About half the problems are variants of problems I have seen elsewhere. At this level it is not easy to be both entertaining and original. Nevertheless even the old favourites appear in acceptable new dress and cannot be dismissed out of hand even by the puzzle-addict. The other half are new to me and involve mathematical ideas that do not feature too often in puzzles. I particularly enjoyed "Bowling Averages" with its use of inequalities and "Wrapping" which makes geometrical sense of what shopkeepers have been doing for years.

The presentation perhaps encourages cheating in that each problem is immediately followed by a very full solution and exposition, but this in itself is far more than half the interest of the book as each problem is placed by the author in its mathematical context, sometimes with ideas for further hard generalizations.

The style is easy to read and in the solutions makes one wonder what the difficulty was—a rare gift. The printing and diagrams are clear with few misprints.

It is an entertaining and well-presented book which would be a good addition to the library of puzzle-collectors and an acceptable present for anyone interested in recreational mathematics.

MAGNUS PETERSON

BAKER, ALAN, *A concise introduction to the theory of numbers* (Cambridge University Press, 1984), xii + 95 pp. £15 cloth, £4.95 paper.

The adjective "concise" is an accurate description of this excellent book. A very bright student could read it without help, but the average honours student will require considerable assistance in following several of the arguments given. As a text for a lecturer to expand on it is admirable. After the usual preliminaries, there are chapters on quadratic residues, forms and fields, and on Diophantine approximation and equations. In the 91 pages of text a surprisingly large amount of material is covered. At the end of each chapter the reader is brought up to date by accounts of recent developments, in many of which the author has played a leading part.

I found the last chapter particularly interesting. In it the equations of Pell, Mordell ($y^2 = x^3 + k$), Fermat, and Catalan ($x^p - y^q = 1$) are discussed. In 15 pages a complete account of these problems is, of course, impossible, but the author has been remarkably skilful in conveying an understanding of the difficulties involved by his comments and choice of examples.

There are exercises at the end of each chapter, some of considerable difficulty. The book is beautifully printed, as one would expect. The only misprint I found was in the index, where, as one knows from experience, printers find it hard to believe that Lebesgue does not have a *q* in place of its *g*.

R. A. RANKIN

CONWAY, J. B. *Subnormal operators* (Research Notes in Mathematics 51, Pitman, 1981), xvii + 476 pp. £15.75.

Of the various special classes of Hilbert space operators which have been studied, normal operators are probably the best understood. Up to unitary equivalence, they are just the operators on L^2 spaces given by multiplication by bounded measurable functions and they can be classified in measure theoretic terms. In 1950, P. R. Halmos generalised the notion of normality by introducing the class of subnormal operators. These are the operators which have normal extensions; that is, an operator S on a Hilbert space H is subnormal if there is a normal operator N on a Hilbert space K containing H such that H is N-invariant and S is the restriction of N to H. The motivating example was the unilateral shift, which is defined as multiplication by z on the Hardy space H^2 and has an obvious normal extension acting on L^2 of the circle.

The early developments in the theory of subnormal operators used mainly the tools of abstract operator theory, whereas more recent work has relied heavily on the techniques of function theory and uniform algebras. The reason for this is that each cyclic subnormal operator S can be represented as multiplication by z on $P^2(\mu)$, where μ is a compactly supported measure on the plane and $P^2(\mu)$ is the closure in $L^2(\mu)$ of the (analytic) polynomials. Furthermore, the ultraweakly

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closed algebra generated by S can be identified with the space $P^\infty(\mu)$, the weak* closure in $L^\infty(\mu)$ of the polynomials. As a result, an understanding of $P^\infty(\mu)$ yields information about S. These ideas were used to great effect in 1978 by S. W. Brown, who showed that subnormal operators always have non-trivial invariant subspaces, thereby settling what was at the time the main open question in the subject.

The theory is now at the stage that a substantial amount of progress has been made, yet there is still much to be understood and many problems remain. The aim of the present book is to give a comprehensive account of the theory to date. It is something of a mixture between a textbook and a research monograph in that the author includes much of the necessary background material from function theory. After some preliminaries and an account of the structure of normal operators, the main part of the book divides roughly in two. The first half consists of an account of the basic theory of subnormal operators, a detailed analysis of the unilateral shift and a discussion of hyponormal operators. The latter half is devoted to the more recent developments involving uniform algebras and rational approximation. An account is given of Sarason's characterisation of $P^\infty(\mu)$ and this is then put to work in the final chapter, where Brown's invariant subspace theorem is proved and a functional calculus for subnormal operators due to the author and R. Olin investigated.

The book has been carefully written, with plenty of examples to illustrate the theory and helpful exercises for the conscientious reader. It will be of great use, both for those wanting to learn about subnormal operators for the first time and for specialists wanting an up-to-date account of the subject.

T. A. GILLESPIE

Lyndon, Roger C. *Groups and geometry* (London Mathematical Society Lecture Note Series 101, Cambridge University Press, 1985), 217 pp. £11.95.

Several books could have been written with this comprehensive title, so let prospective readers first be informed of the scope and aim of this one. It culminates with an account of Fuchsian groups, this following a penultimate chapter on hyperbolic groups (mapping the upper half of the complex plane onto itself) which, in turn, follows a chapter on inversive geometry. The logical sequence of these chapters is patent and the strategy of their unfolding is well conceived. So much should suffice to warn any reader seeking other groups or geometries.

The opening of the first chapter, on symmetries and groups, presents the elementary facts about groups of order not exceeding 6 and dihedral groups in a way that any expositor would do well to follow; an account (p. 7) of Cayley's theorem and an excursus (pp. 13–14) on free groups provide a contrast. The author's intention is to "describe groups in two ways: either a geometric description or a presentation by generators and relations." Tietze transformations of finite presentations of the same group into one another are described, and the chapter ends with the caution that the word problem is undecidable.

The second chapter (isometries of the Euclidean plane) and the third (subgroups of the group of isometries of the plane) deal with those matters expounded by Coxeter in the third chapter of his *Introduction to geometry*. The group of isometries has (p. 29) a chain of normal subgroups with successive quotients all abelian. Coordinates, matrices, complex numbers are called in aid and the affine group noticed. The three regular tesselations of the Euclidean plane afford the opportunity of introducing their fundamental regions, and presentations, involving three generators, of their groups of symmetries are given. The finite symmetry of the regular polyhedra, operating also on their circumscribing sphere S^2, are found, and hyperbolic triangle groups are mentioned with their presentations in terms of involutory generators—these last acting as inversions in Euclidean circles mapping lines of the hyperbolic plane.

The fourth chapter studies the crystallographic groups; any that are conjugate in the affine group are isomorphic and there are, to within isomorphism, 17 types. The first move (p. 63) is to establish the crystallographic restriction (cf. p. 61 of Coxeter's book) and some very closely argued details fill the next 10 pages. It is interesting to contrast these with the declaration (p. 82) in the