

Research Article

Nucleation regions in the Large-Scale Structure II: Morphology and dynamical state of supercluster cores

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Abstract

This work explores the morphology and dynamical properties of *cores* within rich superclusters, highlighting their role as transitional structures in the large-scale structure of the Universe. Using projected and radial velocity distributions of member galaxies, we identify *cores* as dense structures that, despite being gravitationally bound, are not yet dynamically relaxed. However, they exhibit a tendency towards virialisation, evolving in a self-similar manner to massive galaxy clusters but on a larger scale. Morphological analysis reveals that *cores* are predominantly filamentary, reflecting quasi-linear formation processes consistent with the Zeldovich approximation. Our estimates of the entropy confirm their intermediate dynamical state, with relaxation levels varying across the sample. Mass estimates indicate efficient accretion processes, concentrating matter into gravitationally bound systems. We conclude that *cores* are important environments where galaxy evolution and hierarchical assembly occur, bridging the gap between supercluster-scale structures and virialised clusters.

Keywords: Large-scale structure of Universe; galaxies:clusters:general; catalogues; galaxies:groups:general

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1. Introduction

The evolution of structures at various scales in the Universe is understood today through the hierarchical formation model (e.g. Peebles 1980; Padmanabhan 1993) in a Λ CDM scenario. The level of substructuring that a galaxy system (cluster or group) or larger structure could exhibit is a consequence of this hierarchical formation process. In fact, the presence and dynamical significance of substructures (i.e. their prominence and measurable impact on the overall properties) in a particular structure can be considered as an observational estimator of its evolutionary state (e.g. Araya-Melo et al. 2009). More dynamically evolved systems, such as virialised rich clusters, exhibit a more regular – spatial and velocity – distribution of galaxies with negligible or no substructures (e.g. Caretta et al. 2023; Zúñiga et al. 2024b, and references therein).

On the other hand, superclusters are generally not considered dynamically relaxed and globally gravitationally bound structures (e.g. Oort 1983; Pearson et al. 2014; Sankhyayan et al. 2023), presenting clear signs of substructuring at various scales ranging from isolated systems to large filaments and dense ‘nodes’ in their inter-sections (e.g. Einasto et al. 2007b; Santiago-Bautista et al. 2020). Superclusters are considered the youngest structures being formed under gravitational influence and still retain the memory of their formation history (e.g. Chon, Böhringer, & Nowak 2013; Einasto et al. 2019), making them important laboratories for testing cosmological models of structure formation and evolution across different scales (e.g. Einasto et al. 2021).

The study of the internal structure of rich superclusters has revealed the presence of ‘central regions’ within most of them (Einasto et al. 2007c, 2008). These regions, known as *cores*, are characterized by high density contrasts in both galaxy number and mass. In our previous work (Zúñiga, Caretta, & Andernach 2024a, hereafter Paper I), a catalogue of such *cores* was compiled for a sample of rich superclusters. Specifically, they were identified as gravitationally bound structures, comprised of two or more clusters and groups, representing significant cosmological overdensities high enough to suggest they will virialise in the future. A supercluster can host more than one *core* depending on its mass; generally, more massive superclusters tend to host a greater number of *cores*. It is more likely to find multiple *cores* within superclusters with masses $\geq 10^{15} h_{70}^{-1} M_{\odot}$, indicating a more complex and densely populated internal structure in these structures.

In Paper I, *cores* are regarded as nucleation regions because they are zones within superclusters where matter accumulates and condenses, forming denser and more compact structures. In these zones, rich clusters act as ‘seeds’ around which additional matter, whether from galaxies, galaxy systems, or filaments connecting different parts of the supercluster, gathers and aggregates. Essentially, *cores* are focal points of dynamic activity and structure formation within superclusters, where processes of accretion and merging of matter are predominant (e.g. Marini et al. 2004). This makes them important regions for the growth and development of large-scale structures. Moreover, the high density and dynamical interactions within *cores* provide unique environments for studying the interplay between dark and baryonic matter, and the influence of cosmic web filaments, thereby offering valuable insights into the mechanisms driving cosmic structure formation and evolution.

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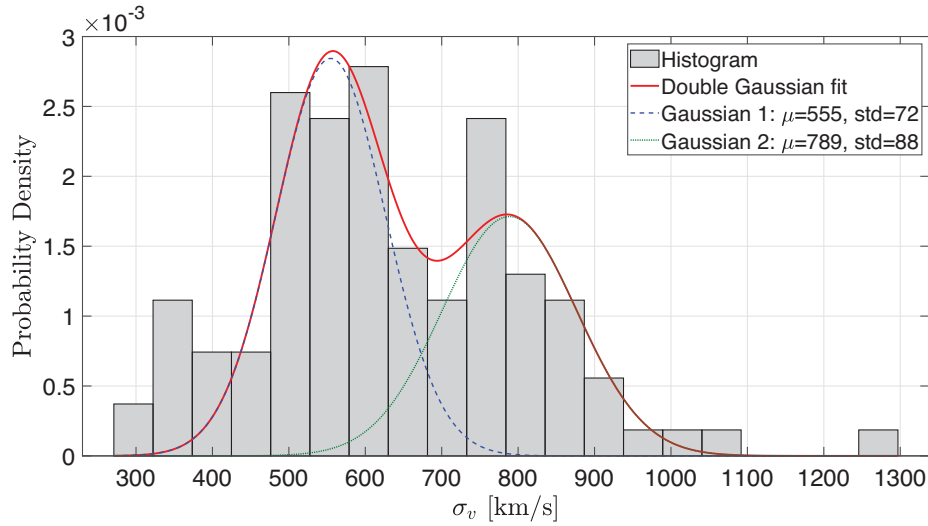


Figure 1. Distribution of line-of-sight galaxy velocity dispersions in DCC cores, along with the double Gaussian fit (red solid line) and its components: Gaussian 1 (blue dashed line) and Gaussian 2 (green dotted line).

In this work, we focus on studying the dynamical state of *cores* through various approaches, including the analysis of the spatial and velocity distributions of their member galaxies, their morphologies, their entropies, and the estimation of their virial parameters. Our goal is to understand the evolutionary state of these structures, apart from finding out whether they are gravitationally bound. Under the hierarchical formation model, we hypothesise that *cores* might be in an intermediate relaxation state between virialised clusters and the superclusters in which they reside.

Throughout this paper a flat Λ CDM cosmology is used with the following parameters: Hubble constant $H_0 = 70 h_{70} \text{ km s}^{-1} \text{ Mpc}^{-1}$ with matter density $\Omega_m = 0.3$, curvature density $\Omega_k = 0$, and dark energy density $\Omega_\Lambda = 0.7$.

2. Data

2.1. Our sample of cores

For this study, we used the full *core* sample from the *Density-based Core Catalogue* (DCC, Paper I), a catalogue containing a total of 105 *cores* in 53 nearby rich superclusters, with redshifts between 0.02 and 0.15. The *cores* were selected from candidate structures that were initially identified using improved percolation techniques (based on the DBSCAN and FoF algorithms, e.g. Ester et al. 1996; Berlind et al. 2006) applied to samples of galaxy systems present in the regions of rich superclusters of the *Main SuperCluster Catalogue* (MSCC, Chow-Martínez et al. 2014) based on Abell/ACO clusters (Abell 1958; Abell, Corwin, & Olowin 1989). The selection of *cores* was based on physically motivated density criteria proposed in the literature (see, for example, Dünner et al. 2006; Chon, Böhringer, & Zaroubi 2015), defining them as structures with a high probability of future collapse and virialisation despite cosmic expansion.

In particular, in Paper I we defined *cores* as galaxy structures with masses $\mathcal{M} \geq 5 \times 10^{14} h_{70}^{-1} \text{ M}_\odot$, $\mathcal{R} \geq 7.86$, and $\Delta_{\text{cr}} \geq 1.36$, where

$$\mathcal{R} \equiv \frac{\rho_{\text{ov}}}{\rho_{\text{b}}}, \quad (1)$$

is the density ratio between the mean mass density of an overdense region, ρ_{ov} , and the mean local background density, ρ_{b} , and

$$\Delta_{\text{cr}} \equiv \frac{\rho_{\text{ov}}}{\rho_{\text{cr}}} - 1, \quad (2)$$

is the density contrast, with $\rho_{\text{cr}} = 3H^2(z)/8\pi G$ being the critical density of the Universe at redshift z . These parameters are used to assess whether a given structure is likely to remain gravitationally bound and eventually virialise in the future.

The DCC catalogue contains *cores* of some well-known superclusters of the Local Universe (such as *Corona Borealis*, *Shapley*, *Ursa Major*, *Coma-Leo*, *Hercules*, *Böotes*, among others), which have already been identified and studied in previous works (e.g. Kopylova & Kopylov 1998; Bardelli et al. 1994; Einasto et al. 2008). Additionally, new *cores* within other superclusters were identified, generating a systematically constructed and statistically significant sample for studying these structures.

For each MSCC supercluster in our sample, we selected galaxy samples with spectroscopic redshift from the Sloan Digital Sky Survey (SDSS DR13, Albareti et al. 2017), the 2dF Galaxy Redshift Survey (2dFGRS, Colless et al. 2001), or the 6dF Galaxy Survey (6dFGS, Jones et al. 2009), depending on the region of the sky where it was located. Galaxy systems were identified from these samples using iterative system identification algorithms based on the works of Biviano et al. (2006) and Santiago-Bautista et al. (2020). Our full sample of systems consisted of a total of 3 337 groups and clusters, including about 527 Abell/ACO clusters, as well as many others that match systems from other published catalogues of galaxy systems.

Beyond the homogeneity in the identification and analysis algorithms (which also include adjustments to local densities), we also limited our samples of superclusters to the ones completely inside the area and restricted to the redshift limits of each survey, as captured in Figure 1 of paper I. SDSS-DR13 contains the Sloan Legacy Survey (DR7), which is complete and deep enough for our aims, despite receiving photometric and spectroscopic improvements through the subsequent DRs. SDSS-DR13 and 2dFGRS have similar depth, which guarantee the coverage to $z_{\text{lim}} \sim 0.15$,

while 6dFGS is shallower ($z_{\text{lim}} \sim 0.08$) and was used only up to this redshift limit.

2.2. Core galaxy sample

The member galaxies of each DCC *core* were defined as all galaxies up to a distance of $3.5R_{\text{vir}}^i$ from the centroid of each i -th member system in the supercluster galaxy sample in 3D rectangular coordinates and corrected for the corresponding Finger-of-God effect (FoG, e.g. Coil 2012). Following Santiago-Bautista et al. (2020), we refer to this as the ‘supercluster box’. The transformations from equatorial to rectangular coordinates were performed in the form

$$\begin{aligned} X &= D_c \cos \delta \cos \alpha, \\ Y &= D_c \cos \delta \sin \alpha, \\ Z &= D_c \sin \delta, \end{aligned} \quad (3)$$

where $\alpha = \text{RA}$, $\delta = \text{Dec}$ and

$$D_c(z) = \frac{c}{H_0} \int_0^z \frac{dz'}{E(z')}, \quad (4)$$

is the line-of-sight comoving distance (e.g. Hogg 2000) of the object defined by its redshift z , c is the speed of light, and

$$E(z) \equiv \sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}. \quad (5)$$

Within a distance of $3.5R_{\text{vir}}^i$ from the centre of each member system, we expect to include galaxies up to their turn-around zone (a region that encompasses galaxies on the zero-velocity surface of each system, as well as those that are decelerating while approaching this surface, and those that have begun moving towards collapse towards the system’s centre), along with galaxies in bridges between them and galaxies in the dispersed component of the *cores*.

3. Galaxy distributions in cores

A first analysis of the dynamical state of the *cores* was carried out for their selection process (see Paper I), making it clear that they are mostly gravitationally bound structures. Furthermore, since the *cores* represent cosmologically significant overdensities, they could already be in the process of, or close to, gravitational collapse, to then reach virial equilibrium in the future. However, although the marginal state of equilibrium of the *cores* is well known – or at least theoretically assumed – due to their self-similarity to any galaxy system, little or nothing is known about their current dynamical states. We do not know how far or close they are from such an equilibrium.

The study of the spatial and velocity distributions of the member galaxies of a system helps to give us an idea of its evolutionary state. In fact, observations reveal that, as the galaxy system approaches dynamical equilibrium, such distributions tend to statistically known shapes (see structure evolution in simulations, e.g. Araya-Melo et al. 2009). For example, galaxies in regular clusters tend to have a normal (or Gaussian) line-of-sight velocity distribution, as well as a projected (spatial) distribution that is very well fit by a King profile (e.g. Adami et al. 1998; Sampaio & Ribeiro 2014; Zúñiga et al. 2024b). Since the dynamics of the galaxy structures that have detached from the Hubble flow is only dominated by gravity, they evolve in a self-similar way to smaller galaxy systems tending towards virialisation. Thus, the spatial and velocity distributions of member galaxies of *cores* are expected to change

throughout the evolutionary processes they undergo to reach the dynamical relaxation like in very rich clusters.

3.1. Velocity distribution of galaxies

The line-of-sight velocity distributions of galaxies in the *cores* were studied from the spectroscopic redshifts $z_i \approx V_{r_i}/c$ of galaxies, where V_{r_i} is the radial velocity of the i -th galaxy and c the speed of light. For each DCC *core*, the set of redshift values of its N_g member galaxies was examined to explore whether it follows an underlying normal distribution. A preliminary analysis of skewness and kurtosis provided an initial qualitative assessment of the redshift distributions, revealing a variety of patterns: some closely resembling normal distributions, while others exhibited features such as flattened shapes, double peaks, or heavy tails. However, this initial exploration was intended only as a guide to identify possible deviations and to gain some insight into the underlying dynamics of the galaxies within the *cores*, as the subsequent analysis focused on quantifying these deviations using more robust statistical tests.

To formally assess the extent of these deviations, we applied a suite of statistical tests, including the Anderson–Darling (AD), Jarque–Bera (JB), and Lilliefors (L) tests. These tests evaluate whether the null hypothesis – that a sample of redshift values follows a normal distribution – can be rejected in favor of an alternative hypothesis suggesting significant deviation from normality. Each test returned a binary result, where $h_{\text{test}} = 1$ indicated rejection of the null hypothesis (when p -values < 0.05), and $h_{\text{test}} = 0$ otherwise. The results of these tests, presented in Columns 2 and 3 of Table 1, provide a more rigorous and quantitative evaluation of the observed patterns, allowing us to distinguish between genuine deviations and random fluctuations from normality.

By combining the results from the three statistical tests, we found that about 30% of the studied *cores* exhibit line-of-sight velocity distributions consistent with an underlying normal distribution within a significance level of 0.05, while the remaining 58% deviate from this behavior. Although the results for another 12% suggest that in some *cores* the radial velocity distributions are not far from normal, this alone is insufficient to conclude that they are dominantly relaxed structures. While a more advanced evolutionary state compared to their host superclusters is probable, the *cores* still remain highly substructured. Since the radial velocity distributions of galaxies in *cores* are mostly non-Gaussian, we infer that these galaxies have not yet reached a Maxwellian^a 3D velocity distribution, which indicates that the *cores* are not dynamically relaxed, as expected. This lack of ‘relaxation’ in the velocity distributions is a key indicator that *cores* are still in the process of forming and evolving.

3.1.1. Bimodality in the distribution of velocity dispersions of core galaxies

Since the line-of-sight velocity distributions of member galaxies deviate from normality in most *cores*, their bulk velocities and velocity dispersions must be calculated using robust statistical estimators. In particular, here the bulk velocities and velocity dispersions in the line of sight (LOS) were estimated using Tukey’s

^aThis arises because each component of the total velocity is treated as a statistically independent random variable, and projecting a Maxwellian distribution (typical of systems in equilibrium) onto a single axis results in a Gaussian distribution (e.g. Binney & Tremaine 2008).

Table 1. Summary of the spatial, velocity, and dynamical properties of DCC cores.

ID	h_{test}			p -values			$\bar{V}_{\text{LOS}}$	σ_v	Σ_0	r_c	γ	$\mathcal{R}_{\text{det}}^2$	R_h	$\mathcal{M}_{\text{vir}}^c$	H_z	$\mathcal{P}_{\text{relax}}$
DCC	AD	JB	L	AD	JB	L	(km s ⁻¹)	(km s ⁻¹)	(gal/h ₇₀ ⁻² Mpc ²)	(h ₇₀ ⁻¹ Mpc)			(h ₇₀ ⁻¹ Mpc)	(10 ¹⁴ h ₇₀ ⁻¹ M _⊙)	(nats)	
(1)	(2)			(3)			(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
001	1	1	0	0.0473	0.0127	0.1056	19 373	831	7.12	2.25	1.42	0.997	6.48	40.92	12.98	0.78
002	1	1	1	0.0005	0.0010	0.0010	18 319	697	17.08	2.83	3.29	0.995	4.15	18.40	13.09	0.78
003	0	0	0	0.3164	0.3793	0.1091	17 931	450	17.57	1.02	1.31	0.984	5.71	10.58	12.17	0.55
004	0	0	0	0.6894	0.5000	0.5000	18 957	372	11.49	2.92	3.33	0.898	3.97	5.01	11.30	0.63
005	1	0	0	0.0214	0.2638	0.0523	31 954	522	9.09	2.63	2.18	0.996	4.20	10.44	12.17	0.81
006	0	0	0	0.9120	0.5000	0.5000	32 576	411	–	–	–	–	4.08	6.29	11.68	0.72
007	1	1	1	0.0012	0.0050	0.0039	33 734	514	–	–	–	–	4.67	11.28	12.60	0.58
008	1	1	1	0.0041	0.0010	0.0227	33 719	370	–	–	–	–	4.01	5.03	11.79	0.65
009	1	1	1	0.0040	0.0010	0.0103	32 360	733	17.57	3.58	6.51	0.972	3.52	17.28	13.40	0.80
010	1	1	1	0.0006	0.0119	0.0013	16 532	903	6.30	1.98	2.29	1.000	2.83	21.11	14.54	0.71
011	1	1	1	0.0005	0.0011	0.0091	16 297	503	8.42	3.62	12.64	0.997	4.77	11.03	12.27	0.48
012	1	1	1	0.0005	0.0052	0.0010	17 809	651	8.61	2.67	2.35	0.953	4.39	16.99	13.05	0.70
013	0	1	0	0.1319	0.0123	0.2099	23 325	271	5.06	1.71	1.06	0.907	4.66	3.12	10.67	0.63
014	1	0	1	0.0156	0.3023	0.0071	23 979	679	29.66	1.54	4.85	0.743	4.76	20.06	12.59	0.49
015	1	0	1	0.0266	0.4693	0.0281	17 987	781	17.24	1.34	2.58	0.960	4.71	26.25	13.05	0.55
016	1	1	1	0.0005	0.0045	0.0010	29 417	746	8.90	4.93	8.47	0.974	3.30	16.79	13.83	0.71
017	1	1	1	0.0038	0.0117	0.0491	31 285	532	9.19	1.21	1.59	0.988	3.90	10.08	13.09	0.52
018	1	0	1	0.0005	0.0512	0.0010	33 739	995	3.50	3.87	2.06	0.803	6.54	59.14	14.10	0.59
019	1	0	0	0.0404	0.4465	0.1962	42 436	1297	9.16	2.20	4.35	0.986	3.44	52.95	14.31	0.72
020	1	0	0	0.0263	0.2754	0.1027	41 067	661	5.68	1.14	1.36	0.994	3.34	13.35	13.98	0.61
021	1	1	1	0.0045	0.0031	0.0190	8836	499	–	–	–	–	3.03	6.89	11.85	0.65
022	1	1	1	0.0005	0.0010	0.0030	9731	276	–	–	–	–	4.52	3.15	10.56	0.53
023	1	1	1	0.0005	0.0010	0.0010	10 547	491	28.01	1.68	3.85	0.986	2.89	6.37	12.48	0.62
024	1	0	1	0.0037	0.0787	0.0060	9901	843	3.03	1.81	0.72	0.961	4.31	28.01	13.08	0.77
025	0	0	0	0.9007	0.5000	0.5000	27 836	696	–	–	–	–	3.22	14.24	13.83	0.66
026	1	1	1	0.0005	0.0336	0.0021	31 806	1058	4.54	3.45	3.20	0.991	5.25	53.63	14.72	0.58
027	1	0	0	0.0270	0.4737	0.1160	36 911	608	11.57	1.45	2.53	0.992	5.53	18.67	12.24	0.63
028	0	0	0	0.2775	0.3001	0.2421	34 346	571	3.49	1.03	0.82	0.780	3.52	10.49	12.55	0.68
029	0	0	0	0.0783	0.5000	0.0915	37 806	761	–	–	–	–	6.83	36.14	13.77	0.51
030	1	1	1	0.0006	0.0227	0.0075	22 611	599	19.40	1.04	0.87	0.973	5.04	16.53	12.79	0.67
031	1	1	1	0.0005	0.0198	0.0010	31 672	744	11.13	2.03	2.97	0.982	5.14	26.00	13.07	0.55
032	0	0	0	0.1792	0.3308	0.2153	33 552	540	4.43	1.71	1.10	0.928	5.32	14.17	12.44	0.54
033	0	0	0	0.5374	0.4145	0.5000	33 168	815	3.91	2.64	2.22	0.964	5.41	32.83	13.39	0.60
034	0	0	0	0.1745	0.5000	0.5000	10 065	726	42.52	1.10	1.36	1.000	2.88	13.85	13.28	0.80
035	0	0	0	0.4181	0.4860	0.5000	10 045	790	6.75	2.64	1.45	0.976	5.28	30.09	13.36	0.75
036	1	1	1	0.0005	0.0010	0.0010	7 226	433	–	–	–	–	3.94	6.74	11.87	0.54
037	0	0	0	0.0621	0.2297	0.0564	38 657	605	4.40	2.38	2.33	0.998	3.95	13.23	13.69	0.72
038	0	0	0	0.1876	0.2511	0.2175	39 895	782	–	–	–	–	4.62	25.82	13.36	0.61
039	1	0	1	0.0030	0.0664	0.0124	41 131	738	5.72	6.63	15.12	0.887	4.23	21.06	13.75	0.72
040	1	1	0	0.0121	0.0343	0.0873	6 790	647	–	–	–	–	4.19	16.03	12.51	0.61
041	1	1	1	0.0005	0.0104	0.0010	6 999	779	–	–	–	–	3.81	21.14	13.18	0.74
042	0	0	1	0.1071	0.2230	0.0337	13 871	465	–	–	–	–	3.55	7.01	12.12	0.63
043	1	0	1	0.0025	0.0806	0.0304	20 837	546	23.77	1.63	3.38	0.948	4.81	13.11	12.20	0.43
044	1	1	1	0.0005	0.0156	0.0010	18 032	722	–	–	–	–	4.02	19.14	13.38	0.64

Table 1. (Continued)

ID	h_{test}			p -values			$\bar{V}_{\text{LOS}}$	σ_v	Σ_0	r_c	γ	$\mathcal{R}_{\text{det}}^2$	R_h	$\mathcal{M}_{\text{vir}}^c$	H_Z	$\mathcal{P}_{\text{relax}}$
DCC	AD	JB	L	AD	JB	L	(km s ⁻¹)	(km s ⁻¹)	(gal/ h_{70}^{-2} Mpc ²)	(h_{70}^{-1} Mpc)			(h_{70}^{-1} Mpc)	(10 ¹⁴ h_{70}^{-1} $\mathcal{M}_{\odot}$)	(nats)	
(1)	(2)			(3)			(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
045	1	1	1	0.0005	0.0011	0.0010	15 343	534	–	–	–	–	4.10	10.68	12.00	0.50
046	0	1	0	0.0500	0.0418	0.0645	19 284	590	29.06	1.48	2.87	0.994	4.28	13.62	12.31	0.64
047	1	1	1	0.0005	0.0089	0.0044	24 294	556	–	–	–	–	6.95	19.61	11.97	0.59
048	0	0	0	0.4211	0.2434	0.5000	24 363	531	6.46	2.80	2.50	0.983	4.08	10.50	12.71	0.69
049	1	1	1	0.0010	0.0010	0.0095	24 945	582	7.18	1.37	0.95	0.952	6.68	20.64	12.62	0.57
050	1	1	1	0.0030	0.0211	0.0116	23 512	613	6.22	1.09	1.30	0.967	4.08	14.03	13.03	0.58
051	0	0	0	0.4699	0.5000	0.4103	36 244	878	5.77	1.36	2.09	0.979	4.07	28.68	14.41	0.56
052	0	0	0	0.0944	0.0847	0.3112	35 356	554	1.80	7.05	11.07	0.703	3.64	10.22	13.67	0.61
053	0	0	0	0.0558	0.1232	0.1039	42 020	792	4.28	4.99	11.22	0.760	6.72	38.52	13.43	0.50
054	0	0	0	0.0650	0.0578	0.2669	41 441	361	1.45	3.82	2.42	0.808	4.24	5.04	11.67	0.62
055	1	0	1	0.0053	0.1314	0.0059	40 899	925	5.77	4.32	10.61	0.958	4.51	35.25	14.46	0.59
056	1	0	1	0.0020	0.1695	0.0114	23 867	378	–	–	–	–	4.42	5.77	10.95	0.42
057	0	1	0	0.0523	0.0011	0.1601	24 120	334	2.08	1.53	0.96	0.984	4.07	4.15	11.33	0.45
058	0	0	0	0.0708	0.5000	0.0816	22 296	503	–	–	–	–	3.35	7.76	11.86	0.80
059	1	0	0	0.0444	0.0550	0.0850	25 372	575	3.50	2.33	1.85	0.627	3.66	11.06	13.07	0.61
060	0	0	1	0.1295	0.5000	0.0289	24 165	625	12.51	1.45	2.13	0.999	2.51	8.98	13.23	0.78
061	0	0	0	0.1246	0.3838	0.1767	24 555	389	–	–	–	–	3.22	4.44	11.52	0.75
062	1	1	1	0.0005	0.0026	0.0010	25 681	514	13.19	4.02	7.56	0.996	3.71	8.94	12.33	0.81
063	0	0	0	0.5827	0.5000	0.5000	31 785	600	–	–	–	–	3.97	13.05	13.39	0.39
064	1	1	1	0.0005	0.0419	0.0010	21 454	772	–	–	–	–	5.82	31.65	13.77	0.63
065	1	0	0	0.0146	0.0807	0.1328	17 861	428	7.36	2.03	1.95	0.721	4.68	7.82	11.78	0.71
066	0	1	0	0.0852	0.0161	0.0990	16 376	637	–	–	–	–	4.82	17.89	12.82	0.67
067	1	1	1	0.0005	0.0010	0.0010	14 376	943	16.26	1.68	1.09	0.998	5.94	48.30	13.99	0.75
068	1	1	1	0.0005	0.0010	0.0010	11 614	607	14.89	1.16	1.20	0.999	3.97	13.40	12.85	0.83
069	1	1	0	0.0051	0.0024	0.0901	22 521	509	–	–	–	–	5.26	12.47	12.63	0.58
070	0	0	0	0.5617	0.3212	0.5000	22 657	734	19.24	1.11	1.44	0.991	6.87	33.79	13.10	0.52
071	1	1	1	0.0005	0.0010	0.0010	18 815	630	25.76	2.06	3.06	0.995	6.84	24.78	12.62	0.47
072	1	0	1	0.0005	0.5000	0.0010	21 155	704	7.42	1.23	1.08	0.996	4.62	20.94	13.55	0.56
073	1	1	1	0.0005	0.0030	0.0011	20 103	626	–	–	–	–	5.18	18.56	12.71	0.42
074	1	1	1	0.0005	0.0128	0.0010	33 844	624	3.03	2.29	1.16	0.838	6.37	22.69	12.87	0.62
075	1	1	1	0.0005	0.0125	0.0010	32 701	557	5.97	1.05	1.14	0.997	4.32	12.24	13.41	0.65
076	0	0	0	0.1106	0.0689	0.2569	33 928	495	1.61	4.78	2.55	0.978	6.27	14.03	12.50	0.67
077	0	0	0	0.1444	0.4597	0.3057	28 660	503	7.74	1.92	2.03	0.995	3.87	8.93	12.01	0.68
078	1	1	0	0.0360	0.0109	0.3012	26 681	558	4.03	3.29	2.32	0.996	3.24	9.23	13.41	0.62
079	1	1	1	0.0005	0.0268	0.0010	35 070	656	3.97	6.01	4.79	0.954	6.53	25.66	13.00	0.58
080	1	1	1	0.0005	0.0117	0.0018	37 600	575	–	–	–	–	5.47	16.56	12.75	0.50
081	0	1	0	0.0926	0.0171	0.4037	15 581	492	–	–	–	–	4.70	10.42	11.69	0.64
082	1	0	1	0.0160	0.5000	0.0467	13 497	532	–	–	–	–	4.22	10.93	12.38	0.59
083	1	1	1	0.0005	0.0010	0.0010	11 425	625	20.06	1.25	1.06	0.948	6.29	22.48	12.89	0.65
084	1	1	1	0.0005	0.0114	0.0010	14 940	817	7.17	2.81	1.88	0.971	5.08	31.00	13.94	0.67
085	0	0	0	0.2138	0.5000	0.2587	23 492	690	6.65	1.33	0.65	0.965	5.74	24.94	13.14	0.61
086	0	0	0	0.0758	0.1727	0.3081	23 557	866	16.96	1.94	1.19	0.996	6.20	42.48	13.82	0.75
087	1	1	1	0.0005	0.0010	0.0180	22 833	419	1.40	2.14	0.57	0.672	7.42	11.92	11.13	0.64
088	1	1	1	0.0005	0.0116	0.0010	25 412	758	7.40	1.75	1.53	0.987	7.06	37.09	13.62	0.43
089	1	0	1	0.0086	0.0785	0.0094	34 033	830	–	–	–	–	6.95	43.76	13.37	0.40

Table 1. (Continued)

ID	h_{test}			p -values			$\bar{V}_{\text{LOS}}$	σ_v	Σ_0	r_c	γ	$\mathcal{R}_{\text{det}}^2$	R_h	$\mathcal{M}_{\text{vir}}^c$	H_z	$\mathcal{P}_{\text{relax}}$
DCC	AD	JB	L	AD	JB	L	(km s ⁻¹)	(km s ⁻¹)	(gal/h ₇₀ ⁻² Mpc ²)	(h ₇₀ ⁻¹ Mpc)			(h ₇₀ ⁻¹ Mpc)	(10 ¹⁴ h ₇₀ ⁻¹ M _⊙)	(nats)	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
090	0	0	0	0.8838	0.5000	0.5000	33 889	675	2.09	3.36	1.28	0.773	5.29	22.00	12.85	0.71
091	0	0	0	0.3112	0.5000	0.3591	34 058	876	14.87	1.35	1.14	0.991	5.70	39.98	13.87	0.77
092	1	1	1	0.0005	0.0047	0.0010	23 157	887	40.54	1.04	1.63	0.968	4.83	34.73	13.68	0.75
093	1	1	1	0.0005	0.0293	0.0010	21 746	856	27.79	1.08	1.18	0.999	6.09	40.74	13.63	0.66
094	1	1	1	0.0005	0.0010	0.0010	19 764	752	–	–	–	–	5.83	30.10	13.14	0.61
095	0	0	0	0.2327	0.5000	0.1497	20 314	818	–	–	–	–	4.47	27.31	13.33	0.73
096	1	1	0	0.0227	0.0335	0.0841	23 343	753	5.71	1.94	1.00	0.982	5.87	30.43	13.96	0.57
097	1	0	1	0.0005	0.0903	0.0010	10 252	479	–	–	–	–	4.31	9.02	11.94	0.56
098	1	1	1	0.0005	0.0015	0.0010	12 132	650	11.16	1.68	1.55	0.999	4.89	18.87	12.78	0.63
099	1	1	1	0.0005	0.0010	0.0010	10 960	873	44.68	2.56	1.78	0.907	5.18	36.08	13.80	0.72
100	0	0	0	0.5517	0.2120	0.5000	40 763	499	–	–	–	–	3.61	8.22	12.33	0.72
101	1	0	1	0.0005	0.1154	0.0010	5 908	349	–	–	–	–	2.26	2.51	11.87	0.63
102	1	1	1	0.0198	0.0237	0.0110	5 677	334	–	–	–	–	3.30	3.36	11.01	0.42
103	0	0	0	0.5577	0.4330	0.5000	33 398	583	7.87	3.15	7.57	0.990	3.69	11.47	13.76	0.65
104	0	0	0	0.1417	0.1488	0.2704	31 790	491	–	–	–	–	5.27	11.59	12.48	0.51
105	1	1	1	0.0005	0.0022	0.0010	12 602	594	29.06	1.52	1.64	0.999	2.78	8.97	13.04	0.81

biweight method (e.g. Beers, Flynn, & Gebhardt 1990). Thus, we take $\bar{V}_{\text{LOS}} = C_{\text{BI}}$ and $\sigma_v = S_{\text{BI}}$, where C_{BI} and S_{BI} are, respectively, the robust estimators for the centre and the scale of the velocity distributions of *core* galaxies. The $\bar{V}_{\text{LOS}}$ and σ_v values obtained for the DCC *cores* are presented respectively in columns 4 and 5 of Table 1. The distribution of velocity dispersions in the DCC *cores* can be seen in Figure 1. The mean and median values of this distribution are 633 and 613 km s⁻¹, respectively. These galaxy velocity dispersion values are lower than those of typical rich galaxy clusters (~ 750 – 1000 km s⁻¹, e.g. Bahcall 1996; Schneider 2015).

In virialised systems, the galaxy velocity dispersion σ_v directly depends on the virial mass $\mathcal{M}_{\text{vir}}$: more massive systems generate stronger gravitational potentials, leading to higher orbital velocities and, consequently, larger velocity dispersions for the member galaxies. According to the virial theorem, $U = -2K$, a greater mass implies a deeper gravitational potential well (U), which in turns results in a higher internal kinetic energy (K), reinforcing this idea (e.g. Zúñiga et al. 2024b).

In non-virialised structures, such as superclusters and their *cores*, the galaxy velocity dispersion σ_v serves primarily as a mere statistical measure of the spread in velocities among member galaxies, rather than as a direct indicator of mass. Unlike virialised systems, where σ_v is proportional to the total mass, the velocity dispersions in *cores* are not completely tied to their mass. This is because non-virialised structures are out of equilibrium, with their dynamics dominated by ongoing processes such as gravitational collapse or cosmic expansion. As a result, the velocity dispersion can be significantly influenced by these large-scale motions, making it a less reliable tracer of mass in these cases. For instance, the velocity dispersion of an expanding or collapsing structure may primarily reflect the impact of bulk flows associated with individual substructures, rather than representing a single cohesive internal dynamics (e.g. Sargent & Turner 1977).

Note that the σ_v -distribution in DCC *cores* exhibits a clear bimodal pattern, as seen from the double Gaussian fit (red solid line) shown in Figure 1, with peaks centred around 555 km s⁻¹ (Gaussian 1 with blue dashed line) and 789 km s⁻¹ (Gaussian 2 with green dotted line). This bimodality remains robust even when varying the binning parameters, suggesting that it is intrinsic to the distribution rather than an artifact of the analysis. To understand the origin of this feature, we consider the role of the most massive cluster (MMC) within each *core*. By definition, each *core* contains one system identified as the MMC. However, the degree to which the MMC dominates the *core*'s dynamics varies significantly.

A separate analysis of the *cores* associated with each Gaussian component confirms this distinction: *cores* associated with the Gaussian 1 (component for low velocity dispersions) typically contain a modest-mass MMC that, while formally the most massive system by the value of its virial mass, is not overwhelmingly dominant. In these cases, the other gravitationally bound systems within each *core* have comparable masses and contribute similarly to the overall internal dynamics of their host *cores*. As a result, the velocity dispersion of *cores* remains relatively low, reflecting a more distributed influence among multiple systems. Conversely, *cores* falling within the Gaussian 2 (component for high velocity dispersions) tend to host an MMC with a significantly larger mass than the other *core* member systems. Here, the velocity dispersion of galaxies within the *core* is strongly influenced by the MMC, resulting in values characteristic of cluster-like systems, where the virialised portion dominates. Furthermore, since *cores* are collapsing structures, their velocity dispersions can be largely influenced by the bulk motions of their member systems (e.g. Sargent & Turner 1977).

On the other hand, we also observe that a few *cores* contain two or even three highly massive systems with comparable masses, making it reasonable to consider that they collectively dominate the dynamics of the host *cores*. Interestingly, these cases also

tend to fall within the Gaussian 2, likely due to the high velocity dispersions of the massive member clusters. This reinforces the assumption that the bimodal velocity dispersion distribution of the DCC *cores* is primarily driven by the varying degrees of dynamical influence exerted by the most massive systems within each *core*.

3.2. Projected distribution of galaxies

To study the spatial distribution of galaxies, we analyzed their projected positions in the plane of the sky to assess whether they exhibit a statistical tendency to follow the projected King profile:

$$\Sigma(r) = \Sigma_0 \left[1 + \left(\frac{r}{r_c} \right)^2 \right]^{-\gamma}, \quad (6)$$

where the parameters r_c , Σ_0 , and γ are determined by fitting the model (6) to the projected distribution of member galaxies in each *core*.

The King profile was selected because, despite being a mass density profile, its projected form provides a good fit to the radial distribution of galaxies in regular clusters (e.g. Rood et al. 1972; Adami et al. 1998). These clusters typically feature a dense core of galaxies surrounded by a sparse halo, where the galaxy number density decreases with distance. In contrast, other profiles, such as the Einasto and NFW profiles (e.g. Einasto 1965; Navarro, Frenk, & White 1996), are better suited to describe the distribution of dark matter within clusters.

Since the MMCs represent the most significant gravitational potential wells in their respective host *cores*, we assume these clusters to be the main physical centres of gravitational attraction within the *cores*. Although in some *cores* (particularly those in Gaussian 1) a clearly dominant MMC is not present, we adopt the coordinates of the most massive cluster as a reference ‘centre’^b for consistency across the sample. MMCs, even when not overwhelmingly dominant in mass, are generally rich systems located in high-density regions, suggesting a strong gravitational influence and a likely site of future collapse. Moreover, they often coincide with local peaks in the spatial distribution of galaxies.

Thus, taking the sky coordinates of each *core*’s MMC as its centre, we calculated the projected *core*-centric distances (in units of h_{70}^{-1} Mpc) of its member galaxies as follows:

$$R_i \simeq \frac{\pi}{180} \frac{D_c(\bar{z})}{(1 + \bar{z})} \left[(\alpha_{\text{MMC}} - \alpha_i)^2 \cos^2 \bar{\delta} + (\delta_{\text{MMC}} - \delta_i)^2 \right]^{1/2}, \quad (7)$$

where α_{MMC} and δ_{MMC} are the RA and Dec coordinates of the MMC (see columns 6 and 7 of Table 4 in Paper I), α_i and δ_i are the coordinates of each member galaxy, and $\bar{z}$ and $\bar{\delta}$ represent the mean redshift and declination of the *core* galaxies, respectively. As before, $D_c(\bar{z})$ is the comoving distance at redshift $\bar{z}$, corresponding to the mean radial distance of each *core*.

We used a binning method to analyze the projected density distribution of galaxies as a function of the *core*-centric distance in the RA-Dec plane. This process involved counting only member galaxies in concentric circular annuli in 2D, centred on the MMC of the given *core*. Each ring was taken to have an area

^bIt is important to mention that studies of the spatial distribution of galaxies in a structure (or system), as well as estimates of certain dynamical parameters, are sensitive to the choice of a ‘centre’. While other definitions such as the centre of mass may provide valuable complementary insights, we defer such analysis to future work.

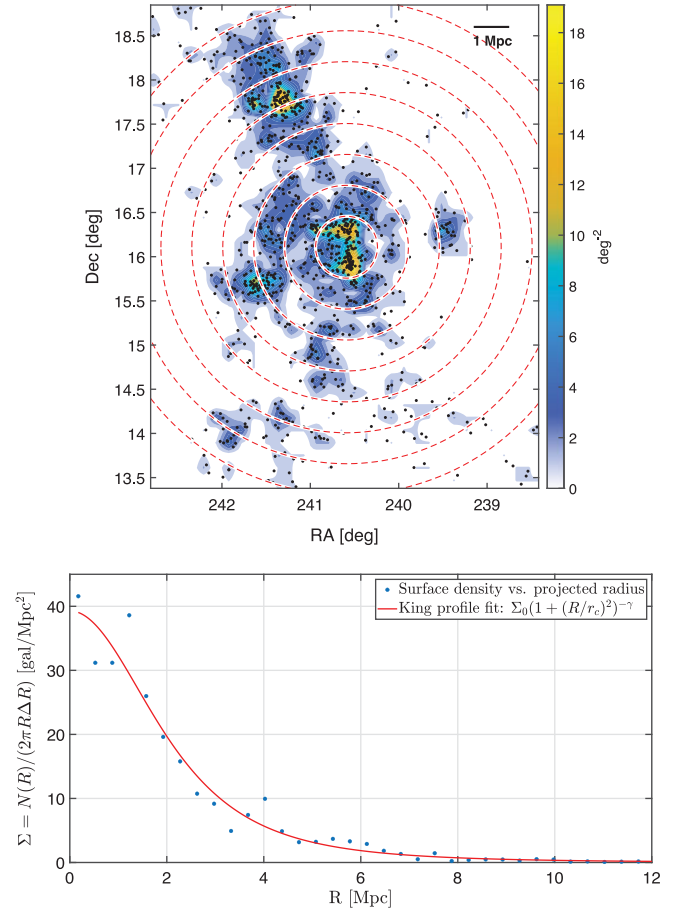


Figure 2. Top: 2D-density map of the projected distribution of galaxies in the DCC 099 core of the Hercules Supercluster (MSCC 474). Each black dot represents a member galaxy of the *core*. The space between the red dashed circles give a schematic representation of the rings (bins in 2D) on which galaxies were counted ($\Delta R = 0.35 h_{70}^{-1}$ Mpc). The width of the annuli plotted was chosen as $1 h_{70}^{-1}$ Mpc to avoid saturation of circles in the graph. Bottom: Σ vs. R plot for the DCC 099 *core*. The blue dots represent the surface number density of galaxies – in the ring – at distance R from the centre (the MMC) of the *core*. The solid red line is the projected King profile fitted to the data using the NLS method. The fit parameters for this case were $\Sigma_0 = 44.68 \text{ gal}/h_{70}^{-2} \text{ Mpc}^2$, $r_c = 2.56 h_{70}^{-1} \text{ Mpc}$, and $\gamma = 1.78$, with a goodness of fit $R_{\text{det}}^2 = 0.91$.

$A_r = 2\pi R \Delta R$, where R is the mean radius of the ring and ΔR is its width. The width ΔR was kept constant for all radii, and all galaxies with *core*-centric distances R_i between $R - \Delta R/2$ and $R + \Delta R/2$ were counted within the corresponding ring. The surface density of galaxies at a given radius R was then calculated as $\Sigma(R) = N_r/A_r$, where N_r is the number of galaxies in that ring.

For all *cores*, we adopted a fixed ring width of $\Delta R = 0.35 h_{70}^{-1}$ Mpc (a value that maximized the goodness-of-fit in most *cores*), while allowing the radius R to vary without overlap between bins. Using the Nonlinear Least Squares (NLS) method, we fitted the projected King profile (6) to the resulting set of (R, Σ) pairs, determining the best-fit parameters (Σ_0, r_c, γ) . Figure 2 provides an example of this procedure applied to the DCC 099 *core* in the Hercules Supercluster (MSCC 474). A similar analysis was performed for all other *cores*.

Columns 6 to 9 of Table 1 present the set of parameters (Σ_0, r_c, γ) obtained by fitting the King profile to the projected galaxy sample of each DCC *core*, along with the respective

goodness of fit measured through the $\mathcal{R}_{\text{det}}^2$ statistic.^c The profile fitting using the NLS method was not successful for approximately 33% of the DCC *cores*, resulting in unphysical parameters that were excluded from the analysis. The mean and median values of the corresponding distributions of parameters (Σ_0 , r_c , γ) in the sample of DCC *cores*, excluding outliers, are presented in Table 2. Notably, the characteristic radius r_c of the *cores*, with a median value of $1.93 h_{70}^{-1} \text{Mpc}$, is comparable to the virial radii of their MMCs (in contrast to the median values for galaxy clusters of about $0.25 h_{70}^{-1} \text{Mpc}$, e.g. Bahcall 1996; Caretta *et al.* 2023). This may suggest that, although DCC *cores* are not yet regular galaxy structures, they have the potential to evolve into ‘core-halo supersystems’, with their central nuclei likely located close to their MMCs. It can be expected that these systems will serve as the formation regions for the future nuclei of marginally virialised (relaxed and regular) *cores*.

Approximately 55% of the DCC *cores* could be fitted by the King profile achieving a goodness of fit $\mathcal{R}_{\text{det}}^2 > 0.9$, while only $\sim 8\%$ of them were fitted with a goodness of fit $\mathcal{R}_{\text{det}}^2 < 0.8$. Thus, most of the *cores* in the sample exhibit a projected distribution of galaxies that can be adequately described by a King profile, suggesting an evolutionary tendency towards dynamical states statistically consistent with this density model. However, the observed density distributions still display ‘humps’ as a function of radius, indicating the presence of substructures within the *cores*. These humps can affect the values of the fit parameters, which, in principle, assuming that the distribution of galaxies follows the distribution of total matter (dark and baryonic) and *vice versa* in these structures, should be related to the equilibrium state of the *cores*. For instance, and as anticipated, the γ values obtained for the distributions of *core* galaxies remain far from $\gamma = 1$, which is the expected terminal value for a projected galaxy distribution in dynamical equilibrium (e.g. Sarazin 1986; Adami *et al.* 1998).

4. Morphology analysis of cores

Studying the morphology of galaxy structures also provides valuable insights into their dynamical states or evolutionary phases. The shape of a structure is primarily influenced by the distribution of matter within it and the gravitational interactions among its member galaxies and galaxy systems. Consequently, the morphology of these structures can reveal information about the spatial distribution of galaxies and dark matter within them, as well as their interactions with one another. Additionally, the shape may be related to their formation and evolutionary history, and it can be influenced by external factors, such as interactions with other large-scale structures in the Universe.

Since *cores* are not virialised structures, they exhibit a wide variety of shapes. In this section, a morphometric analysis of the DCC *cores* will be conducted using statistical and geometrical methods.

4.1. Minkowski functionals and shapefinders

A comprehensive morphological study of bodies in n dimensions requires both topological and geometrical descriptors to characterise their connectivity, content, and shape (e.g. Mecke, Buchert, & Wagner 1994). Minkowski functionals (MFs) constitute a family of $n + 1$ morphological descriptors for extended bodies, grounded

Table 2. Mean (with standard deviation) and median values for the Σ_0 , r_c and γ parameters (excluding outliers^d) of the King profile fit to the projected distributions of galaxies in the sample of DCC *cores*. Median values are given as asymmetric ranges with $\Delta Q_1 = \text{Median} - Q_1$ and $\Delta Q_3 = Q_3 - \text{Median}$ as lower and upper indices, where Q_1 and Q_3 are the 25th and 75th percentiles, respectively.

Parameter	Mean $\pm$ std	Median $^{+\Delta Q_3}_{-\Delta Q_1}$
Σ_0 (gal/ $h_{70}^{-2} \text{Mpc}^2$)	10.24 ± 7.69	$7.38^{+7.50}_{-2.89}$
r_c ($h_{70}^{-1} \text{Mpc}$)	2.17 ± 1.04	$1.93^{+0.81}_{-0.57}$
γ	1.93 ± 0.99	$1.64^{+0.78}_{-0.48}$

in well-established principles of integral geometry (e.g. Wiegand, Buchert, & Ostermann 2014). The morphological properties of an n -dimensional body are determined by the $(n - 1)$ -dimensional hypersurface that encloses it (e.g. Sheth *et al.* 2003). Thus, the morphology of a closed two-dimensional surface embedded in three-dimensional Euclidean space is comprehensively characterized by four MFs (e.g. Einasto *et al.* 2007a; Bag *et al.* 2019):

1. The volume enclosed by the surface: V ,
2. The surface area: S ,
3. The integrated mean curvature of the surface:

$$C = \frac{1}{2} \oint \left(\frac{1}{R_1} + \frac{1}{R_2} \right) dS, \quad (8)$$

4. The integrated Gaussian curvature (or Euler characteristic) of the surface:

$$\chi = \frac{1}{2\pi} \oint \frac{1}{R_1 R_2} dS, \quad (9)$$

where in Equations (8) and (9), R_1 and R_2 denote the two local principal radii of curvature at any point on the surface. Furthermore, the Euler characteristic can be expressed in terms of the genus, which quantifies the number of topological handles that the surface exhibits and provides a measure of the connectivity of the structure, distinguishing between isolated underdense regions (voids) and interconnected features (Sheth *et al.* 2003). Essentially, it describes the number of holes in a closed surface or three-dimensional object and can be defined as follows:

$$\mathcal{G} = 1 - \frac{\chi}{2}. \quad (10)$$

Both χ and $\mathcal{G}$ serve as measures of the surface topology (multiply connected surfaces have $\mathcal{G} > 0$, while those that are simply connected have $\mathcal{G} = 0$, e.g. Sahni, Sathyaprakash, & Shandarin 1998). Thus, while the genus provides insight into the connectivity of a surface, the other three MFs are sensitive to local surface deformations, effectively characterizing the geometry and shape of the bodies (Sheth *et al.* 2003).

To characterise the shape and determine the characteristic dimensions of a structure, we employ ‘shapefinders’ (Sahni *et al.* 1998), which are defined from the MFs as follows (e.g. Sheth *et al.* 2003; Bag *et al.* 2019):

1. Thickness:

$$\mathcal{T} = 3V/S, \quad (11)$$

^c Also known as the coefficient of determination. This statistic assesses the quality of a model in replicating results and indicates the proportion of variation in the results that can be explained by the fitted model.

^d Throughout this work, outliers were identified using the standard interquartile range (IQR) rule, that is, data points outside $[Q_1 - 1.5 \times \text{IQR}, Q_3 + 1.5 \times \text{IQR}]$ were excluded from the analysis.

2. Breadth:

$$\mathcal{B} = S/C, \quad (12)$$

3. Length:

$$\mathcal{L} = \frac{C}{4\pi(1 + |\mathcal{G}|)}, \quad (13)$$

4. Planarity:

$$\mathcal{P} = \frac{\mathcal{B} - \mathcal{T}}{\mathcal{B} + \mathcal{T}} \quad (14)$$

5. Filamentarity:

$$\mathcal{F} = \frac{\mathcal{L} - \mathcal{B}}{\mathcal{L} + \mathcal{B}}, \quad (15)$$

This set of five shapefinders includes three quantities with dimensions of length ($\mathcal{T}$, $\mathcal{B}$, and $\mathcal{L}$) and two dimensionless ratios ($\mathcal{P}$ and $\mathcal{F}$), providing a robust framework for quantifying the morphology of structures.

The shapefinders $\mathcal{T}$, $\mathcal{B}$, and $\mathcal{L}$ estimate the three principal physical extensions of a three-dimensional structure. These shapefinders are spherically normalized, ensuring that $V = (4\pi/3)\mathcal{T}\mathcal{B}\mathcal{L}$, where V is the volume enclosed by the structure. Generally, for a convex surface, the relation $\mathcal{T} \leq \mathcal{B} \leq \mathcal{L}$ holds; if not, the smallest dimension is designated as $\mathcal{T}$ and the largest as $\mathcal{L}$ to maintain order (Bag et al. 2019). In cases where $C < 0$, it is possible to redefine $C \rightarrow |C|$ to ensure that both $\mathcal{B}$ and $\mathcal{L}$ remain positive. Oblate ellipsoids (pancake-like shapes) are characterized by $\mathcal{T} < \mathcal{B} \approx \mathcal{L}$, while prolate ellipsoids (filamentary structures) are described by $\mathcal{T} \approx \mathcal{B} < \mathcal{L}$ (Einasto et al. 2007a).

On the other hand, the shapefinders $\mathcal{P}$ and $\mathcal{F}$ are dimensionless quantities that allow us to determine the shape of an object. For example, in some works (e.g. Sahni et al. 1998; Einasto et al. 2007a; Bag et al. 2019) these shapefinders have been characterized so that:

- For spheres: $\mathcal{T} = \mathcal{B} = \mathcal{L}$, that is, $\mathcal{P} = \mathcal{F} = 0$,
- For ideal filaments $\mathcal{P} \approx 0$, $\mathcal{F} \approx 1$,
- For real filaments: $\mathcal{F} \gg \mathcal{P}$,
- For ideal pancakes: $\mathcal{P} \approx 1$, $\mathcal{F} \approx 0$,
- For planar objects (sheets or pancakes): $\mathcal{P} \gg \mathcal{F}$,
- For ideal ribbons: $\mathcal{P} \approx \mathcal{F} \approx 1$,
- For ribbon-like objects: $\mathcal{P}/\mathcal{F} \approx 1$.

Note that in this context, the word ‘ideal’ refers to a theoretical or simplified representation of the structure, as commonly used in the literature.

4.2. Morphometry of cores

The MFs are typically defined for extended bodies with well-established (smooth and differentiable) boundary surfaces. However, they can also be applied to galaxy distributions by constructing an extended object from the point set of galaxy coordinates. This is achieved by defining a limiting surface that encloses the member galaxies of a structure, thus allowing its morphology to be characterized by the four MFs (V , S , C , χ) that describe the enclosing surface (e.g. Einasto et al. 2007a, 2008). Given the global and additive nature of MFs, their applicability can be extended to surfaces with singular edges and corners (Mecke et al. 1994). This versatility makes them suitable for analyzing irregular or discontinuous structures, providing valuable insights into the geometry

and topology of *cores*. In this work, we employ two methods to generate enclosing surfaces around the *core* member galaxies, adjusting the definition of the MFs (and shapefinders) in each case to match the specific characteristics of the generated surfaces.

4.2.1. Main method: Polyhedral surfaces

Polyhedral surfaces that envelop the DCC *cores* were constructed by triangulating boundary points from the three-dimensional distribution of their member galaxies. The triangulation was performed using the 3D alpha-shape algorithm (e.g. Edelsbrunner & Mücke 1994), implemented through the MATLAB boundary function. This function identifies and returns the set of boundary points of a distribution for a given compactness level, modulated by the shrink factor s_f , and subsequently allows the generation of a polyhedral envelope, ranging from the *convex hull*, for $s_f = 0$, to a tightly fitted *compact boundary*, for $s_f = 1$ (see MATLAB 2023). The left panel of Figure 3 illustrates an example of the polyhedral surface fitted to member galaxies in the main *core* (DCC 099) of the *Hercules Supercluster* (MSCC 474).

The obtained triangulated polyhedral surface defines a single well-defined boundary around the member galaxies. Morphological properties of these surfaces, including their geometric and topological characteristics, are determined by the MFs adapted for triangulated surfaces (e.g. Sheth et al. 2003; Bag et al. 2019):

- The total surface area is computed as

$$S = \sum_{k=1}^{N_T} S_k, \quad (16)$$

where S_k is the area of the k -th triangle, and N_T is the total number of triangles forming the surface.

- The volume enclosed by the surface is the sum of contributions from N_T tetrahedra:

$$V = \sum_{k=1}^{N_T} V_k, \quad V_k = \frac{1}{3} S_k (n_k \cdot P_k), \quad (17)$$

where V_k is the volume of the k -th tetrahedron with base S_k in the k -th triangle, with normal vector n_k and centroid vector position P_k , and apex in an arbitrary origin. Note that the sums of S and V are maximum for $s_f = 0$ and minimum for $s_f = 1$.

- The integrated mean curvature is calculated as

$$C = \frac{1}{2} \sum_{j,k} l_{jk} \phi_{jk} \epsilon, \quad (18)$$

where l_{jk} is the length of the common edge between adjacent triangles j and k , ϕ_{jk} is the angle between their normals, and $\epsilon = 1$ for convex and $\epsilon = -1$ for concave edges (Sheth et al. 2003).

- The Euler Characteristic and Genus are defined as

$$\chi = N_T - N_E + N_V, \quad \mathcal{G} = 1 - \chi/2, \quad (19)$$

where N_E and N_V are the number of edges and vertices, respectively.

This approach ensures a detailed and versatile morphological analysis of *cores*, enabling the characterisation of both their geometry and topology.

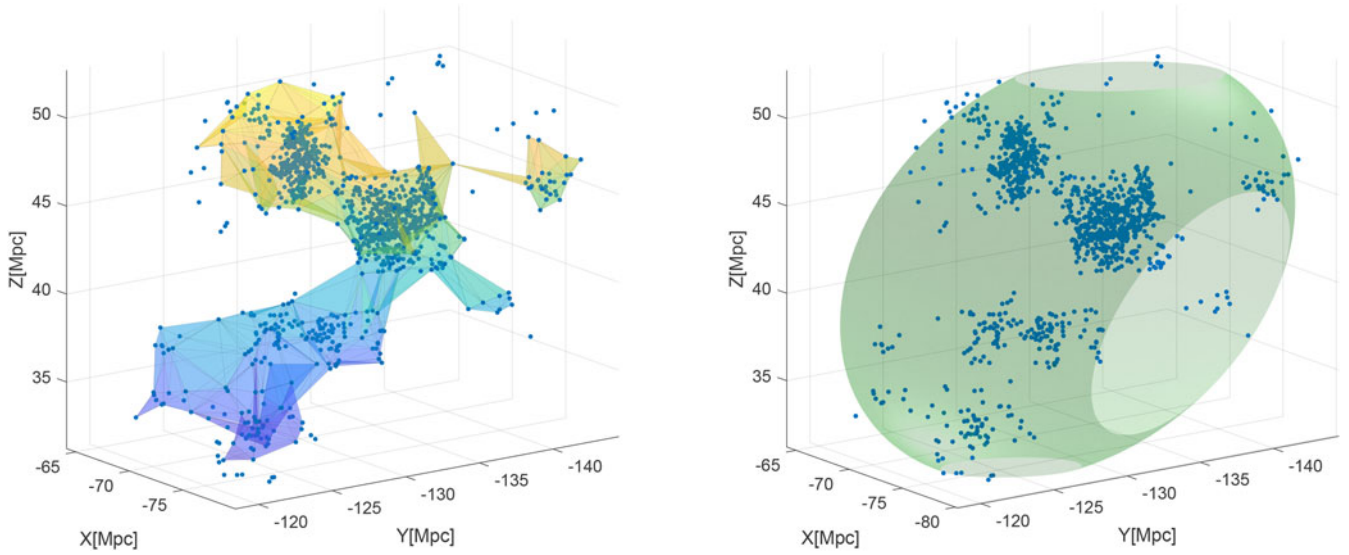


Figure 3. Left: Polyhedral fit of the optimal surface (with shrink factor, $s_f = 1$) enclosing the distribution of sampled member galaxies in DCC 099, the main core (A2147-A2151-A2152A-A2153A) of the *Hercules Supercluster* (MSC 474). Right: The alternative ellipsoidal fit for the same galaxy distribution. The lighter-colored regions on the ellipsoid correspond to automatically generated cut planes in the upper XY and right YZ sections, enhancing the 3D visualisation of the surface. The polyhedral and ellipsoidal surfaces shown are best-fit models of the main overdense body of the core, not a strict envelope of all its member galaxies. Due to the complex and unrelaxed nature of the structures, some galaxies, particularly those in the more diffuse or peripheral regions, may fall outside these fitted surfaces.

4.2.2. Alternative method: Ellipsoidal fitting

To validate the polyhedral surface method described above, continuous ellipsoidal surfaces, whose MFs can be calculated analytically, were fitted to the same galaxy samples for each core. The algorithm proposed by Petrov (2015) was used to estimate the semi-major axes (a , b , c) of the ellipsoid that best represents the 3D galaxy distribution in rectangular coordinates. The fitting process was performed iteratively by extracting boundary points using the boundary function for different values of the shrink factor ($0 \leq s_f \leq 1$). For each s_f , a new set of boundary points was obtained and used to fit an ellipsoidal surface. The optimal ellipsoid was then selected based on the maximum goodness of fit, measured by the Chi-square statistic.

The right panel of Figure 3 illustrates an example of the best ellipsoidal surface fit for the DCC 099 core. The parameters of the best fits for each core are presented in columns 2–5 of Table 3. The parametric equation for an ellipsoid with semi-major axes a , b , and c is given by:

$$r(\theta, \phi) = a(\sin \theta \cos \phi)i + b(\sin \theta \sin \phi)j + c(\cos \theta)k, \quad (20)$$

where $0 \leq \phi \leq 2\pi$ and $0 \leq \theta \leq \pi$.

The four MFs for an ellipsoidal body are expressed as follows (e.g. Lipschutz 1969; Sahni et al. 1998):

$$V = \frac{4}{3}\pi abc, \quad (21)$$

$$S = \oint \sqrt{EG - F^2} d\theta d\phi, \quad (22)$$

$$C = \frac{1}{2} \oint \left[\frac{EN + GL - 2FM}{EG - F^2} \right] dS, \quad (23)$$

$$\chi = \frac{1}{2\pi} \oint \left[\frac{LN - M^2}{EG - F^2} \right] dS, \quad (24)$$

where:

$$\begin{aligned} E &= r_\theta \cdot r_\theta, & F &= r_\theta \cdot r_\phi, & G &= r_\phi \cdot r_\phi, \\ L &= r_{\theta\theta} \cdot n, & M &= r_{\theta\phi} \cdot n, & N &= r_{\phi\phi} \cdot n, \end{aligned}$$

n is the unit normal vector to the surface at any point, defined as $n = r_\theta \times r_\phi / |r_\theta \times r_\phi|$. The differential area is $dS = \sqrt{EG - F^2} d\theta d\phi$, and $r_\theta = \partial r / \partial \theta$, $r_\phi = \partial r / \partial \phi$, $r_{\theta\theta} = \partial^2 r / \partial \theta^2$, $r_{\phi\phi} = \partial^2 r / \partial \phi^2$, $r_{\theta\phi} = \partial^2 r / \partial \theta \partial \phi$ are the first- and second-order partial derivatives of r with respect to θ and ϕ .

This approach provides an analytical framework for validating the polyhedral surfaces, enabling a comparative morphological analysis of the cores.

4.2.3. Dimensions and global topology of cores

Once the MFs have been determined using both the polyhedral and ellipsoidal surface methods, the shapefinders can be directly computed using the definitions in Equations (11)–(15). The resulting values of V , S , C , χ , $\mathcal{G}$, and the shapefinders $\mathcal{T}$, $\mathcal{B}$, $\mathcal{L}$, $\mathcal{P}$, and $\mathcal{F}$ for each DCC core, based on polyhedral (with $s_f = 1$) and ellipsoidal surface fits, are summarized in Tables 3 and 4, respectively.

The shapefinders $\mathcal{T}$, $\mathcal{B}$, and $\mathcal{L}$, which have units of length, are particularly useful for estimating the dimensions of the cores. The smallest shapefinder, $\mathcal{T}$, represents the thickness of the cores; the intermediate shapefinder, $\mathcal{B}$, is analogous to the breadth (width); and the largest shapefinder, $\mathcal{L}$, characterizes the length. It should be noted that $\mathcal{L}$ is not the actual physical length of the structure, but rather a morphological measure related to the integrated curvature of its surface (Einasto et al. 2007a). Thus, $\mathcal{L}$ can become significantly large for irregularly shaped or curved surfaces, without necessarily corresponding directly to a linear extension, as in elongated structures. The mean and median values of these dimensions are presented in Table 5. As shown in the table, the extension in any dimension of the DCC cores (as measured by $\mathcal{T}$, $\mathcal{B}$, and $\mathcal{L}$)

Table 3. Minkowski functional and shapefinders for DCC cores (ellipsoidal fits). In this fit all cores have $\chi = 2$, i.e., $G = 0$.

ID	a	b	c	χ^2	V	S	C	$\mathcal{T}$	$\mathcal{B}$	$\mathcal{L}$	$\mathcal{P}$	$\mathcal{F}$	$\mathcal{P}/\mathcal{F}$
DCC	$(h_{70}^{-1} \text{ Mpc})$	$(h_{70}^{-1} \text{ Mpc})$	$(h_{70}^{-1} \text{ Mpc})$		$(h_{70}^{-3} \text{ Mpc}^3)$	$(h_{70}^{-2} \text{ Mpc}^2)$	$(h_{70}^{-1} \text{ Mpc})$	$(h_{70}^{-1} \text{ Mpc})$	$(h_{70}^{-1} \text{ Mpc})$	$(h_{70}^{-1} \text{ Mpc})$			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
001	5.20	6.37	12.75	9.4	1772.8	783.5	108.5	6.78	7.21	8.63	0.0306	0.0897	0.341
002	3.67	5.88	10.09	11.0	914.3	517.5	88.6	5.30	5.83	7.05	0.0484	0.0941	0.514
003	3.55	3.85	11.83	11.0	679.7	448.4	87.1	4.54	5.14	6.93	0.0618	0.1479	0.418
004	2.52	5.95	8.34	12.2	525.2	387.8	76.5	4.06	5.06	6.09	0.1100	0.0916	1.200
005	2.99	7.35	8.80	10.2	811.5	508.8	81.9	4.78	6.20	6.52	0.1295	0.0246	5.251
006	2.98	5.57	9.39	19.7	654.1	431.4	81.4	4.54	5.30	6.47	0.0763	0.0999	0.763
007	5.25	6.04	7.64	10.3	1017.6	498.3	80.9	6.12	6.15	6.44	0.0025	0.0226	0.110
008	2.53	4.84	12.62	14.4	649.1	484.0	90.4	4.02	5.35	7.19	0.1418	0.1467	0.966
009	4.32	8.69	11.72	10.8	1845.7	840.6	110.1	6.58	7.63	8.76	0.0736	0.0689	1.067
010	2.71	4.78	7.58	4.0	412.8	307.4	69.1	4.02	4.44	5.50	0.0492	0.1063	0.462
011	4.07	7.18	16.28	12.3	1995.8	958.2	122.4	6.24	7.82	9.74	0.1121	0.1090	1.028
012	3.76	5.92	9.38	20.6	877.3	493.0	86.1	5.33	5.72	6.85	0.0349	0.0897	0.389
013	3.48	4.66	12.25	13.4	833.8	514.7	92.0	4.85	5.59	7.32	0.0702	0.1339	0.523
014	6.60	8.71	10.70	7.5	2582.3	939.3	111.0	8.24	8.46	8.83	0.0128	0.0214	0.601
015	4.63	8.55	9.35	9.9	1554.3	705.3	96.2	6.61	7.32	7.65	0.0514	0.0220	2.332
016	3.26	6.27	8.42	11.8	723.9	443.6	76.9	4.89	5.76	6.12	0.0815	0.0301	2.699
017	2.78	5.85	7.19	6.2	492.3	346.9	67.8	4.25	5.11	5.40	0.0911	0.0278	3.274
018	7.01	9.54	11.98	14.1	3361.2	1128.1	126.1	8.93	8.94	10.03	0.0003	0.0573	0.006
019	3.67	9.05	14.73	4.6	2054.1	1015.2	121.9	6.07	8.32	9.70	0.1567	0.0762	2.056
020	3.24	4.63	21.88	5.3	1377.5	864.6	132.4	4.77	6.52	10.54	0.1545	0.2351	0.657
021	2.77	7.08	9.70	6.4	798.6	526.2	88.1	4.55	5.96	7.01	0.1345	0.0807	1.665
022	3.03	5.86	11.41	14.4	849.9	538.2	91.5	4.73	5.87	7.28	0.1074	0.1068	1.006
023	2.09	6.65	12.55	8.6	732.9	596.8	95.8	3.68	6.22	7.62	0.2565	0.1011	2.535
024	4.88	8.50	18.43	4.0	3210.1	1294.6	140.6	7.43	9.20	11.19	0.1060	0.0975	1.086
025	2.04	7.65	14.64	25.4	959.1	779.0	108.8	3.69	7.15	8.65	0.3193	0.0948	3.367
026	4.88	8.20	12.82	5.3	2152.0	909.7	115.1	7.09	7.89	9.16	0.0535	0.0742	0.721
027	4.63	6.56	14.26	10.5	1820.0	837.3	113.5	6.52	7.37	9.03	0.0615	0.1010	0.609
028	2.74	4.06	10.49	4.3	490.0	368.9	78.8	3.98	4.68	6.27	0.0803	0.1454	0.552
029	3.58	9.21	13.93	9.4	1930.0	973.0	118.7	5.95	8.19	9.45	0.1584	0.0715	2.216
030	3.93	5.86	10.24	10.3	991.5	538.2	90.2	5.52	5.96	7.18	0.0378	0.0930	0.406
031	3.75	7.72	16.40	16.0	1993.3	993.1	123.8	6.02	8.01	9.85	0.1421	0.1030	1.379
032	3.30	6.00	11.24	18.6	934.2	555.9	92.5	5.04	6.00	7.36	0.0875	0.1012	0.864
033	5.62	6.43	12.78	4.1	1939.5	821.4	110.7	7.08	7.41	8.81	0.0230	0.0857	0.269
034	2.58	5.24	12.45	16.0	708.0	510.8	91.5	4.15	5.57	7.28	0.1458	0.1328	1.097
035	5.06	6.82	14.57	6.3	2110.9	909.9	117.7	6.95	7.73	9.36	0.0524	0.0957	0.548
036	4.12	5.84	6.74	10.9	681.6	388.0	71.5	5.27	5.42	5.69	0.0145	0.0237	0.613
037	3.19	4.46	16.68	8.2	996.6	645.5	109.0	4.63	5.91	8.68	0.1219	0.1892	0.644
038	4.06	4.55	17.10	9.1	1325.4	743.6	114.9	5.34	6.47	9.14	0.0952	0.1711	0.556
039	3.16	6.41	17.99	4.2	1528.7	894.5	122.7	5.12	7.28	9.76	0.1739	0.1456	1.194
040	4.59	6.62	10.21	17.4	1304.7	626.2	96.1	6.25	6.51	7.65	0.0206	0.0803	0.256
041	5.37	5.91	11.96	22.7	1592.0	718.8	103.9	6.64	6.91	8.26	0.0201	0.0890	0.226
042	2.55	4.14	10.62	15.5	471.5	368.4	78.9	3.84	4.66	6.28	0.0972	0.1474	0.659
043	4.45	6.97	22.20	11.9	2886.9	1295.0	148.6	6.68	8.71	11.83	0.1313	0.1519	0.864
044	3.16	6.37	10.71	12.0	904.6	549.0	91.1	4.94	6.02	7.25	0.0982	0.0931	1.054

Table 3. (Continued)

ID	<i>a</i>	<i>b</i>	<i>c</i>	chi ²	<i>V</i>	<i>S</i>	<i>C</i>	<i>T</i>	<i>B</i>	<i>L</i>	<i>P</i>	<i>F</i>	<i>P/F</i>
DCC	(<i>h</i> ₇₀ ^{−1} Mpc)	(<i>h</i> ₇₀ ^{−1} Mpc)	(<i>h</i> ₇₀ ^{−1} Mpc)		(<i>h</i> ₇₀ ^{−3} Mpc ³)	(<i>h</i> ₇₀ ^{−2} Mpc ²)	(<i>h</i> ₇₀ ^{−1} Mpc)	(<i>h</i> ₇₀ ^{−1} Mpc)	(<i>h</i> ₇₀ ^{−1} Mpc)	(<i>h</i> ₇₀ ^{−1} Mpc)			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
045	3.85	4.64	10.74	12.4	805.1	476.6	86.9	5.06	5.48	6.92	0.0392	0.1159	0.338
046	4.84	5.56	13.01	10.5	1469.6	707.7	104.7	6.22	6.75	8.33	0.0405	0.1048	0.386
047	4.10	8.26	13.89	15.7	1976.0	923.7	116.8	6.41	7.90	9.29	0.1040	0.0805	1.291
048	2.29	4.90	10.50	25.9	494.9	399.0	80.4	3.72	4.95	6.40	0.1424	0.1273	1.118
049	4.55	6.22	37.33	23.4	4436.1	2003.7	211.2	6.64	9.48	16.81	0.1762	0.2786	0.632
050	3.95	5.17	14.64	4.2	1253.6	685.0	106.4	5.48	6.43	8.46	0.0794	0.1362	0.583
051	4.35	4.59	7.15	4.1	599.5	356.4	73.3	4.85	5.04	5.83	0.0190	0.0728	0.260
052	2.16	4.96	24.22	3.6	1092.5	890.6	139.4	3.67	6.38	11.09	0.2689	0.2693	0.998
053	4.38	9.25	18.12	9.1	3082.4	1312.7	140.3	7.04	9.35	11.17	0.1407	0.0886	1.588
054	2.36	4.43	6.39	2.5	281.0	237.7	56.7	3.54	4.19	4.51	0.0836	0.0366	2.279
055	4.63	6.58	56.54	1.4	7221.0	3142.8	295.9	6.89	10.61	23.55	0.2127	0.3785	0.562
056	4.46	10.21	29.55	5.9	5645.3	2264.3	193.9	7.47	11.67	15.43	0.2189	0.1389	1.575
057	2.69	6.61	12.14	9.0	907.6	606.8	96.4	4.48	6.29	7.67	0.1674	0.0990	1.690
058	3.39	6.54	8.52	8.0	794.0	469.8	79.0	5.07	5.94	6.29	0.0791	0.0286	2.758
059	2.06	5.01	8.04	8.7	348.3	308.2	69.3	3.39	4.44	5.51	0.1344	0.1079	1.246
060	2.73	3.93	7.44	4.6	335.5	263.8	65.0	3.81	4.05	5.17	0.0301	0.1220	0.247
061	2.69	5.32	8.16	6.5	489.8	354.8	73.8	4.14	4.80	5.87	0.0743	0.0999	0.743
062	2.68	4.42	8.96	9.0	446.0	336.0	73.5	3.98	4.57	5.84	0.0689	0.1225	0.562
063	2.94	4.53	20.94	9.2	1170.7	787.7	126.7	4.45	6.21	10.08	0.1647	0.2371	0.694
064	4.53	6.66	17.50	22.3	2219.3	1011.8	127.5	6.57	7.93	10.14	0.0933	0.1222	0.763
065	1.84	5.32	12.39	14.3	510.9	475.3	88.5	3.22	5.36	7.04	0.2493	0.1352	1.844
066	5.34	5.96	12.71	13.1	1699.3	760.5	107.2	6.70	7.08	8.53	0.0278	0.0927	0.300
067	5.37	8.16	11.13	11.8	2044.6	836.5	109.7	7.33	7.62	8.73	0.0194	0.0679	0.285
068	4.81	5.66	7.31	8.6	835.3	438.8	76.0	5.71	5.76	6.05	0.0050	0.0240	0.211
069	3.71	5.88	10.13	15.7	928.6	521.6	88.9	5.34	5.86	7.08	0.0466	0.0940	0.495
070	5.94	9.10	15.81	11.0	3588.9	1276.1	136.3	8.43	9.36	10.84	0.0518	0.0736	0.704
071	5.92	9.58	11.42	19.0	2720.2	1004.9	114.9	8.12	8.74	9.14	0.0370	0.0223	1.660
072	4.15	6.27	9.36	10.0	1021.5	534.3	89.1	5.73	5.99	7.09	0.0223	0.0835	0.267
073	3.55	6.78	13.28	11.9	1344.5	727.9	105.6	5.54	6.88	8.41	0.1083	0.0996	1.087
074	3.21	8.87	14.64	10.3	1748.3	961.2	118.8	5.45	8.08	9.45	0.1943	0.0779	2.492
075	2.36	5.80	11.46	5.9	659.6	500.1	88.7	3.95	5.63	7.06	0.1752	0.1120	1.563
076	2.52	8.00	13.39	3.0	1135.1	770.7	106.9	4.41	7.20	8.50	0.2398	0.0828	2.894
077	3.46	6.66	11.97	4.4	1159.2	647.5	99.1	5.37	6.53	7.89	0.0975	0.0942	1.034
078	2.66	5.52	11.49	7.0	710.2	497.7	88.9	4.28	5.59	7.07	0.1331	0.1170	1.137
079	5.53	7.54	24.08	7.5	4215.3	1604.0	163.7	7.88	9.79	13.03	0.1081	0.1416	0.763
080	3.18	7.40	12.84	0.7	1269.2	731.1	104.8	5.20	6.97	8.33	0.1452	0.0889	1.632
081	5.36	6.88	7.63	13.4	1182.6	550.6	84.9	6.44	6.48	6.75	0.0032	0.0205	0.157
082	2.65	9.83	24.82	23.1	2715.5	1686.5	164.4	4.83	10.25	13.08	0.3595	0.1213	2.961
083	5.48	7.38	11.44	18.5	1942.6	806.2	108.3	7.22	7.44	8.61	0.0145	0.0732	0.199
084	3.83	6.77	16.83	12.9	1830.0	927.4	122.0	5.91	7.59	9.71	0.1241	0.1223	1.014
085	6.77	7.55	8.39	11.6	1801.7	720.4	96.9	7.43	7.50	7.71	0.0045	0.0137	0.332
086	6.37	9.83	13.13	22.0	3447.7	1184.9	129.5	8.72	9.14	10.31	0.0232	0.0598	0.388
087	4.74	8.88	13.82	14.2	2444.0	1020.8	121.7	7.18	8.38	9.69	0.0770	0.0724	1.064
088	5.59	6.94	12.95	10.4	2109.1	869.0	113.4	7.28	7.65	9.02	0.0253	0.0820	0.309

Table 3. (Continued)

ID	a	b	c	χ^2	V	S	C	$\mathcal{T}$	$\mathcal{B}$	$\mathcal{L}$	$\mathcal{P}$	$\mathcal{F}$	$\mathcal{P}/\mathcal{F}$
DCC	$(h_{70}^{-1} \text{ Mpc})$	$(h_{70}^{-1} \text{ Mpc})$	$(h_{70}^{-1} \text{ Mpc})$		$(h_{70}^{-3} \text{ Mpc}^3)$	$(h_{70}^{-2} \text{ Mpc}^2)$	$(h_{70}^{-1} \text{ Mpc})$	$(h_{70}^{-1} \text{ Mpc})$	$(h_{70}^{-1} \text{ Mpc})$	$(h_{70}^{-1} \text{ Mpc})$			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
089	5.68	10.76	13.70	14.7	3514.5	1253.8	133.0	8.40	9.42	10.58	0.0568	0.0582	0.977
090	4.67	6.91	9.30	16.0	1260.8	601.4	93.7	6.28	6.41	7.46	0.0100	0.0752	0.133
091	7.20	8.63	9.46	13.2	2466.3	892.0	107.8	8.27	8.29	8.57	0.0012	0.0168	0.072
092	6.13	9.23	11.98	9.3	2844.1	1032.3	121.1	8.26	8.52	9.63	0.0154	0.0613	0.251
093	6.73	7.67	12.68	14.4	2745.7	1000.4	120.1	8.23	8.32	9.55	0.0057	0.0687	0.083
094	6.02	9.98	13.07	6.9	3295.6	1165.1	128.4	8.48	9.06	10.22	0.0331	0.0600	0.551
095	4.59	8.08	12.91	9.4	2012.1	885.2	113.9	6.81	7.77	9.06	0.0653	0.0767	0.851
096	4.63	8.55	11.58	17.0	1925.0	842.9	110.2	6.85	7.64	8.77	0.0547	0.0688	0.795
097	4.35	5.70	7.99	17.4	833.1	449.0	77.2	5.56	5.80	6.15	0.0214	0.0285	0.751
098	2.28	8.71	13.37	15.2	1115.2	816.0	108.8	4.10	7.50	8.65	0.2930	0.0716	4.089
099	5.59	9.59	14.49	20.3	3261.4	1198.7	131.2	8.16	9.13	10.44	0.0563	0.0667	0.843
100	2.32	6.00	16.48	5.5	965.7	724.8	111.0	3.99	6.52	8.83	0.2403	0.1504	1.597
101	1.52	4.18	6.46	4.9	173.1	201.4	52.4	2.57	3.84	4.17	0.1969	0.0413	4.763
102	2.42	3.37	9.69	5.2	332.5	288.1	71.1	3.46	4.04	5.66	0.0781	0.1659	0.470
103	3.98	5.03	7.93	2.8	668.0	392.5	77.0	5.09	5.10	6.13	0.0012	0.0915	0.013
104	4.18	5.05	11.66	5.8	1033.7	563.0	94.0	5.50	5.98	7.48	0.0415	0.1112	0.373
105	2.97	5.87	13.15	6.6	963.6	610.8	98.8	4.73	6.18	7.86	0.1326	0.1199	1.105

does not exceed $10 h_{70}^{-1} \text{ Mpc}$ for either surface method. These results are consistent with those of Einasto et al. (2007a), who reported similar scales for the densest central regions of superclusters constrained to high-mass fraction areas, which coincide with their *cores*.

Although the polyhedral and ellipsoidal surface fits share the same boundary-point selection process, the mean (and median) values of $\mathcal{T}$, $\mathcal{B}$, and $\mathcal{L}$ obtained with these two methods differ significantly. These differences stem from the intrinsic nature of each approach. The polyhedral fit with $s_f = 1$ yields a more compact representation of the surface enclosing the member galaxies of the *cores*, capturing (via triangulation) finer substructures and morphological details that are typical of dynamically evolving regions. Consequently, this compactness produces smaller values of $\mathcal{T}$, $\mathcal{B}$, and $\mathcal{L}$, since, by construction, surface area and volume are minimal at $s_f = 1$ (and increase toward $s_f = 0$). In contrast, the ellipsoidal fit enforces a smoother, global shape that encloses most of the *core* member galaxies within an ellipsoidal boundary, which systematically returns larger values for the shapefinders.

This dual strategy (compact polyhedral fits for structural detail and ellipsoidal fits for global extent) provides a robust morphological characterisation of the *cores*: the results of the polyhedral fit can be interpreted as a lower bound on the dimensions of these structures, while those of the ellipsoidal fits serve as an upper bound. The ellipsoidal method is included for comparison because it is a conservative technique widely used in the literature. Furthermore, exploratory tests (not included in the manuscript for brevity) employing intermediate shrink factors (e.g. $0 \leq s_f < 1$) in polyhedral fits confirm that the qualitative classification, in particular the dominance of filamentary morphologies, remains consistent and does not critically depend on the specific choice of s_f .

Figure 4 shows scatterplots of the three characteristic dimensions ($\mathcal{T}$, $\mathcal{B}$, and $\mathcal{L}$ from Table 4) of the DCC *cores* versus their

extensive mass ($\mathcal{M}_{\text{ext}}^c$, defined as the sum of the virial masses of the member galaxy systems of a *core*, and presented in Table 5 of Paper I). Statistical correlations were evaluated using a Pearson test with a 95% confidence level. The correlation coefficients for $\mathcal{T}$, $\mathcal{B}$, and $\mathcal{L}$ with mass were 0.70, 0.64, and 0.40, respectively. These correlations are statistically significant with a significance level of $\alpha_s = 0.05$. These results suggest that the dimensions of the *cores* are positively correlated with their mass: more massive structures tend to exhibit greater thickness, breadth, and length. This finding is consistent with previous studies by Shandarin et al. (2004), which reported similar trends in superclusters.

On the other hand, a standard interpretation of the genus in the cosmology literature defines it as (e.g. Shandarin et al. 2004):

$$\mathcal{G} = (\text{number of holes}) - (\text{number of isolated regions}) + 1, \quad (25)$$

where ‘holes’ are considered complex mathematical objects that, in three-dimensional structures, typically manifest as tunnels crossing the structure from one side to the other, as in a toroidal shape. The term ‘isolated regions’ refers to the number of disconnected parts of the fitted surface that define the structure’s boundaries. Thus, Equation (26) can be rewritten as:

$$\mathcal{G} = N_{\text{tunn}} - N_{\text{is,surf}} + 1, \quad (26)$$

where N_{tunn} and $N_{\text{is,surf}}$ represent the number of tunnels and isolated surfaces, respectively (e.g. Bag et al. 2019).

Visual inspections using basic rotation in 3D and rendering tools available in standard scientific software (e.g. MATLAB and Python libraries) revealed that the DCC *cores* exhibit significant substructure patterns, frequently containing tunnels and isolated systems. The top panel of Figure 5 shows the distribution of $\mathcal{G}$ values obtained from the fitted polyhedral surfaces of the DCC *cores*, as presented in Table 4. The wide range of $\mathcal{G}$ values (from -1 to 8) reflects the diversity of complex three-dimensional shapes

Table 4. Minkowski functional and shapefinders for DCC cores (polyhedral fits).

ID	V	S	C	χ	$\mathcal{G}$	$\mathcal{T}$	$\mathcal{B}$	$\mathcal{L}$	$\mathcal{P}$	$\mathcal{F}$	$\mathcal{P}/\mathcal{F}$
DCC	$(h_{70}^{-3} \text{ Mpc}^3)$	$(h_{70}^{-2} \text{ Mpc}^2)$	$(h_{70}^{-1} \text{ Mpc})$			$(h_{70}^{-1} \text{ Mpc})$	$(h_{70}^{-1} \text{ Mpc})$	$(h_{70}^{-1} \text{ Mpc})$			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
001	474.5	567.2	125.0	1	0.5	2.50	4.53	6.63	0.28	0.18	1.53
002	185.0	344.6	94.6	−2	2	1.61	2.51	3.63	0.21	0.18	1.19
003	94.1	268.2	156.9	−13	7.5	1.05	1.46	1.70	0.16	0.07	2.18
004	60.3	187.0	111.4	−3	2.5	0.96	1.67	2.53	0.26	0.20	1.31
005	79.4	212.2	132.7	1	0.5	1.12	1.59	7.04	0.17	0.62	0.27
006	98.2	210.5	88.4	−2	2	1.40	2.34	2.38	0.25	0.00	34.38
007	170.7	354.4	113.7	−4	3	1.44	2.26	3.11	0.22	0.15	1.39
008	40.3	137.2	93.4	0	1	0.88	1.46	3.71	0.24	0.43	0.57
009	156.9	267.3	110.3	3	−0.5	1.76	2.42	5.85	0.15	0.41	0.38
010	106.4	208.9	72.0	−4	3	1.43	1.52	2.90	0.03	0.30	0.10
011	59.9	212.2	142.7	4	−1	0.84	1.48	5.67	0.27	0.58	0.46
012	131.3	316.2	132.4	−4	3	1.24	2.38	2.63	0.31	0.04	6.37
013	65.0	180.7	101.6	1	0.5	1.07	1.77	5.39	0.24	0.50	0.48
014	241.0	378.5	149.3	0	1	1.91	2.53	5.94	0.14	0.40	0.34
015	408.8	507.1	111.8	0	1	2.41	4.44	4.53	0.29	0.00	31.30
016	76.5	179.3	93.7	3	−0.5	1.28	1.91	4.97	0.19	0.44	0.44
017	59.7	172.8	88.7	1	0.5	1.03	1.94	4.70	0.30	0.41	0.73
018	567.5	757.6	194.7	−7	4.5	2.24	2.81	3.89	0.11	0.15	0.70
019	233.0	328.0	121.8	3	−0.5	2.13	2.69	6.46	0.11	0.41	0.28
020	79.1	170.9	67.4	−3	2.5	1.38	1.53	2.53	0.04	0.24	0.19
021	118.2	216.1	85.3	3	−0.5	1.64	2.53	4.52	0.21	0.28	0.75
022	49.0	191.9	152.2	−5	3.5	0.76	1.26	2.69	0.24	0.36	0.67
023	33.1	129.4	93.2	0	1	0.76	1.38	3.71	0.28	0.45	0.63
024	525.9	600.6	153.1	2	0	2.62	3.92	12.18	0.19	0.51	0.38
025	91.7	191.8	95.6	1	0.5	1.43	2.00	5.07	0.16	0.43	0.38
026	315.8	411.2	118.5	3	−0.5	2.30	3.46	6.29	0.20	0.28	0.69
027	219.7	399.3	182.1	−2	2	1.65	2.19	4.83	0.14	0.37	0.37
028	63.1	178.7	78.9	2	0	1.05	2.26	6.28	0.36	0.47	0.77
029	202.8	437.7	164.7	0	1	1.38	2.65	6.55	0.31	0.42	0.74
030	267.5	431.1	111.6	−5	3.5	1.86	1.97	3.86	0.02	0.32	0.09
031	313.1	520.0	149.6	1	0.5	1.80	3.47	7.94	0.31	0.39	0.80
032	151.2	332.3	151.4	−1	1.5	1.36	2.19	4.82	0.23	0.37	0.62
033	242.3	465.2	119.5	2	0	1.56	3.89	9.51	0.42	0.41	1.01
034	124.8	252.9	92.8	0	1	1.48	2.72	3.69	0.29	0.15	1.94
035	213.9	374.8	99.4	0	1	1.71	3.76	3.95	0.37	0.02	15.13
036	101.6	247.7	102.2	0	1	1.23	2.42	4.06	0.32	0.25	1.28
037	102.5	179.3	68.5	1	0.5	1.71	2.61	3.63	0.20	0.16	1.26
038	306.0	370.0	100.6	0	1	2.48	3.67	4.00	0.19	0.04	4.56
039	173.3	271.6	93.0	3	−0.5	1.91	2.91	4.93	0.20	0.25	0.80
040	210.7	516.7	176.3	−11	6.5	1.22	1.87	2.93	0.20	0.22	0.94
041	440.0	595.9	120.7	−2	2	2.21	3.20	4.93	0.18	0.21	0.85
042	76.1	202.6	94.4	1	0.5	1.12	2.14	5.01	0.31	0.40	0.77
043	143.8	313.7	151.5	2	0	1.37	2.07	12.05	0.20	0.70	0.28
044	124.6	280.6	122.4	−6	4	1.33	1.94	2.29	0.18	0.08	2.31
045	230.2	363.8	95.9	−1	1.5	1.89	3.05	3.79	0.23	0.10	2.16

Table 4. (Continued)

ID	V	S	C	χ	$\mathcal{G}$	$\mathcal{T}$	$\mathcal{B}$	$\mathcal{L}$	$\mathcal{P}$	$\mathcal{F}$	$\mathcal{P}/\mathcal{F}$
DCC	$(h_{70}^{-3} \text{ Mpc}^3)$	$(h_{70}^{-2} \text{ Mpc}^2)$	$(h_{70}^{-1} \text{ Mpc})$			$(h_{70}^{-1} \text{ Mpc})$	$(h_{70}^{-1} \text{ Mpc})$	$(h_{70}^{-1} \text{ Mpc})$			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
046	121.1	324.1	130.8	−1	1.5	1.12	2.47	4.16	0.37	0.25	1.48
047	288.0	556.8	205.8	−6	4	1.55	2.70	3.27	0.27	0.09	2.83
048	97.5	217.1	98.7	−2	2	1.34	2.19	2.62	0.23	0.08	2.72
049	206.5	460.3	196.5	−14	8	1.34	1.73	2.34	0.12	0.14	0.85
050	194.4	328.0	92.5	1	0.5	1.77	3.54	4.90	0.33	0.16	2.06
051	146.9	227.8	78.2	2	0	1.93	2.91	6.22	0.20	0.36	0.55
052	9.7	69.2	76.0	−4	3	0.42	0.91	1.51	0.36	0.24	1.47
053	507.0	521.8	144.1	1	0.5	2.91	3.62	7.64	0.10	0.35	0.30
054	30.9	106.3	77.6	0	1	0.87	1.36	3.09	0.22	0.38	0.57
055	142.3	330.8	140.5	0	1	1.29	2.35	5.59	0.29	0.40	0.71
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
056	78.7	244.6	139.0	2	0	0.96	1.76	11.06	0.29	0.72	0.40
057	34.7	140.8	114.2	−2	2	0.73	1.23	3.03	0.24	0.42	0.59
058	127.6	303.0	96.4	−6	4	1.26	1.53	3.14	0.09	0.34	0.28
059	66.8	171.7	87.1	1	0.5	1.16	1.97	4.62	0.25	0.40	0.63
060	81.3	162.8	55.5	1	0.5	1.49	2.93	2.94	0.32	0.00	116.18
061	64.5	142.9	75.6	2	0	1.35	1.89	6.01	0.16	0.52	0.31
062	77.9	190.1	100.2	−5	3.5	1.22	1.77	1.89	0.18	0.03	5.40
063	68.7	227.9	123.4	−6	4	0.90	1.84	1.96	0.34	0.03	11.02
064	313.0	532.5	159.8	−3	2.5	1.76	3.33	3.63	0.30	0.04	7.05
065	55.5	163.6	107.0	2	0	1.01	1.52	8.52	0.20	0.69	0.28
066	122.6	282.7	134.5	3	−0.5	1.30	2.10	7.13	0.23	0.54	0.43
067	733.0	684.7	112.9	2	0	3.21	6.06	8.99	0.30	0.19	1.57
068	234.4	360.0	82.9	1	0.5	1.95	4.34	4.40	0.37	0.00	56.21
069	148.9	317.5	132.3	−1	1.5	1.40	2.39	4.21	0.26	0.27	0.95
070	590.2	716.5	144.8	2	0	2.47	4.94	11.52	0.33	0.39	0.83
071	393.1	667.1	168.6	−2	2	1.76	3.95	4.47	0.38	0.06	6.21
072	126.0	284.2	107.0	1	0.5	1.33	2.65	5.68	0.33	0.36	0.91
073	134.6	317.7	129.6	−2	2	1.27	2.45	3.43	0.31	0.16	1.88
074	282.4	402.4	131.7	3	−0.5	2.10	3.05	6.98	0.18	0.39	0.46
075	48.5	127.7	87.0	0	1	1.13	1.46	3.46	0.12	0.40	0.31
076	169.8	287.3	109.5	3	−0.5	1.77	2.62	5.81	0.19	0.37	0.51
077	75.9	191.3	88.5	1	0.5	1.19	2.16	4.69	0.28	0.36	0.78
078	66.1	160.2	78.4	−2	2	1.23	2.04	2.08	0.24	0.00	27.03
079	415.6	568.3	128.0	2	0	2.19	4.44	10.18	0.33	0.39	0.86
080	96.6	205.3	99.4	0	1	1.41	2.06	3.95	0.18	0.31	0.59
081	136.5	277.3	121.9	−2	2	1.47	2.27	3.23	0.21	0.17	1.21
082	123.9	307.6	142.1	−1	1.5	1.20	2.16	4.52	0.28	0.35	0.80
083	254.0	486.2	154.9	−7	4.5	1.56	2.24	3.13	0.17	0.16	1.06
084	167.5	362.4	163.0	−3	2.5	1.38	2.22	3.70	0.23	0.25	0.92
085	214.8	417.1	153.4	−8	5	1.54	2.03	2.71	0.13	0.14	0.94
086	416.3	589.0	175.5	−1	1.5	2.12	3.35	5.58	0.22	0.24	0.90
087	451.9	590.4	140.1	0	1	2.29	4.21	5.57	0.29	0.13	2.11
088	227.7	466.2	187.2	−1	1.5	1.46	2.48	5.96	0.25	0.41	0.63
089	310.6	577.7	214.1	0	1	1.61	2.69	8.52	0.25	0.51	0.48

Table 4. (Continued)

ID	V	S	C	χ	$\mathcal{G}$	$\mathcal{T}$	$\mathcal{B}$	$\mathcal{L}$	$\mathcal{P}$	$\mathcal{F}$	$\mathcal{P}/\mathcal{F}$
DCC	$(h_{70}^{-3} \text{ Mpc}^3)$	$(h_{70}^{-2} \text{ Mpc}^2)$	$(h_{70}^{-1} \text{ Mpc})$			$(h_{70}^{-1} \text{ Mpc})$	$(h_{70}^{-1} \text{ Mpc})$	$(h_{70}^{-1} \text{ Mpc})$			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
090	185.3	328.0	140.5	1	0.5	1.69	2.33	7.45	0.15	0.52	0.30
091	418.6	534.7	153.6	1	0.5	2.34	3.48	8.15	0.19	0.40	0.48
092	445.6	643.3	157.0	-2	2	2.07	4.09	4.16	0.32	0.00	37.72
093	471.3	682.7	178.6	-13	7.5	1.67	2.07	3.82	0.10	0.29	0.35
094	271.7	471.7	164.5	0	1	1.72	2.86	6.54	0.24	0.39	0.63
095	403.3	567.3	166.7	0	1	2.13	3.40	6.63	0.22	0.32	0.71
096	221.2	423.5	160.8	1	0.5	1.56	2.63	8.53	0.25	0.52	0.48
097	192.5	384.6	130.8	-12	7	1.30	1.50	2.94	0.07	0.32	0.22
098	89.0	256.5	146.0	1	0.5	1.04	1.75	7.74	0.25	0.63	0.40
099	424.6	743.9	226.8	-13	7.5	1.71	2.12	3.27	0.10	0.21	0.50
100	102.2	191.2	84.3	0	1	1.60	2.26	3.35	0.17	0.19	0.88
101	16.1	66.5	56.8	3	-0.5	0.72	1.17	3.01	0.23	0.44	0.52
102	29.0	131.6	84.0	-3	2.5	0.66	1.56	1.91	0.40	0.09	4.09
103	50.6	147.0	92.7	1	0.5	1.03	1.58	4.91	0.21	0.51	0.41
104	76.2	236.6	114.0	-6	4	0.96	1.81	2.07	0.30	0.06	4.54
105	96.0	184.9	98.3	4	-1	1.55	1.88	3.91	0.09	0.35	0.26

Table 5. Mean (with standard deviation) and median values for the $\mathcal{T}$, $\mathcal{B}$, and $\mathcal{L}$ shapefinders estimated from polyhedral and ellipsoidal surface fits to the DCC *cores*. Median values are given as asymmetric ranges with $\Delta Q_1 = \text{Median} - Q_1$ and $\Delta Q_3 = Q_3 - \text{Median}$ as lower and upper indices, where Q_1 and Q_3 are the 25th and 75th percentiles, respectively.

Shapefinder	Polyhedral surfaces		Ellipsoidal surfaces	
	Mean $\pm$ sth	Median ^{$+\Delta Q_3$ $-\Delta Q_1$}	Mean $\pm$ std	Median ^{$+\Delta Q_3$ $-\Delta Q_1$}
$\mathcal{T}(h_{70}^{-1} \text{ Mpc})$	1.52 ± 0.50	$1.43^{+0.34}_{-0.24}$	5.70 ± 1.51	$5.45^{+1.34}_{-0.92}$
$\mathcal{B}(h_{70}^{-1} \text{ Mpc})$	2.47 ± 0.91	$2.26^{+0.61}_{-0.43}$	6.75 ± 1.52	$6.48^{+1.33}_{-0.83}$
$\mathcal{L}(h_{70}^{-1} \text{ Mpc})$	4.88 ± 2.31	$4.40^{+1.67}_{-1.19}$	8.07 ± 1.80	$7.89^{+1.42}_{-1.06}$

exhibited by these structures. Interpreting the genus as a structural feature of such surfaces is not unique and becomes non-trivial when the regions have highly intricate geometries (e.g. Shandarin et al. 2004).

For this analysis, the genus obtained from ellipsoidal surface fits was not considered, since ellipsoids are topologically homeomorphic to spheres, which have genus $\mathcal{G} = 0$. As a result, a meaningful topological study based on this type of surface fit is not feasible.

Various non-integer values of $\mathcal{G}$ were obtained because the genus was computed using the formula $\mathcal{G} = 1 - \chi/2$, where χ is the Euler characteristic. Since χ can take odd values depending on the discretisation and resolution of the surface representation, $\mathcal{G}$ is not always an integer. However, genus is fundamentally a topological invariant that describes the number of handles or holes in a surface, and it is conventionally defined for integer values. Non-integer values are difficult to interpret in this context, as they do not correspond to well-defined topological classes. Therefore, we restrict our analysis to integer values as defined in Equation (26). Under this criterion, about 10% of the DCC *cores* are topologically isomorphic to a sphere (i.e. $\mathcal{G} = 0$), $\sim 18\%$ to a toroid (i.e.

$\mathcal{G} = 1$), and $\sim 10\%$ to a pretzel (i.e. $\mathcal{G} = 2$). Higher genus values indicate increasingly complex topologies. Structures with negative genus (about 11% of the *cores*) can be interpreted as coarser configurations with multiple isolated member systems.

Additionally, the bottom panel of Figure 5 shows the distribution of all genus values (both integer and non-integer) versus the extensive mass of *cores*, though no significant statistical correlation can be inferred from this relationship. This suggests that the topological complexity of *cores* is not directly governed by their mass, possibly due to the influence of other structural and dynamical factors.

4.2.4. The spectrum of shapes of the cores

In addition to MFs and shapefinders, Tables 3 and 4 present the morphological parameter $\mathcal{P}/\mathcal{F}$, which enables the statistical analysis of the shapes of the *cores*. The shapefinder formalism allows for classifying *cores* into two basic morphological types:

1. Pancakes (oblate structures), with $\mathcal{P}/\mathcal{F} > 1$, and
2. Filaments (prolate structures), with $0 \leq \mathcal{P}/\mathcal{F} \leq 1$.

Here, ribbon-like objects were excluded, as defining the range $\mathcal{P}/\mathcal{F} \approx 1$ for this morphology is somewhat arbitrary (e.g. Costa-Duarte et al. 2011). The distribution of the morphological parameter $\mathcal{P}/\mathcal{F}$, referred to as the *shape spectrum*, for the DCC *cores* is shown in Figure 6. Outliers (15 data points from polyhedral fits and 9 from ellipsoidal fits) were excluded from the analysis. The polyhedral surface fits reveal that about 75% of the *cores* can be classified as filaments, while the remaining $\sim 25\%$ can be identified as pancakes. The ellipsoidal fits corroborate these findings, indicating that approximately 62% of the *cores* have filamentary morphologies, while $\sim 38\%$ exhibit pancake-like shapes. The shape spectrum obtained from both methods confirms a statistical tendency of DCC *cores* towards filamentary morphologies.

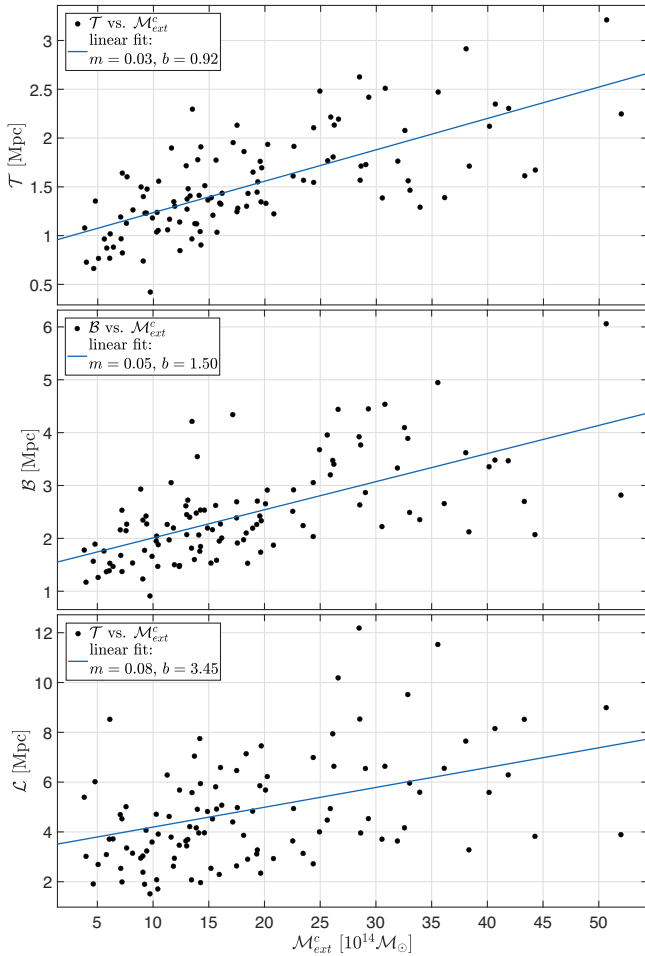


Figure 4. Length, breadth, and thickness ($\mathcal{T}$, $\mathcal{B}$, and $\mathcal{L}$ shapefinders) versus extensive mass $\mathcal{M}_{\text{ext}}^c$ for DCC cores. The three solid blue lines correspond to the best linear fit in each case. The slope m and vertical intercept b of each fit are displayed in the upper-left inset of the corresponding panel. The units of each axis must be understood in terms of h_{70}^{-1} .

Despite the differences in the values of the characteristic dimensions (see Table 5) obtained from polyhedral and ellipsoidal surface fitting, the analyses of planarity and filamentarity remain consistent for both methods. Since these quantities are defined as combined ratios of $\mathcal{T}$, $\mathcal{B}$, and $\mathcal{L}$ (see Equations 14 and 15), their stability suggests that both fitting approaches preserve the overall structural characteristics of the cores (see Section 4.2.4). This reinforces the robustness of our morphological classification, regardless of the specific method used to estimate absolute dimensions. Therefore, while the polyhedral fit provides a more precise morphological description, the ellipsoidal fit remains a useful comparative reference within the framework of large-scale structure studies.

Since filaments are the most common features in rich superclusters (e.g. Einasto et al. 2007a; Cautun et al. 2014), it is reasonable to expect that cores, being internal substructures of superclusters, tend towards these morphologies. This is consistent with the hierarchical growth these structures experience, as cores are primarily ‘nourished’ through the filaments to which they are connected. Filaments act as massive channels for transporting

matter in the Universe (e.g. Libeskind et al. 2018, and references therein), primarily along directions of maximum anisotropic gravitational attraction (e.g. Zeldovich 1970), in this case towards cores, which are the densest regions within superclusters.

On the other hand, pancake-like cores are less common in rich superclusters but represent another class of structures that can evolve into compact, virialised objects. Pancake cores are more prevalent in lumpy, less filamentary superclusters and are often isolated or disconnected from other parts of their host superclusters.

The predominantly anisotropic morphologies (flattened and elongated) of cores are indicative of their current quasi-linear dynamical stage, as predicted by the Zeldovich model (e.g. Araya-Melo et al. 2009). As cores evolve, their morphologies change, leading to various intermediate shapes over time. According to the Zeldovich formalism, which defines a morphological sequence linked to stages of an anisotropic gravitational collapse (see, e.g. Cautun et al. 2014), cores that are mostly filaments are the largest bound structures closest to becoming virialised objects, as they only need to collapse along a single axis (the longest of the three). In contrast, pancake-shaped cores still need to collapse along two axes before reaching virialisation. However, since they are already bound structures with significant overdensities, their collapse and future virialisation are inevitable.

The evolution of cores rarely occurs in isolation within the LSS. As previously noted, most cores are dense filaments or located at filament intersections within superclusters, constantly accreting matter from their surroundings. The boxplot in Figure 7 shows that filamentary cores ($0 \leq \mathcal{P}/\mathcal{F} \leq 1$) are relatively more massive than pancakes ($\mathcal{P}/\mathcal{F} > 1$). The mass distribution of filaments extends towards higher values, while pancake masses are distributed within a narrower, lower-mass range. Pancake-like cores are often surrounded by less dense regions, as evidenced by density contrast analyses and connectivity studies between cores and their host superclusters. As core richness increases with mass, these results align with Costa-Duarte et al. (2011), who found that filaments in superclusters tend to be richer than pancakes on average.

Finally, we preestimate the fraction of cores with approximately spherical shapes (i.e. $\mathcal{P} \approx \mathcal{F} \approx 0$). Allowing a tolerance of up to 0.05 in both $\mathcal{P}$ (planarity) and $\mathcal{F}$ (filamentarity), only $\sim 8\%$ of the DCC cores are approximately spherical based on ellipsoidal fits. None of these cores fall within this range when using polyhedral surface fits. Decreasing the tolerance further reduces the fraction of cores with comparable axes ($a \approx b \approx c$). The virtual absence of spherical objects underscores the relative youth of the cores, which may reflect the fact that the primordial density field did not contain spherical overdense regions (e.g. Bardeen et al. 1986; Kravstov & Borgani 2012), and that the early stages of contraction and gravitational collapse occur in strongly flattened and elongated geometries (e.g. Zeldovich 1970; Araya-Melo et al. 2009; Cautun et al. 2014).

5. Entropy analysis

According to the standard model of structure formation, gravitational collapse occurs anisotropically, progressing from flattened and elongated configurations to a more compact triaxial structure in a virialised state (e.g. Shandarin et al. 2004; Cautun et al.

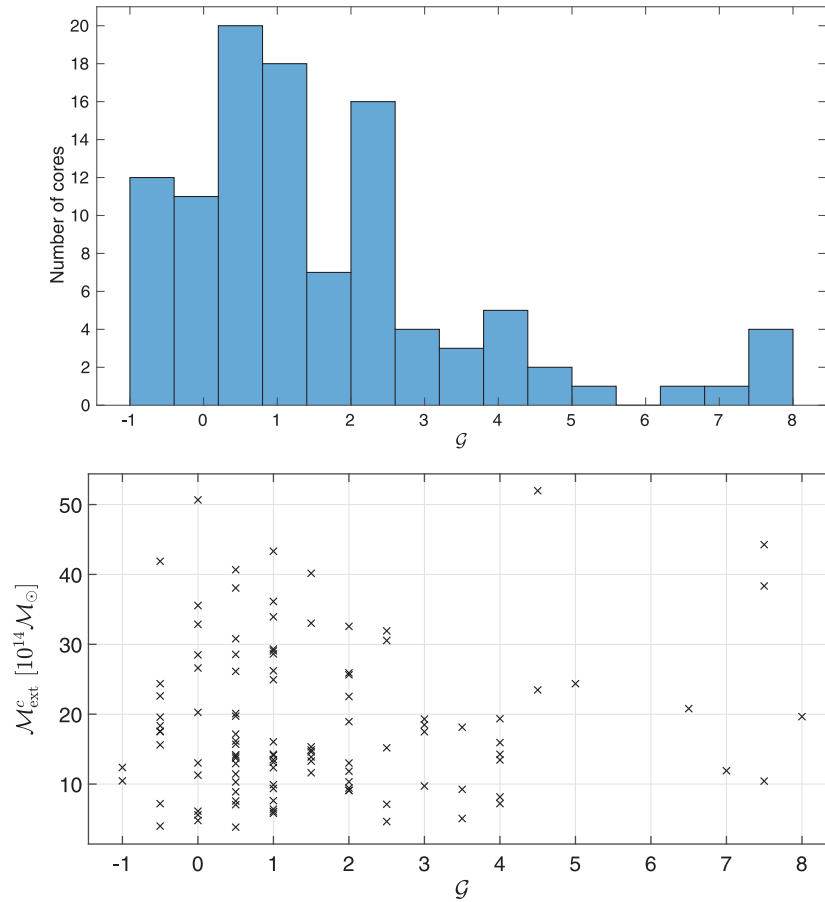


Figure 5. *Top:* Distribution of (genus) $\mathcal{G}$ values for DCC cores. *Bottom:* Distribution of $\mathcal{G}$ values versus the extensive mass $\mathcal{M}_{\text{ext}}^c$ of DCC cores.

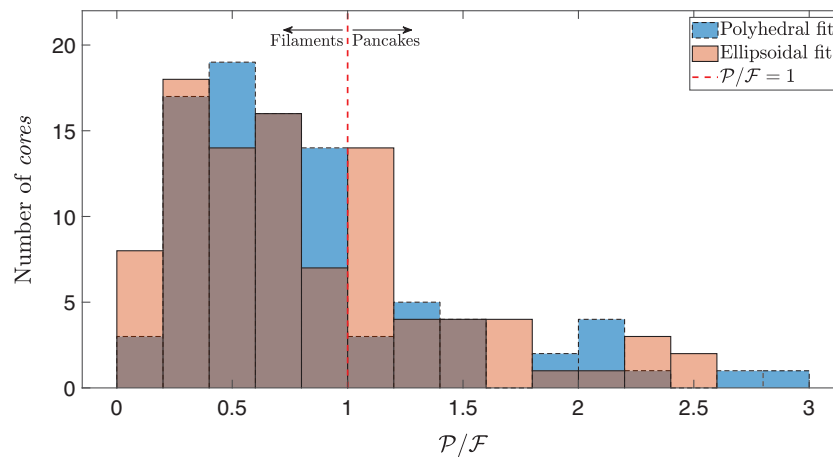


Figure 6. The ‘shape spectrum’ of DCC cores. Distribution of the shape statistic $\mathcal{P}/\mathcal{F}$ in the sample of cores excluding outliers and using two surface fit (polyhedral and ellipsoidal) methods. The structures are classified as filaments if $0 \leq \mathcal{P}/\mathcal{F} \leq 1$ or as pancakes if $\mathcal{P}/\mathcal{F} > 1$.

2014). This collapse along different axes leads to a continuous redistribution of mass, shaping the evolving density profiles of these structures (e.g. Araya-Melo *et al.* 2009). In the case of cores – highly overdense regions within superclusters – their internal

structure is primarily shaped by the spatial distribution of their member galaxy systems, as traced in optical observations. As cores dynamically evolve, it is expected that smaller groups accrete onto richer clusters, which in turn merge, gradually dissolving

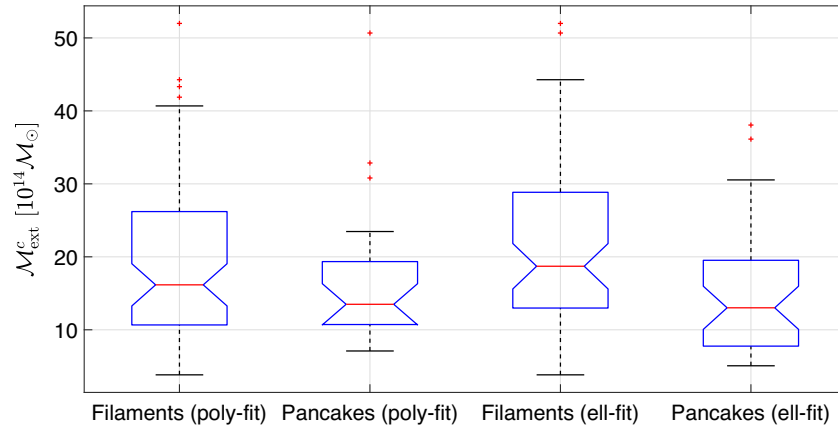


Figure 7. Boxplots of extensive mass $\mathcal{M}_{\text{ext}}^c$ for the two morphological classifications of DCC *cores*, filaments ($0 \leq \mathcal{P}/\mathcal{F} \leq 1$) and pancakes ($\mathcal{P}/\mathcal{F} > 1$), based on the polyhedral (poly-fit) and ellipsoidal (ell-fit) fits surfaces. Filaments have a relatively greater mass than pancakes.

substructures and leading to a more homogeneous (spatial and velocity) galaxy distributions similar to those in relaxed massive clusters. The most massive cluster (MMC) within each *core* could eventually become the main gravitational centre, around which the virialised structure of the *core* may form.

The evolution of structures in the Universe is shaped by a complex combination of fundamental physical laws and stochastic processes, often challenging our understanding (e.g. Lynden-Bell 1967; Saslaw 1980; Padmanabhan 1990; Binney & Tremaine 2008). This complexity increases further when considering environmental interactions, as matter and energy exchanges can significantly influence their dynamical evolution. One of the most significant principles influencing the evolution of any physical system is the second law of thermodynamics, which asserts that the entropy of an isolated system increases over time, reaching its maximum at equilibrium – the most evolved state possible for a system of given mass and energy. In this context, entropy directly correlates with the degree of dynamical evolution of a physical system, making it a relevant parameter for characterizing the dynamical state of the *cores* (or any other galaxy system or structure).

Entropy measures the randomness and absence of macroscopic motions or special configurations such as substructures within a system (e.g. Landau & Lifshitz 1980). In this sense, it indicates how close a system is to dynamical relaxation (equilibrium). However, entropy remains a state function related to the thermodynamic properties of macroscopic systems, which are still challenging to describe and interpret in systems dominated by gravity due to their unique behaviors (see Zúñiga et al. 2024b, for further discussion).

5.1. The H_Z -entropy estimator

To quantitatively characterise the evolutionary state of galaxy systems within an entropy-increasing framework, a continuous entropy estimator has been introduced (see Zúñiga et al. 2024b), defined as:

$$H_Z \equiv \ln \left(\frac{\mathcal{M}_{\text{vir}}}{\frac{4}{3}\pi R_{\text{vir}}^3 \rho_0} \right) + \frac{\beta}{2} \ln \left(\frac{\beta \sigma_v^2}{2 \sigma_v^2 \rho_0} \right), \quad (27)$$

where $\mathcal{M}_{\text{vir}}$, R_{vir} , and σ_v denote the virial mass, virial radius, and line-of-sight galaxy velocity dispersion of a system, respectively. The parameter β accounts for the velocity anisotropy of galaxies

($\beta = 3$ for Maxwellian distributions, e.g. Tully 2015). The constants $\rho_0 = (10^{14} \mathcal{M}_{\odot} / \text{Mpc}^3) h_{70}^2$ and $\sigma_{v_0} = 1 \text{ km s}^{-1}$ are fiducial reference values for density and velocity dispersion used to render the arguments of the logarithms dimensionless. The H_Z estimator is a physically motivated, dimensionless quantity that traces the net change in specific entropy (Δs) experienced by a system from its formation to the present epoch. Since it is defined in terms of (optical) observational parameters, it can be readily computed once a sufficient number of member galaxies are identified.

The H_Z estimator was first tested on a sample of 70 well-sampled galaxy clusters in the Local Universe (the *Top70* cluster sample,^c Caretta et al. 2023). The test results suggest a strong correlation between H_Z and the dynamical state of clusters (Zúñiga et al. 2024b). Specifically, H_Z correlates with the level of gravitational assembly, yielding lower values for dynamically younger, substructured clusters and higher values for more relaxed, evolved systems. Furthermore, H_Z exhibits a robust correlation with other continuous dynamical indicators derived from both optical and X-ray observations.

5.2. Entropy of cores

The use of the H_Z estimator can be extended to the study of structures larger than clusters due to self-similarity. Gravity-dominated systems are expected to follow a comparable evolutionary path on a global scale (e.g. simulation results indicate that supercluster-scale structures tend to evolve into very rich cluster-like systems, Araya-Melo et al. 2009). Based on its successful application to characterise the dynamical state of galaxy clusters (Zúñiga et al. 2024b), we propose here to extend the use of the H_Z estimator to larger structures, such as superclusters and their *cores*. To this end, we generalise the H_Z estimator as follows:

$$H_Z = \ln \left(\frac{\bar{\rho}}{\rho_0} \right) + \frac{\beta}{2} \ln \left(\frac{\beta \sigma_v^2}{2 \sigma_v^2 \rho_0} \right), \quad (28)$$

where $\bar{\rho}$ and σ_v represent the average mass density (e.g. $\bar{\rho} = \mathcal{M}_{\text{ext}}/V$) and the line-of-sight galaxy velocity dispersion of the structure under study, respectively. This generalisation avoids

^cSee www.astro.ugto.mx/recursos/HP_SCLs/Top70.html.

using virial parameters when estimating entropy for non-relaxed systems.

Using the average densities and galaxy velocity dispersions estimated for the *cores* (see Table 5 of Paper I and Table 1) and superclusters (see Table 3 of Paper I and Table 6), we computed the H_Z -entropy for each of these structures. The H_Z values obtained for the complete MSCC-supercluster sample are listed in column 6 of Table 6, while the H_Z values for DCC *cores* are shown in column 12 of Table 1.

To compare evolutionary states of galaxy systems and structures at different scales, we took the H_Z -entropy values for the *Top70* cluster sample from Table 2 of Zúñiga et al. (2024b). These clusters, among the best-sampled galaxy systems in the nearby Universe, belong to MSCC superclusters and cover a broad range from poor to rich systems (with ICM temperatures from 1 to 12 keV).

The top panel of Figure 8 displays boxplots of H_Z -entropy distributions for clusters, *cores*, and superclusters. It is evident that clusters exhibit higher entropies (median value of 15.4), consistent with systems near virial equilibrium. In contrast, superclusters show the lowest entropies (median value of 11.1), indicating their less advanced evolutionary state among the studied galaxy structures. Meanwhile, *cores* present intermediate entropy values (median value of 12.9), supporting the hypothesis that these regions are dynamically more evolved than their rich host superclusters.

5.3. Relaxation probability of cores

To complement the entropy-based approach, we evaluate the relaxation probability parameter, $\mathcal{P}_{\text{relax}}$, as defined in Zúñiga et al. (2024b). This parameter provides an independent method for assessing the dynamical state of galaxy structures based on the distribution of their member galaxies in phase space. virialised *cores* are expected to exhibit spatial and velocity distributions similar to those of relaxed galaxy clusters. These distributions are typically modeled using profiles derived from equilibrium assumptions, such as the King profile for radial distributions, uniform azimuthal distributions, and Gaussian velocity distributions (e.g. Saslaw & Hamilton 1984; Sarazin 1986; Adami et al. 1998; Sampaio & Ribeiro 2014).

The $\mathcal{P}_{\text{relax}}$ parameter is determined by comparing the observed probability density functions (PDFs) of galaxies within a *core* to equilibrium models. This approach mirrors that used in Zúñiga et al. (2024b) for galaxy clusters. Briefly, the raw observational coordinates of member galaxies, namely the triples (RA, Dec, z) of right ascension, declination, and redshift, are distributed within a solid angle that can be approximated by a cylinder with a circular base in the plane of the sky and depth along the line of sight. Within this cylinder, each galaxy's position can be expressed as (r, θ, z) , where r represents its projected distance from the *core* centroid, θ its azimuthal angle relative to the local north direction in the projected sky distribution, and z its redshift, serving as a proxy for radial velocity. Assuming statistical independence,^f the galaxy distribution in each *core* can be described by an empirical joint PDF, $\tilde{f}_{r\theta z} = \tilde{f}_r(r)\tilde{f}_\theta(\theta)\tilde{f}_z(z)$. Here, $\tilde{f}_r(r)$, $\tilde{f}_\theta(\theta)$, and $\tilde{f}_z(z)$

represent the observed PDFs for the radial- r , azimuthal- θ , and redshift- z variables, respectively.

To estimate these observed PDFs, we apply a smoothing technique using a kernel density estimator over normalized galaxy counts within bins of width $\Delta r = 0.35 h_{70}^{-1}$ Mpc, $\Delta\theta = 12^\circ$, and $c\Delta z = 250 \text{ km s}^{-1}$ for the r , θ , and z variables, respectively. These bin widths were chosen from a range of test values to optimise the fit of the distribution functions during histogram smoothing. A standard Gaussian smoothing kernel is applied using the same bin widths over the intervals $[0, R_h]$, $[0, 360^\circ]$, and $[z_{\min}, z_{\max}]$ for $\tilde{f}_r$, $\tilde{f}_\theta$, and $\tilde{f}_z$, respectively. Here, $z_{\min}$ and $z_{\max}$ are the minimum and maximum redshifts of the galaxies in the structure, and

$$R_h \equiv \frac{2N(N-1)}{\sum_{i \neq j} R_{ij}^{-1}}, \quad (29)$$

represents the harmonic radius of the structure calculated from the projected distances R_{ij} (in Mpc) between its N sampled galaxies (see column 10 of Table 1). Since $\tilde{f}_r$, $\tilde{f}_\theta$, and $\tilde{f}_z$ depend on the dynamical state of the structure, they describe the current distributions of its member galaxies.

To define the equivalent relaxed PDFs for each variable in each *core* (or supercluster), we adopt a simple reference equilibrium model. virialised *cores* are assumed to exhibit spherical galaxy distributions with a homogeneous core-halo configuration and isotropic velocities, lacking net angular momentum. For simplicity, we use the King-type radial, continuous uniform, and normal distributions $f_r^{\text{eq}}(r)$, $f_\theta^{\text{eq}}(\theta)$, and $f_z^{\text{eq}}(z)$ from Zúñiga et al. (2024b) as equilibrium models for the r , θ , and z variables, respectively. The similarity between observed and relaxed PDFs is quantified using the Hellinger distance ($0 \leq H(\tilde{f}_k, f_k^{\text{eq}}) \leq 1$; Hellinger 1909), where lower Hellinger distances indicate distributions closer to equilibrium, corresponding to higher relaxation probabilities since $\mathcal{P}_{\text{relax}} \equiv 1 - H$.

The $\mathcal{P}_{\text{relax}}$ values computed for DCC *cores* are shown in column 13 of Table 1. Additionally, the bottom panel of Figure 8 illustrates the $\mathcal{P}_{\text{relax}}$ distributions for the *Top70* clusters, DCC *cores*, and MSCC superclusters. For superclusters, these values were estimated similarly (see column 7 of Table 6), while the *Top70* cluster values are from Zúñiga et al. (2024b). Consistent with the H_Z -entropy analysis, the relaxation probability results also support the hypothesis that *cores*, with an average $\langle \mathcal{P}_{\text{relax}} \rangle = 0.64$, represent galaxy structures in an intermediate evolutionary state. Clusters, being the most relaxed structures, exhibit $\langle \mathcal{P}_{\text{relax}} \rangle = 0.83$, while superclusters, the least relaxed, have $\langle \mathcal{P}_{\text{relax}} \rangle = 0.56$.

6. Estimations of core virial masses

To quantify the mass content of *cores*, two complementary approaches were employed. First, the extensive mass $\mathcal{M}_{\text{ext}}^c$ (presented in Table 5 of Paper I) that provide a lower limit for the total mass of *cores*. This is because $\mathcal{M}_{\text{ext}}^c$ excludes contributions from the dispersed component^g and additional dark matter potentially present at the scale of these structures beyond that contained in clusters.

^fThis assumption is supported by the low fraction of galaxies exhibiting a strong correlation between r and z , which carry negligible statistical weight and show no significant correlations between r , z , and θ (see Zúñiga et al. 2024b).

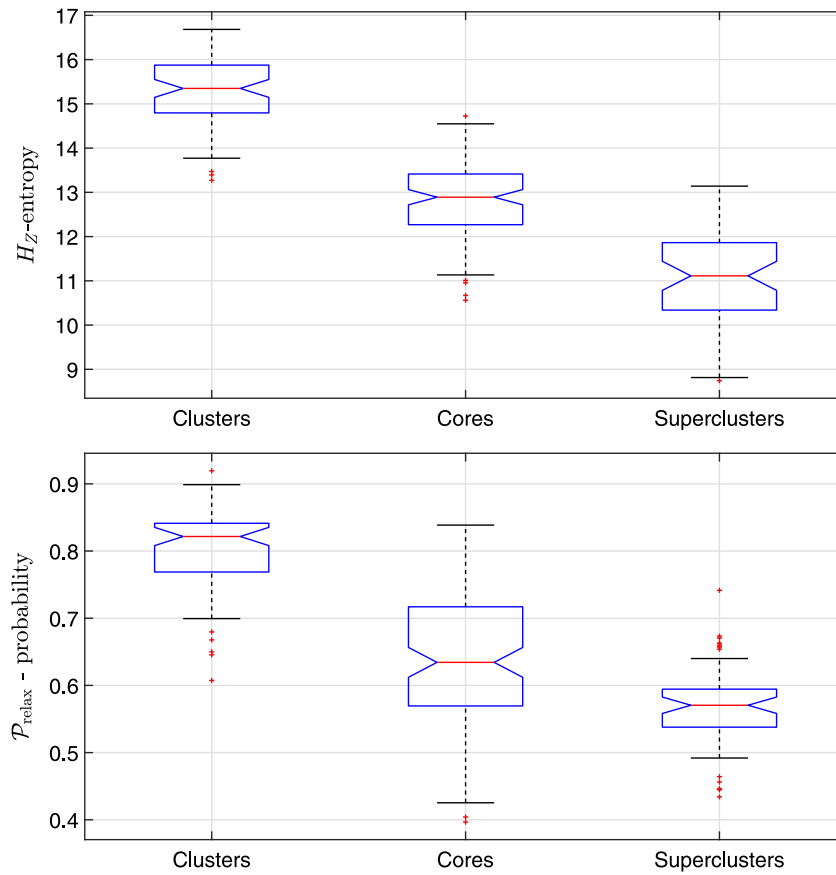
^gThe dispersed component refers to those galaxies that, while located within the *cores*, are not members of galaxy systems.

Table 6. General properties of the MSCC superclusters in the sample.

ID	$\mathcal{M}_{\text{ext}}^{\text{sc}}$	V_{sc}	$\rho_{\text{sc}} = \mathcal{M}_{\text{ext}}^{\text{sc}}/V_{\text{sc}}$	σ_v	H_z	$\mathcal{P}_{\text{relax}}$
MSCC	$(10^{14} h_{70}^{-1} \mathcal{M}_{\odot})$	$(10^3 h_{70}^{-3} \text{Mpc}^3)$	$(10^{10} h_{70}^2 \mathcal{M}_{\odot} \text{Mpc}^{-3})$	(km s^{-1})	(nats)	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	143.24	150.4	9.5	2 155	12.51	0.45
27	19.59	73.8	2.6	1 159	9.68	0.53
33	201.42	268.3	7.5	1 685	11.65	0.54
39	41.84	57.0	7.3	858	9.94	0.55
55	32.53	24.7	13.1	1 240	11.45	0.65
72	66.25	60.5	10.9	1 074	10.90	0.44
75	72.06	100.0	7.2	2 121	12.19	0.58
76	59.68	450.6	1.3	2 636	11.04	0.65
117	91.11	129.3	7.0	1 849	11.82	0.50
175	65.23	122.5	5.3	1 522	11.06	0.57
184	37.19	204.1	1.8	1 927	10.57	0.59
211	7.44	27.6	2.6	792	8.74	0.46
219	77.87	257.6	3.0	1 949	11.11	0.58
222	47.46	107.8	4.4	1 547	10.91	0.61
223	7.23	12.9	5.6	982	10.01	0.44
229	30.21	368.0	0.8	2 450	10.38	0.58
236	155.02	158.6	9.7	1 720	11.97	0.52
238	177.01	1412.2	1.2	4 230	12.16	0.63
248	16.37	197.8	0.8	2 012	9.89	0.57
264	16.77	202.4	0.8	1 760	9.56	0.55
266	43.29	29.5	14.6	1 136	11.33	0.52
272	23.93	36.3	6.5	813	9.70	0.74
277	103.33	103.8	9.9	1 602	11.81	0.57
278	107.07	268.7	3.9	1 625	10.93	0.57
283	80.42	290.3	2.7	1 408	10.21	0.49
295	159.78	153.8	10.3	938	10.51	0.55
310	251.39	351.5	7.1	3 108	13.14	0.66
311	142.23	294.4	4.8	1 468	10.87	0.55
314	26.85	3.4	78.5	1 104	12.94	0.57
317	48.89	77.2	6.3	2 745	12.70	0.55
323	106.49	174.0	6.1	1 646	11.39	0.54
333	33.33	12.8	25.9	932	11.41	0.53
335	55.74	57.8	9.6	1 731	11.97	0.56
343	66.17	21.8	30.3	1 121	12.03	0.43
360	39.88	299.3	1.3	1 142	8.95	0.49
386	77.39	146.9	5.2	1 895	11.5	0.59
389	171.33	191.0	8.9	2 062	12.34	0.67
407	12.81	92.1	1.3	1 593	9.83	0.54
414	298.68	242.0	12.3	2 285	12.91	0.59
419	96.06	70.5	13.6	1 287	11.57	0.58
422	7.35	199.9	0.3	1 806	8.81	0.49
430	48.33	95.3	5.0	1 505	10.98	0.66
440	83.57	244.3	3.4	1 581	10.71	0.57
441	15.35	130.7	1.1	1 205	8.96	0.56
454	172.79	249.3	6.9	1 720	11.63	0.67

Table 6. Continued.

ID	$\mathcal{M}_{\text{ext}}^{\text{sc}}$	V_{sc}	$\rho_{\text{sc}} = \mathcal{M}_{\text{ext}}^{\text{sc}}/V_{\text{sc}}$	σ_v	H_z	$\mathcal{P}_{\text{relax}}$
MSCC	$(10^{14} h_{70}^{-1} \mathcal{M}_{\odot})$	$(10^3 h_{70}^{-3} \text{Mpc}^3)$	$(10^{10} h_{70}^2 \mathcal{M}_{\odot} \text{Mpc}^{-3})$	(km s^{-1})	(nats)	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
457	173.07	177.1	9.7	1 348	11.36	0.60
460	169.83	163.9	10.3	1 490	11.67	0.58
463	312.36	326.1	9.5	2 194	12.56	0.65
474	120.34	64.2	18.7	1 182	11.68	0.63
484	24.74	4.4	56.3	556	10.90	0.54
509	39.05	176.9	2.2	2 029	10.89	0.57
574	104.17	412.6	2.5	3 349	12.28	0.55
579	24.52	121.2	2.0	1 201	9.50	0.53

**Figure 8.** Distributions of H_z -entropy (top panel) and probability $\mathcal{P}_{\text{relax}}$ (bottom panel) values for clusters (the *Top70* sample), cores (from DCC), and superclusters (from MSCC).

The second approach estimates *core* masses using the virial mass estimator for clusters (e.g. Biviano et al. 2006):

$$\mathcal{M}_{\text{vir}}^c = \frac{\beta\pi}{2G} \sigma_v^2 R_h. \quad (30)$$

These calculations were based on the harmonic radii R_h and line-of-sight velocity dispersions σ_v of the galaxy distributions within the *cores* (see columns 5 and 10 of Table 1), assuming weak anisotropy $\beta = 2.5$. The resulting $\mathcal{M}_{\text{vir}}^c$ values, shown in column 11 of Table 1, provide an estimate of the total dynamical mass.

However, given that *cores* are not fully virialised structures, the computed $\mathcal{M}_{\text{vir}}^c$ may deviate from the true values, particularly in the presence of substructures (e.g. Biviano et al. 2006). The most reliable estimates arise when *cores* approach dynamical relaxation.

Figure 9 illustrates the relationship between $\mathcal{M}_{\text{vir}}^c$ and $\mathcal{M}_{\text{ext}}^c$ for the DCC *cores*. The correlation between these parameters is quasi-linear, with a Pearson correlation coefficient of 0.93. This trend suggests that most of the matter within *cores* resides in their member galaxy systems. The dynamical evolution of *cores* appears to be driven by the gravitational influence of these systems, which

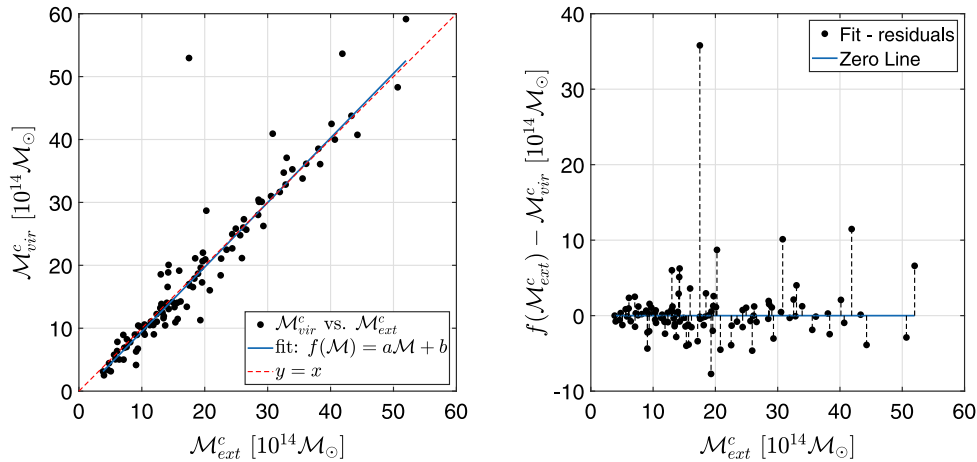


Figure 9. Relationship between the extensive masses $\mathcal{M}_{ext}^c$ and the virial masses $\mathcal{M}_{vir}^c$ (in units of h_{70}^{-1}) of the DCC cores. *Left:* $\mathcal{M}_{vir}^c$ vs. $\mathcal{M}_{ext}^c$ plot. The solid blue line represents the best linear fit $f(\mathcal{M}) = a\mathcal{M} + b$ to the data, with $a = 1.027$, $b = -0.917$, and a goodness of fit $R_{square} = 0.995$. *Right:* residual plot. The length of the vertical dashed lines expresses the distance between the data and the fit (zero) line.

act as attractors, accreting surrounding matter such as individual galaxies, gas, and dark matter. This mechanism explains why the dispersed component of galaxies and external dark matter contribute minimally to the total dynamical mass. Consequently, the relationship $\mathcal{M}_{ext}^c \approx \mathcal{M}_{vir}^c$ generally holds, indicating that, before a single virialised structure is formed, all the matter in the *cores* falls into the potential wells of their member systems, which dominate the macroscopic dynamics towards equilibrium.

One notable outlier, corresponding to DCC 019 (a *core* of MSCC 222), deviates significantly from the linear trend. Unlike other structures such as DCC 016 and DCC 077, which exhibit pairwise gravitational binding among some member systems, DCC 019 appears unbound under any of the analyses conducted in Paper I. Its virial mass is significantly overestimated due to this lack of cohesion. Nonetheless, DCC 019 was included in the DCC catalogue owing to its high density contrast ($\mathcal{R} = 46.6$, $\Delta_{cr} = 14.1$), suggesting a high probability of eventual binding and virialisation.

7. Discussion and conclusions

The results presented in this study provide a comprehensive exploration of the physical and dynamical properties of *cores* within the large-scale structure of the Universe. By analyzing their – projected and velocity – galaxy distributions, morphology, entropy, and mass estimates, we have deepened our understanding of these intermediate structures and their role in the cosmic web. Below, we summarise and discuss the key findings with respect to these aspects.

The study of the galaxy distributions and velocity dispersions within *cores* shows that these structures are still far from being isotropic or dynamically relaxed. The radial velocity distributions indicate a mix of bound and infalling galaxies, supporting the view that *cores* are transitional regions between virialised galaxy clusters and unbound superclusters. These dynamical states underscore the importance of the local gravitational potential and environmental effects in shaping galaxy orbits within *cores*.

The morphological analysis of *cores* reveals a diversity in their shapes and internal configurations, ranging from irregular and flattened structures to elongated and nearly spherical distributions (less frequent). Notably, a significant proportion of *cores*

exhibit filamentary morphologies, consistent with theoretical predictions from the Zeldovich approximation, which describes the anisotropic collapse of structures along preferred axes in the cosmic web. These filamentary configurations suggest that many *cores* are in an intermediate evolutionary stage, where matter accretion and interactions along the connected filaments dominate their dynamics. The influence of the large-scale environment is further evident in the presence of substructures, which reflect recent mergers, accretion processes or ongoing dynamical evolution. These findings underscore the role of morphology as a diagnostic tool for understanding both the relaxation state and the formation history of *cores*.

We note, however, that part of the observed tendency towards filamentary morphologies could, in principle, arise even in randomly distributed galaxy samples when analysed with the same polyhedral and ellipsoidal fitting procedures. Although a quantitative assessment of this effect lies beyond the scope of the present work, such a comparison would provide a useful benchmark for future studies aimed at disentangling intrinsic morphological features from those induced by statistical fluctuations or projection effects.

The entropy analysis reveals a dynamical evolution of cosmic structures, with entropy values increasing from superclusters to *cores* and galaxy clusters. This evolutionary trend, as reflected in the distributions of our entropy estimator H_Z , aligns with the hierarchical formation model, in which structures evolve anisotropically and gradually move towards dynamical relaxation. Clusters exhibit the highest median entropy – and relaxation probability – consistent with their more advanced evolutionary stage and near-virialised states. In contrast, superclusters, still in early collapse stages and characterized by strong substructure and anisotropy, display the lowest median entropy values. The intermediate entropy values observed for *cores* suggest that they occupy an evolutionary stage between their less-evolved supercluster hosts and highly evolved galaxy clusters. This interpretation is consistent with the fact that *cores*, as overdense regions within superclusters, are dynamically evolving structures where member galaxy systems (clusters and groups) undergo continuous merging and accretion of smaller ones, gradually dissolving substructures. Moreover, the generalized H_Z estimator highlights the importance of average density and velocity dispersion in quantifying the

entropy of non-virialised systems, offering a practical and robust tool for exploring the dynamical states of large-scale structures.

Mass estimates for *cores* were obtained using both extensive mass calculations, based on the sum of the virial masses of member galaxy systems, and virial mass estimates derived from the velocity dispersion and projected radii of galaxies. The strong correlation between these mass estimates ($\mathcal{M}_{\text{ext}}^c \approx \mathcal{M}_{\text{vir}}^c$) suggests that most of the matter in *cores* is contained within their galaxy systems, with limited contributions from dispersed components or additional dark matter. This result supports the idea that *cores* are dominated by their constituent systems, which drive the local dynamics.

Taken together, these findings provide a unified picture of *cores* as dynamically evolving structures that bridge the gap between smaller, virialised systems and the larger, globally unbound supercluster environment. The interplay between gravitational forces, matter accretion, and dynamical relaxation governs their evolution, making them key sites for studying the hierarchical assembly of cosmic structures.

The main conclusions of this work are:

- More than half of the *cores* exhibit projected galaxy distributions consistent with the King density profile, indicating a tendency to evolve towards core-halo structures. This highlights their intermediate evolutionary status, as they begin to resemble the density profiles of more relaxed clusters.
- Although *cores* are gravitationally bound structures, analysis of the spatial and velocity distribution of their member galaxies reveals that these structures are not yet dynamically relaxed. Only about 30% of the studied *cores* show line-of-sight velocity distributions consistent with an underlying normal distribution, while the remaining $\sim 70\%$ do not. Since a relaxed Maxwellian distribution of galaxy velocities in three dimensions requires each velocity component to follow a normal distribution, the lack of Gaussianity in the radial velocities suggests that *cores* have not yet reached dynamical equilibrium, as expected.
- The velocity dispersions of *cores* are systematically lower than those of rich galaxy clusters, even though they have comparable or greater masses. This reflects the fact that *cores* are not virialised systems, so they do not exhibit a direct mass-velocity relationship like relaxed clusters.
- Morphological studies reveal that *cores* are mainly filamentary structures which, according to the Zeldovich approximation for large-scale structure growth, are currently in a quasi-linear dynamical stage of evolution. Given their high density contrasts, *cores* have a high probability of undergoing gravitational collapse along their remaining uncollapsed dimension (i.e. the last axis of contraction), eventually transforming into virialised structures.
- The morphology of *cores* exhibits significant diversity, reflecting their dynamical states and formation histories. Many *cores* display anisotropic accretion patterns and substructures, which suggest that they are still in the process of consolidating mass and transitioning towards more relaxed configurations.
- The *cores* are in a transition stage between structures in the linear evolutionary phase and fully developed virialised objects in the nonlinear stage. These results are supported by the entropy and relaxation probability analyses, which confirm the hypothesis that the *cores* are structures in an

evolutionary stage intermediate between superclusters and relaxed rich clusters.

- This work emphasises the relevance of entropy as a diagnostic tool for understanding the dynamical state and evolutionary history of cosmic structures. It provides a quantitative framework to compare galaxy systems and larger structures across different scales and stages of evolution.
- Mass estimates indicate that most of the matter within *cores* is concentrated in their member galaxy systems. This highlights the efficiency of accretion processes in consolidating matter into gravitationally bound structures capable of surviving cosmic expansion.
- Within most *cores*, member galaxy systems may have cleaned up their surroundings, accreting the surrounding matter (e.g. individual galaxies, gas, and dark matter). This would explain why neither the dispersed component of galaxies nor the dark matter outside the clusters contribute significantly to the total dynamical mass, so that in general $\mathcal{M}_{\text{ext}}^c \approx \mathcal{M}_{\text{vir}}^c$ holds.

This study contributes to the broader understanding of *cores* within the cosmic web and lays the groundwork for future investigations into their role in galaxy evolution and large-scale structure formation.

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Data availability statement. Data resulting from the present work are available in the manuscript, both printed and digitally, and codes can be requested to the corresponding author.

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