t} = \mathrm{e}^{\lambda t} - 1 = 0.06$, $t = 3.8 \cdot 10^8$ years.

16.3 Total power radiated by the Sun: $P = 4\pi R^2 \sigma T_\mathrm{S}^4$, where σ – Boltzmann's constant, T_S – Sun's surface temperature ($\approx 6000\,\mathrm{K}$), R – Sun's radius. The satellite will absorb the power

$$P_1 = \frac{4\pi R^2 \sigma T_\mathrm{S}^4}{4\pi D^2} \cdot \pi r^2 \varepsilon = \frac{R^2}{D^2} \sigma T_\mathrm{S}^4 \pi r^2 \varepsilon \ ,$$

where D – distance Sun–satellite, r – radius of the satellite, ε – absorption coefficient. Since the emissivity is equal to the absorption, one gets

$$P_2 = 4\pi r^2 \sigma T^4 \cdot \varepsilon$$

for the radiated power by the satellite. At equilibrium one has $P_1 = P_2$, and therefore

$$\frac{R^2}{D^2} \sigma T_\mathrm{S}^4 \pi r^2 \cdot \varepsilon = 4\pi r^2 \sigma T^4 \cdot \varepsilon$$

yielding

$$T = T_\mathrm{S} \cdot \left(\frac{R^2}{4D^2} \right)^{1/4} \ ;$$

with $R \approx 700\,000\,\mathrm{km}$ and $D \approx 150\,000\,000\,\mathrm{km}$ one obtains $T = 290\,\mathrm{K}$.

16.4

$$E_\mathrm{Li} + E_\alpha = 2.8\,\mathrm{MeV} \ , \quad E = \frac{p^2}{2m} \ \rightarrow \ \sqrt{2m_\mathrm{Li} E_\mathrm{Li}} = \sqrt{2m_\alpha E_\alpha}$$

https://doi.org/10.1017/9781009401531.022 Published online by Cambridge University Press

because the lithium nucleus and the α particle are emitted back to back;

$$E_\alpha = \frac{m_{\mathrm{Li}}}{m_\alpha} \cdot (Q - E_\alpha) \;\to\; E_\alpha = \frac{m_{\mathrm{Li}}}{m_{\mathrm{Li}} + m_\alpha} \cdot Q = 1.78\,\mathrm{MeV} \;\; .$$

16.5 $\mathrm{d}\sigma/\mathrm{d}\Omega \propto 1/\sin^4\theta/2 \propto 1/\theta^4$ for Bhabha scattering. The count rate is determined by the lower acceptance boundary,

$$\sigma_{\mathrm{Bhabha}}(\theta_0) = \int_{\theta_0} (\mathrm{d}\sigma/\mathrm{d}\Omega) 2\pi \, \mathrm{d}\theta \propto 1/\theta_0^3 \;\; .$$

Doubling the accuracy of $\sigma(e^+e^- \to Z)$ by a factor of 2 means

$$\sigma_{\mathrm{Bhabha}}(\theta_{\mathrm{new}}) = 4 \cdot \sigma_{\mathrm{Bhabha}}(\theta_0) \;\; , \quad 1/\theta_{\mathrm{new}}^3 = 4 \cdot 1/\theta_0^3 \;\; .$$

This leads to

$$\theta_{\mathrm{new}} = \theta_0 \cdot \sqrt[3]{1/4} = 0.63\,\theta_0 \approx 19\,\mathrm{mrad} \;\; .$$

16.6 A 100 GeV γ-induced shower starts at an altitude of $d \approx 20\,\mathrm{km}$ and has just about 100 energetic secondaries which emit Cherenkov light over a distance of $\approx 20\,X_0 (= 6000\,\mathrm{m})$. The photon yield in air is ≈ 20 photons/m, leading to a total number of Cherenkov photons of

$$N_\gamma \lesssim 100 \cdot 20 \cdot 6000 = 1.2 \cdot 10^7 \;\; .$$

These photons will be distributed at sea level over a circular area

$$A = \pi \cdot (d \cdot \tan\theta)^2 \;\; ,$$

where θ is the Cherenkov angle of relativistic electrons in air at 20 km altitude ($\approx 1.2°$):

$$A = 550\,000\,\mathrm{m}^2 \;\; .$$

With an absorption coefficient in air of $\epsilon \approx 30\%$ one gets

$$n = N_\gamma/A \cdot (1 - \epsilon) \lesssim 15/\mathrm{m}^2 \;\; .$$

References

[1] E. Fenyves & O. Haimann, *The Physical Principles of Nuclear Radiation Measurements*, Akadémiai Kiadó, Budapest (1969)

[2] G. Hertz, *Lehrbuch der Kernphysik*, Bd. 1, Teubner, Leipzig (1966)

[3] C. Grupen, *Grundkurs Strahlenschutz*, Springer, Berlin (2003)

[4] M. Mandò, Non-focused Cherenkov Effect Counters, *Nuovo Cim.* **12** (1954) 5–27; in J.V. Jelley, *Cherenkov Radiation and Its Applications*, Pergamon Press, London, New York (1958)

[5] Particle Data Group, Review of Particle Physics, S. Eidelman *et al*, *Phys. Lett.* **B592 Vol. 1–4** (2004) 1–1109; W.-M. Yao *et al*, *J. Phys.* **G33** (2006) 1–1232; http://pdg.lbl.gov

[6] Donald E. Groom, Nikolai V. Mokhov & Sergei I. Striganov, Muon Stopping Power and Range Tables 10-MeV to 100-TeV, *Atom. Data Nucl. Data Tabl.* **78** (2001) 183–356; A. van Ginneken, Energy Loss and Angular Characteristics of High-Energy Electromagnetic Processes, *Nucl. Instr. Meth.* **A251** (1986) 21–39

[7] B. Rossi, *High Energy Particles*, Prentice-Hall, Englewood Cliffs (1952)

[8] C. Grupen, Electromagnetic Interactions of High Energy Cosmic Ray Muons, *Fortschr. der Physik* **23** (1976) 127–209

[9] C. Bernardini, B. Touschek *et al.*, Lifetime and Beam Size in a Storage Ring, *Phys. Rev. Lett.* **10** (1963) 407–9

[10] S. Brandt, *Datenanalyse*, 4. Auflage; Spektrum Akademischer Verlag, Heidel-berg/Berlin (1999); *Data Analysis: Statistical and Computational Methods for Scientists and Engineers*, 3rd edition, Springer, New York (1998)