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CORRESPONDENCE

The Editor,

Journal of Glaciology

SIR, *Dielectric relaxation in temperate glaciers: comments on Dr P. W. F. Gribbon's paper*

Gribbon (1967) measured the electrical capacitance between two wires laid on the surface of a glacier and related the results to changing ice properties with depth. The theory of the experiment (in section 3.1 of his paper) and the derivation of relaxation frequency from "Cole–Cole" plots are in error and we show here how a better formulation of the problem raises doubts about his conclusions.

We may determine the capacitance C per unit length between two infinitely long cylinders of radius a, separation h, and distance d from the plane boundary between two dielectrics of relative permittivity ϵ_1 and ϵ_2 by simple electrostatic theory (see Fig. 1). To a good approximation

$$C = \pi\epsilon_0\,\epsilon_1\left[\ln\frac{h}{a}+\frac{\epsilon_2-\epsilon_1}{\epsilon_2+\epsilon_1}\ln\frac{d+(d^2-a^2)^{\frac{1}{2}}}{(h^2+4d^2)^{\frac{1}{2}}}\right]^{-1} \tag{1}$$

valid for positive values of d greater than, or equal to a.

The vacuum capacitance per unit length between the cylinders is $C_{\text{vac}} = \pi\epsilon_0/\ln(h/a)$, so we define an apparent permittivity by the ratio of the measured capacitance to the vacuum capacitance.

$$C = \epsilon_{\text{app}}\,C_{\text{vac}}. \tag{2}$$

1. THE EFFECT OF THE GLACIER SURFACE

For cylinders close to the boundary, that is d much less than h, Equation (1) reduces to

$$C = \pi\epsilon_0\,\epsilon_1\left[\frac{2\epsilon_1}{\epsilon_1+\epsilon_2}\ln\left(\frac{h}{a}\right)+\frac{\epsilon_2-\epsilon_1}{\epsilon_2+\epsilon_1}\cosh^{-1}\left(\frac{d}{a}\right)\right]^{-1}. \tag{3}$$

Let the boundary be the surface separating the air from a homogeneous glacier and consider two experimental situations:

(I) The wire electrodes resting on, or above, a wet melting glacier as in Gribbon's measurements from Greenland. Then $\epsilon_1 = 1$ and $\epsilon_2 = \epsilon$.

(II) The wire electrodes resting on, or melting into, a cold *névé* as in France. Then invert Figure 1, and $\epsilon_1 = \epsilon$ refers to the *névé* and $\epsilon_2 = 1$ refers to the air above.

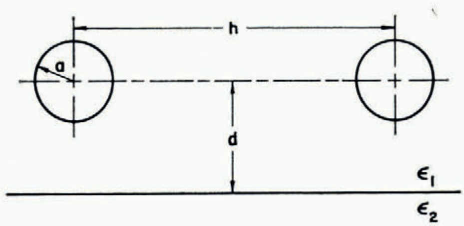

Fig. 1. Geometry of the electrodes

In neither case can the capacitance be represented by a constant geometrical factor times the glacier permittivity except that when the cylinders are in contact with the boundary, $d = a$ and the apparent permittivity

$$\epsilon_{\text{app}} = (\epsilon+1)/2 \tag{4}$$

in both (I) and (II). This is the same value as is obtained when the cylinder lies symmetrically in the boundary surface.

Now suppose the glaciers behave as ideal Debye dielectrics with properties tabulated below.

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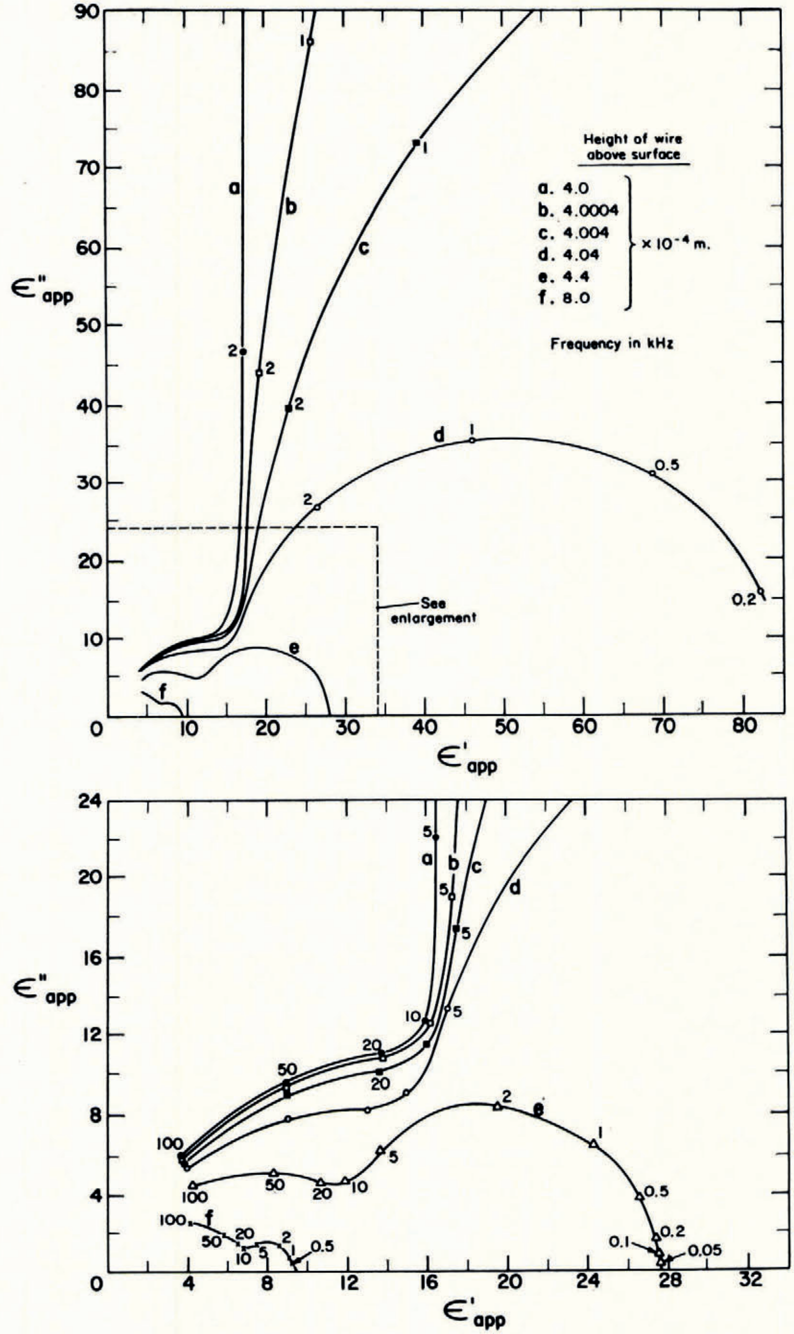

Fig. 2. Cole–Cole plots of the real and imaginary parts of apparent permittivity for wires raised above the surface of a warm wet glacier

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TABLE I. PARAMETERS FOR THREE TYPES OF ICE OR NÉVÉ

Location	*Greenland, surface*	*France, surface*	*Deep ice*
Temperature, °C	0	−22	0
Density, g cm^{-3}	0.77	0.46	0.92
Limiting permittivities			
static, ϵ_s	33.8	12	95
high frequency, ϵ_∞	2.8	2	3.17
Relaxation frequency f_r, kHz	18.3	4	8
Conductivity σ, Ω^{-1} m^{-1}	10^{-5}	10^{-7}	10^{-6}
Formzahl u	10	10	(homogeneous)

The wire radius a is given as 4×10^{-4} m, and let us consider a separation h of 80 m.

The Cole–Cole diagrams of the apparent permittivity, ϵ_{app}, for raised wires, for buried wires, and for several heights d have been plotted and the results can be summarized as follows:

(I) *Raised wires* (Fig. 2)

The low frequency points fall on the arc of a circle. This is a Maxwell–Wagner polarization phenomenon. A circle results since at frequencies less than one-tenth of f_r the dielectric has constant permittivity and conductivity. The low frequency limit of capacitance

$$C_s = \pi\epsilon_0/\cosh^{-1}(d/a) \tag{5}$$

which is the capacitance between the wire and its image in a perfect conductor.

An arc of another circle may be made to fit the high frequency values, but the apparent relaxation frequency f_m (where ϵ'' is a maximum), is higher than the true value, and if no allowance is made for the conductivity (see later) then

$$f_m/f_r = 1 + \frac{\epsilon_s - \epsilon_\infty}{\epsilon_\infty + 2C_s/C_{vac}}, \tag{6}$$

which gives apparent relaxation frequencies shown in Table II.

TABLE II. APPARENT RELAXATION FREQUENCIES FOR WIRES 80 m APART OVER PURE ICE WITH NEGLIGIBLE CONDUCTIVITY AND A TRUE RELAXATION FREQUENCY OF 8.0 kHz

d/a	f_m
1.0001	8.3 kHz
1.001	9.3 kHz
1.01	12.8 kHz
1.1	20.6 kHz

(II) *Buried wires* (Fig. 3)

The apparent permittivity $\epsilon_{app} = (\epsilon + 1)/2$ for wires buried within 1 cm of the surface and the apparent relaxation frequency is the same as that of the homogeneous glacier to a good approximation.

2. DETECTION OF DISCONTINUITIES WITHIN THE GLACIER

Because of the uncertainties described in the previous section, we now imagine the wire electrodes to be buried at such a depth in the glacier that the air surface does not affect the capacitance, and we consider instead a surface of discontinuity between upper and lower layers of ice. Let ϵ_1 refer to the surface layer, and ϵ_2 refer to deep ice with the properties tabulated in column 3 of Table I. For d much greater than a,

$$\epsilon_{app} = \epsilon_1 \left[1 + \frac{\epsilon_1 - \epsilon_2}{\epsilon_1 + \epsilon_2} \cdot \frac{\ln(1 + (h/2d)^2)}{2\ln(h/a)}\right]^{-1}. \tag{7}$$

Taking the radius of the wire to be 4×10^{-4} m and the separation 80 m as before, then Figure 4 shows a Cole–Cole plot of apparent permittivity for a wet (Greenland) surface and Figure 5 for the cold *névé* surface (as in France).

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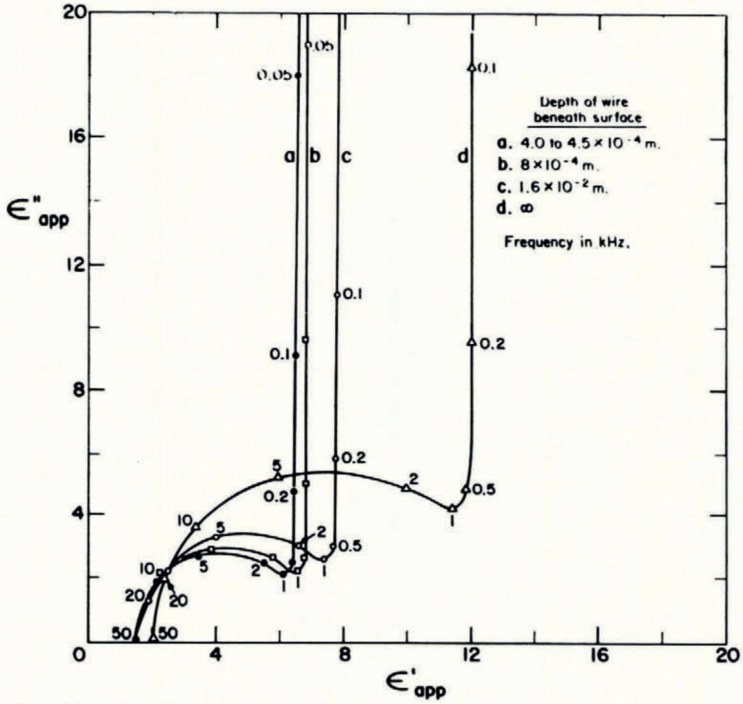

Fig. 3. Cole–Cole plots of the real and imaginary parts of apparent permittivity for wires buried beneath the surface of a cold névé

The result to be noted is that even if the ice layer is only 1 m below the wires, the apparent permittivity is essentially that of the surface layer, for high frequencies. However, at low frequencies the apparent conductivity tends towards the conductivity of the deep ice and relaxation frequencies deduced from either Cole–Cole plots or the variation of conductivity will be wrong.

3. Effect of Conductivity

With a high d.c. conductivity σ, the high-frequency values fall onto an arc which gives a false impression of the static permittivity. Following Gränicher and others (1957) the value obtained is

$$\epsilon_\infty + \Delta\epsilon(s+1)^2 \tag{8}$$

where $\Delta\epsilon = \epsilon_s - \epsilon_\infty$ and $s = \sigma/2\pi f_r \epsilon_0 \Delta\epsilon$.

If the maximum value of ϵ'' occurs at a frequency f_m then

$$f_m/f_r = [2/(s+1)^2] - 1 \tag{9}$$

and the observed value of f_m will differ from f_r as shown in Table III.

Table III. Frequency f_m for Maximum Loss Factor ϵ'' for the Deep Ice in Table I, but Various Conductivities

Conductivity σ, $\Omega^{-1}\,m^{-1}$	0	10^{-6}	5×10^{-6}	10^{-5}	1.65×10^{-5}
Parameter s, defined in Equation (8)	0	0.025	0.125	0.25	$\sqrt{2}-1$
Frequency for maximum f_m, kHz	8	7.2	4.6	2.2	—
Nature of maximum	real	real	point of inflection	extrapolated	extrapolated

For measurements covering a wide frequency range the best value of the relaxation frequency would be determined by plotting the conductivity against frequency and obtaining the frequency for which $\sigma = (\sigma_s + \sigma_\infty)/2$ where σ_s and σ_∞ are the static and high-frequency limiting values.

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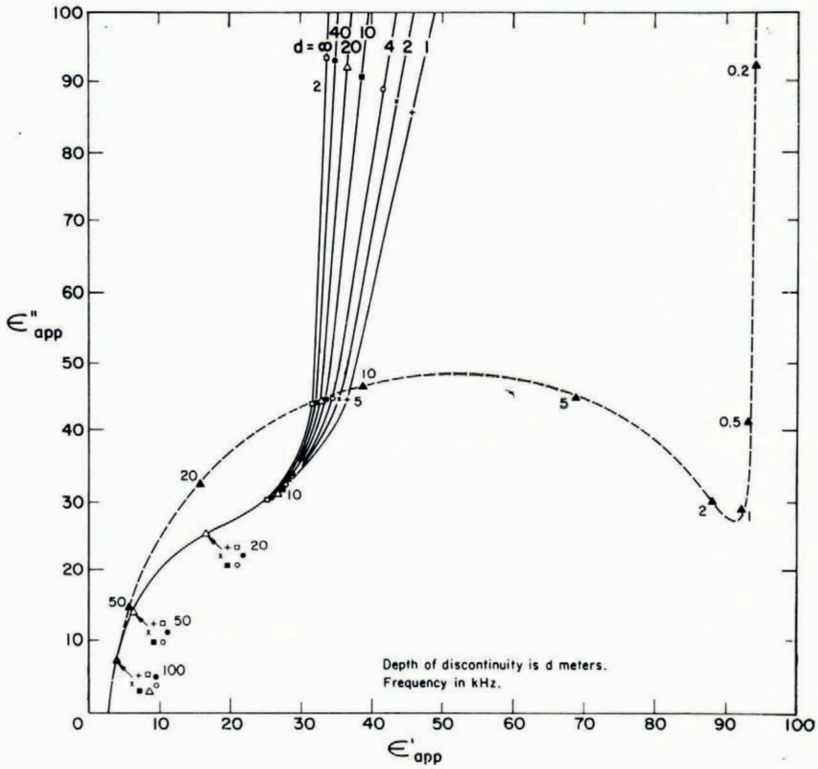

Fig. 4. Cole–Cole plots of the real and imaginary parts of apparent permittivity for warm wet ice overlying deep pure dense ice. Superimposed is a plot for deep ice alone

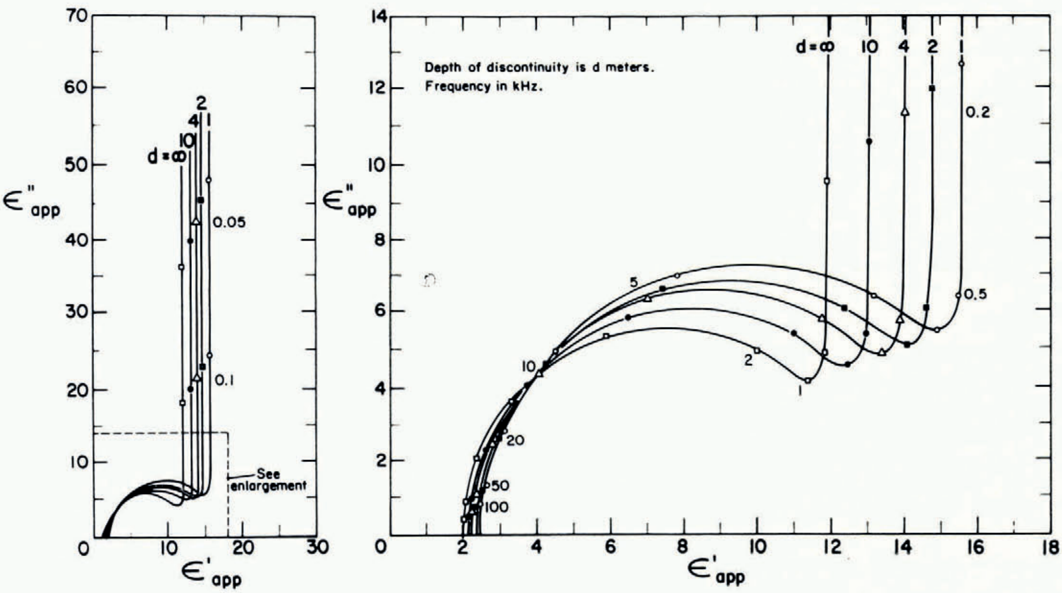

Fig. 5. Cole–Cole plots of the real and imaginary parts of apparent permittivity for cold névé overlying dense warm ice

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4. Interpretation of Gribbon's Results

The general result of this discussion is that it is possible to obtain information only about the surface layers of the glacier. For Gribbon's wires lying near the surface and melting into the glacier in places, situations (I) and (II) could occur in parallel. Then we expect the apparent relaxation frequencies to be higher than the true values, and the f_m and α values (his figures 6 and 7) to be random, depending on surface configurations.

However, Gribbon's f_m values are lower than currently accepted values for pure ice (Auty and Cole, 1952) and snow (Ozawa and Kuroiwa, 1958) and impurities tend to increase the relaxation frequency (Gränicher and others, 1957) so we suppose that this is due to his method of obtaining f_m in the presence of high d.c. conductivity.

High conductivity snow exhibits an increased static permittivity (Ozawa and Kuroiwa, 1958) and this cannot be fitted to the Debye equations. However, the calculations given here for ideal snows are altered only slightly, and it may be found that $\alpha > 0$ at high frequencies, and lower f_m values may be obtained than with the ideal Debye snow.

Probably, Gribbon's figures 3 and 4 do not show pronounced surface effects. If we determine the relaxation frequency by the conductivity method the results are both higher than 15 kHz and a more lengthy analysis suggests that the relaxation frequency is greater than 20 kHz for Gribbon's figure 5.

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15 September 1967

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Sir, *Discussion of the theory of pingo formation by water expulsion in a region affected by subsidence*

A novel theory of pingo formation has recently been proposed by R. C. Bostrom in the *Journal of Glaciology*, Vol. 6, No. 46, 1967, p. 568–72. According to Bostrom, "Pingos are of sparse occurrence in the Arctic as a whole but they occur in hundreds in the Mackenzie River delta. In a region of subsidence, as recent sediments pass through the base of permafrost, compaction becomes possible. The resulting water expulsion produces an artesian head responsible for building pingos" (p. 568). As this theory is completely at variance with the closed-system theory for the Mackenzie type of pingo (Porsild, 1938; Müller, 1959; Shumskiy, 1959, p. 17–27) and implies pingo concentrations in other permafrost regions affected by subsidence, the hypothesis is discussed below.

Pingo Distribution

The distribution of pingos in Northwest Territories, Canada, is shown in Figure 1, which is in turn generalized from the writer's map referred to by Bostrom (Mackay, 1962, fig. 1). The hundreds of pingos, discussed by Bostrom, are obviously those of group A. These pingos have grown in Pleistocene (or older) sediments in a rolling glaciated terrain with altitudes rising to more than 200 ft (61 m) above sea-level. These are the classic Mackenzie type closed-system pingos. These pingos are *not* in: the "Mackenzie River delta"; the "present delta surface"; the "alluvial surface"; or in an area where "sediment is added at the surface by the waters of the Mackenzie River". The deposits may pre-date the Illinoian glacier advance.

Group B comprises a little-studied pingo cluster which *is* located in the Mackenzie Delta. These diminutive Delta pingos differ in size, origin and numbers from the group A pingos of the Pleistocene area (Mackay, 1963, p. 88–94; Mackay and Stager, 1966).

Therefore: