Trans-oceanic Passages by Rhumbline Sailing

Captain Ivica Tijardović

1. INTRODUCTION. In this paper it is suggested that a practical method of trans-oceanic navigation between two positions in middle latitudes is to follow a track consisting of two legs. One leg is a rhumbline track from the position nearer the equator to the latitude of the position further from the equator. The other is along the parallel of the position further from the equator. Such a composite track is illustrated in Figure 1. The problem is to find the rhumbline course such that the total distance on the two legs is a minimum, and this is considered in the following section.

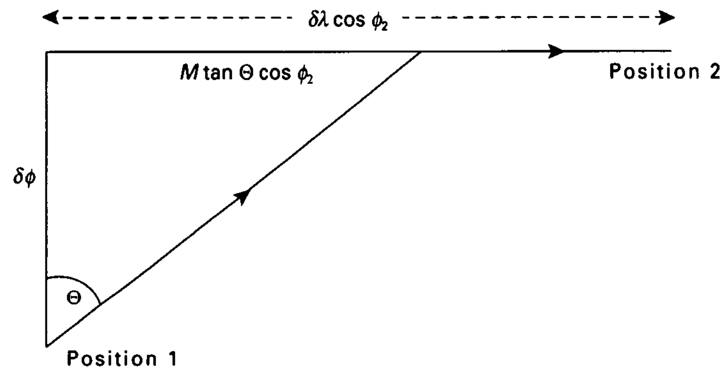

Fig. 1. Voyage legs

2. ANALYSIS. Referring to Figure 1, the distance, D, from position 1 to position 2 is the sum of the rhumbline leg and the parallel sailing leg, so that:

$$D = \delta\phi \sec\theta + (\delta\lambda - M \tan\theta)\cos\phi_2 \tag{1}$$

where

$\delta\phi$ is the difference in latitude between positions 1 and 2 (minutes of arc).
$\delta\lambda$ is the difference in longitude between positions 1 and 2 (minutes of arc).
M is the difference in meridional parts between positions 1 and 2.
ϕ_2 is the latitude of the second position.
θ is the rhumbline course.

To find the value of θ which gives the minimum distance, we differentiate the expression for D with respect to θ and equate it to zero. Thus:

$$\delta\phi \tan\theta \sec\theta - M\cos\phi_2 \sec^2\theta = 0$$

$$\delta\phi \sin\theta - M\cos\phi_2 = 0$$

$$\sin\theta = \frac{M\cos\phi_2}{\delta\phi} \tag{2}$$

Using the example of a voyage from position 1 in 35° N, 140° 30′ E to position 2 in 46° 20′ N, 124° 30′ W, we have:

$$\delta\phi = 680' \qquad \delta\lambda = 5700'$$
$$\phi_2 = 46°\ 20' \quad M = 896{\cdot}63'$$

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Substituting these values in equation 2, gives:

$$\sin\theta = \frac{896{\cdot}63 \cos 46^\circ 20'}{680}$$

$$\theta = 65^\circ 34'$$

Substituting in equation 1, gives:

$$D = 680 \sec 65^\circ 34' - 896{\cdot}63 \cos 46^\circ 20' \tan 65^\circ 34' + 5700 \cos 46^\circ 20'$$
$$= 4216{\cdot}9 \text{ n.m.}$$

Note that the values of D for $\theta = 60^\circ$ and $\theta = 70^\circ$ are 4223·3 and 4222·8 respectively, which confirms that the above value of 4216·9 is a minimum.

3. CONCLUSION. This method is applicable when the vertex of a great circle is between the position of departure and the destination and where the destination is nearer the pole. The ship follows a rhumbline to the destination parallel and thence steers due east or west to the destination. The two legs of the track may be followed in the reverse direction when the departure position is nearer the pole.

The method makes possible considerable savings in relation to direct rhumbline sailing and requires only one alteration of course. There are a number of additional points to note:

(i) The method is applicable on a mercator chart.

(ii) The overall distance is shorter than a direct rhumbline and, in practice, can be shorter by over a hundred miles. The saving is about 160 miles in the example given.

(iii) The distance is longer than by a great circle track but that may lead a ship to higher latitudes than is desirable and requires frequent alterations of heading.

(iv) The calculations required are very simple and become trivial when using a computer or even a hand-held calculator.

It is hoped that the simplicity and practicality of the method described in this paper will prove attractive to shipmasters planning oceanic passages.

KEY WORDS

1. Marine navigation. 2. Voyage planning.