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RIGID AND FINITELY V-DETERMINED GERMS OF C^∞-MAPPINGS

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1. The result. Let $\mathscr{E}$ (respectively $\mathscr{E}_{[\mu]}$, $0 \leqq \mu \leqq \infty$) denote the ring of germs at $0 \in \mathbf{R}^n$ of all C^∞ functions (respectively C^μ functions) from $\mathbf{R}^n$ to $\mathbf{R}$. For a given $\varphi = (\varphi_1, \ldots, \varphi_p) \in \mathscr{E}^p$, $p \leqq n$, where $\mathscr{E}^p$ is the space of all germs of C^∞ mappings $\mathbf{R}^n \to \mathbf{R}^p$, let $J(\varphi)$ denote the ideal in $\mathscr{E}$ generated by $\varphi_1, \ldots, \varphi_p$ and the Jacobian determinants

$$\frac{D(\varphi_1 \ldots, \varphi_p)}{D(x_{i_1}, \ldots, x_{i_p})},$$

where $1 \leqq i_1 < \ldots < i_p \leqq n$. Let

$$\mathscr{M}^\infty = \{\varphi \in \mathscr{E} : D^\alpha \varphi(0) = 0, |\alpha| = 0, 1 \ldots\}.$$

Clearly, $\mathscr{M}^\infty$ is an ideal in $\mathscr{E}$ and $\mathscr{M}^\infty = \bigcap_{s=1}^\infty \mathscr{M}^s$, where $\mathscr{M}$ is the (unique) maximal ideal of $\mathscr{E}$. For $\varphi \in \mathscr{E}^p$ and $s \leqq \infty$ denote by

$$j^s(\varphi) = \left(\sum_{|\alpha|=0}^{s} \frac{1}{\alpha!} D^\alpha \varphi_1(0) x^\alpha, \ldots, \sum_{|\alpha|=0}^{s} \frac{1}{\alpha!} D^\alpha \varphi_p(0) x^\alpha \right)$$

the Taylor's expansion of φ at 0 up the order s, called the *s-jet* of φ, and for $\varphi = (\varphi_1, \ldots, \varphi_p) \in \mathscr{E}_{[\mu]}{}^p$ let $\mathscr{E}_{[\mu]}(\varphi)$ denote the ideal in $\mathscr{E}_{[\mu]}$ generated by $\varphi_1, \ldots, \varphi_p$.

Definition. We call a given germ $\varphi \in \mathscr{E}^p$ *finitely V-determined* (respectively *C^μ-rigid*) if there exists a positive integer s for which the following holds: for any $\psi \in \mathscr{E}^p$ with the same s-jet $j^s(\varphi) = j^s(\psi)$, the germs of the varieties $\varphi^{-1}(0)$ and $\psi^{-1}(0)$ are homeomorphic (respectively, one can find a local C^μ diffeomorphism $\tau : (\mathbf{R}^n, 0) \to (\mathbf{R}^n, 0)$ such that $\mathscr{E}_{[\mu]}(\varphi \circ \tau) = \mathscr{E}_{[\mu]}(\psi)$).

For $\varphi \in \mathscr{E}^p$ write

$$Z_\varphi(x) = \sum_{i=1}^{p} \varphi_i{}^2(x) + \sum \left[\frac{D(\varphi_1, \ldots, \varphi_p)}{D(x_{i_1}, \ldots, x_{i_p})}(x) \right]^2,$$

the second summation being taken for $1 \leqq i_1 < \ldots < i_p \leqq n$.

THEOREM 1. *For $\varphi \in \mathscr{E}^p$, the following conditions are equivalent:*
(a) *For each $\mu \in \mathbf{N}$, φ is C^μ-rigid;*
(b) *φ is finitely V-determined;*
(c) *$Z_\varphi(x) \geqq c|x|^\alpha$ in a neighborhood of 0, where c and α are positive constants;*
(d) $\mathscr{M}^\infty \subset J(\varphi)$.

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We are merely interested in those φ with $\varphi(0) = 0$; Theorem 1 reduces to triviality if $\varphi(0) \neq 0$.

Observe that if $\varphi \in \mathscr{E}^p$ is finitely V-determined then, by definition, the germ of $\varphi^{-1}(0)$ is homeomorphic to the germ of an algebraic variety; if φ is C^μ-rigid then the variety $\varphi^{-1}(0)$ can be transformed under a local C^μ diffeomorphism of $\mathbf{R}^n$ onto an algebraic variety. For general φ, however, there is no criterion for (the germ of) $\varphi^{-1}(0)$ to be homeomorphic with an algebraic variety. Thom has conjectured that if φ is analytic then this is the case.

The problems concerning sufficiency of jets and finitely determined mappings have been considered by several authors [**1**; **2**; **3**; **4**; **5**; **6**; **7**; **8**; **9**; **12**; **13**]. We recall the definition. Denote by $J^r(n, p)$ the space of r-jets of mappings from $\mathbf{R}^n$ to $\mathbf{R}^p$ (this space can be identified with the space of all p-tuples $w = (w_1, \ldots, w_p)$ of polynomials w_i of degree $\leqq r$ in n variables). A jet $w \in J^r(n, p)$ is called *V-sufficient* (respectively *C^μ-sufficient*, $r \leqq \mu \leqq \infty$) in $\mathscr{E}^p$ if for any $\varphi \in \mathscr{E}^p$ with $j^r(\varphi) = w$, the germs of varieties $w^{-1}(0)$ and $\varphi^{-1}(0)$ are homeomorphic (respectively, there exists a local C^μ diffeomorphism τ, such that $\varphi \circ \tau = w$). A germ $\varphi \in \mathscr{E}^p$ is called *finitely C^μ determined* if there exists a positive integer r such that $j^r(\varphi)$ is C^μ-sufficient in $\mathscr{E}^p$.

Many problems concerning V- and C^μ-sufficiency in $J^r(n, 1)$ ($p = 1$) have been solved. In particular, for $\varphi \in \mathscr{E}$, it has been proved in [**3**] (compare also [**1**] and [**13**]) the equivalence of the following four conditions:

(α) For each $\mu \in \mathbf{N}$, φ is finitely C^μ-determined;

(β) φ is finitely V-determined;

(γ) $|\operatorname{grad} \varphi(x)| \geqq c|x|^\alpha$ for $|x| < \delta$, where c, α, δ are positive constants;

(δ) $\mathscr{M}^\infty \subset \mathscr{P}(\varphi)$, where $\mathscr{P}(\varphi)$ is the ideal in $\mathscr{E}$ generated by the partial derivatives $\partial\varphi/\partial x_1, \ldots, \partial\varphi/\partial x_m$.

Observe that for $\varphi \in \mathscr{E}^p$, $p > 1$, conditions (α) and (β) are not equivalent. The fact that φ is finitely V-determined does not even imply that φ is finitely C^0-determined.

Counterexample (Mather). Let $\varphi(x, y) = (x, y^3)$. It is easy to see that $j^3(\varphi)$ is V-sufficient. But φ is not finitely C^0-determined. For any $s \in \mathbf{N}$, $\psi_s = (x, y^3 + yx^{2s+1})$ is a realization of $j^3(\varphi)$. But φ and ψ_s have different topological types, since φ is a homeomorphism, while ψ_s is not.

Remarks 1. One can prove that $\varphi \in \mathscr{E}^p$ is C^∞-rigid if and only if $J(\varphi)$ is the ideal of definition of $\mathscr{E}$, i.e. there exists a positive integer s such that $\mathscr{M}^s \subset J(\varphi)$ [**4**, Theorem 4(b); **13**].

2. Observe that if $\varphi \in \mathscr{E}^p$ is an analytic mapping then each of the conditions (a), (b), (c) and (d) in Theorem 1 is equivalent (by Lojasiewicz inequality) to the condition.

(e) *In a neighborhood of zero*, $Z_\varphi(x) = 0$ *implies* $x = 0$.

This generalizes Theorem 5 in [**8**].

2. Proof of Theorem 1. We shall assume that $p \geqq 2$. The case $p = 1$ was explained above. In fact, our proof would not work in the case $p = 1$.

(a) $\Rightarrow$ (b). This is trivial.

(b) $\Rightarrow$ (c). Observe firstly that if $\varphi \in \mathscr{E}^p$ is finitely V-determined then the (germ of) $\varphi^{-1}(0)\backslash\{0\}$ is either empty or a topological submanifold of codimension p in $\mathbf{R}^n$. Indeed, suppose the s-jet $w = j^s(\varphi)$ of φ is V-sufficient, choose a system of p homogeneous polynomials $h = (h_1, \ldots, h_p)$ of degree $s + 1$ in such a way that $Z_{w+h}(x) \neq 0$ for all x in a neighborhood of $0 \in \mathbf{R}^n$, $x \neq 0$ (this is possible, for example by [**4**, Proposition 1(b)]). By assumption, the germs of $\varphi^{-1}(0)$ and $(w + h)^{-1}(0)$ are homeomorphic, but it is clear that $(w + h)^{-1}(0)\backslash\{0\}$ is either empty or a smooth submanifold of $\mathbf{R}^n$ of codimension p.

Now assume that (c) is false, we shall derive a contradiction. We shall find an application $\tilde{\varphi} \in \mathscr{E}^p$ such that $j^\infty(\varphi) = j^\infty(\tilde{\varphi})$ and the germ of $\tilde{\varphi}^{-1}(0)\backslash\{0\}$ is not a topological manifold (and is not empty). The idea is similar to that in [**1**] and [**8**] and is due to S. Lojasiewicz.

Since (c) is false, we can find a sequence $\{a_i\}_{i\in\mathbf{N}}$, $a_i \in \mathbf{R}^n$, $a_i \neq 0$, $a_i \rightarrow 0$, such that for each $s \in \mathbf{N}$,

$$(*) \qquad Z_\varphi(a_i) = o(|a_i|^s).$$

For p vectors $u_1, \ldots, u_p$ in $\mathbf{R}^n$ write $d(u_1, \ldots, u_p) = \min\{\alpha_1, \ldots, \alpha_p\}$, where α_k denotes the distance from u_k to the linear subspace of $\mathbf{R}^n$ spanned by the vectors u_j, $j \neq k$, and let $\mathrm{Vol}(u_1, \ldots, u_p)$ denote the p-dimensional volume of the parallelotope with edges $u_1, \ldots, u_p$. Then $\mathrm{Vol}(u_1, \ldots, u_p) \geqq (d(u_1, \ldots, u_p))^p$. Moreover if we write $u_k = (u_{k1}, \ldots, u_{kp})$ then

$$\sum \begin{vmatrix} u_{1i_1}, \ldots, u_{1i_p} \\ \cdots\cdots\cdots\cdots \\ u_{pi_1}, \ldots, u_{pi_p} \end{vmatrix}^2 = (\mathrm{Vol}(u_1, \ldots, u_p))^2,$$

where $1 \leqq i_1 < \ldots < i_p \leqq n$. The above formula can be verified by checking the axioms for a volume (see for example [**11**]).

Now consider $u_k = \operatorname{grad} \varphi_k$ at a_i; without loss of generality, we may assume that for all $i \in \mathbf{N}$

$$d(\operatorname{grad} \varphi_1(a_i), \ldots, \operatorname{grad} \varphi_p(a_i)) = \delta_i,$$

where $\delta_i =$ the distance from $\operatorname{grad} \varphi_p(a_i)$ to the subspace spanned by $\operatorname{grad} \varphi_1(a_i), \ldots, \operatorname{grad} \varphi_{p-1}(a_i)$. Since

$$Z_\varphi(a_i) = \sum_{k=1}^p \varphi_k^2(a_i) + (\mathrm{Vol}(\operatorname{grad} \varphi_1(a_i), \ldots, \operatorname{grad} \varphi_p(a_i))^2 \geqq \sum_{k=1}^p \varphi_k^2(a_i) + \delta_i^{2p},$$

condition (*) implies that

(1) $|\varphi_k(a_i)| = o(|a_i|^s)$, for all $s \in \mathbf{N}$ and $1 \leqq k \leqq p$;

(2) $\delta_i = o(|a_i|^s)$, for all $s \in \mathbf{N}$.

To complete the proof we need the following

LEMMA. *Let $\{u_i^{(1)}, \ldots, u_i^{(p)}\}_{i\in \mathbf{N}}$ be a sequence of p-tuples of vectors in $\mathbf{R}^n$. Suppose there is a sequence of positive numbers α_i, $\alpha_i \to 0$ such that for all $s \in \mathbf{N}$*

$$\delta_i = o(\alpha_i^s),$$

where δ_i = the distance from $u_i^{(p)}$ to the linear subspace spanned by $u_i^{(1)}, \ldots, u_i^{(p-1)}$.

Then we can find a sequence $\{\lambda_i^{(2)}, \ldots, \lambda_i^{(p)}\}_{i\in \mathbf{N}}$ of $(p-1)$-tuples of vectors in $\mathbf{R}^n$, satisfying the following three conditions:

(i) *For all $s \in \mathbf{N}$, $|\lambda_i^{(k)}| = o(\alpha_i^s)$, $2 \leqq k \leqq p$;*

(ii) *For each $i \in \mathbf{N}$, $u_i^{(2)} + \lambda_i^{(2)}, \ldots, u_i^{(p)} + \lambda_i^{(p)}$ are linearly independent;*

(iii) *For each $i \in \mathbf{N}$, u_i^1 belongs to the subspace spanned by $u_i^{(k)} + \lambda_i^{(k)}$, $2 \leqq k \leqq p$.*

Proof of the Lemma. Let $v_i^{(p)}$ denote the orthogonal projection of $u_i^{(p)}$ to the subspace spanned by $u_i^{(1)}, \ldots, u_i^{(p-1)}$, and let $v_i^{(k)} = u_i^{(k)}$, $k \leqq p-1$. Then $|v_i^{(k)} - u_i^{(k)}| = o(\alpha_i^s)$, $1 \leqq k \leqq p$. For each i, the vectors $v_i^{(1)}, \ldots, v_i^{(p)}$ are linearly dependent; the subspace L_i spanned by them has dimension $\leqq p-1$. Now we can choose $w_i^{(1)}, \ldots, w_i^{(p)}$, where $w_i^{(1)} = v_i^{(1)}$, $|w_i^{(k)} - v_i^{(k)}| = o(\alpha_i^s)$, such that a subset of linearly independent vectors $\{w_i^{(2)}, \ldots, w_i^{(p)}\}$ is a basis of L_i. Consequently, $w_i^{(1)} = u_i^{(1)}$ is a linear combination of $w_i^{(2)}, \ldots, w_i^{(p)}$. Now put $\lambda_i^{(k)} = w_i^{(k)} - u_i^{(k)}$, $2 \leqq k \leqq p$.

With this Lemma, we now complete the proof that (b) $\Rightarrow$ (c). With $u_i^{(k)} = \operatorname{grad} \varphi_k(a_i)$ and $\alpha_i = |a_i|$, choose $\lambda_i^{(k)}$ as in the above Lemma. We may assume $|a_{i+1}| < \frac{1}{2}|a_i|$. Let $\psi : \mathbf{R}^n \to [0, 1]$ be C^∞, $\psi(x) = 1$ in a neighborhood of $0 \in \mathbf{R}^n$ and $\psi(x) = 0$ for $|x| \geqq \frac{1}{4}$. Put

$$\eta_1(x) = \sum_{i=1}^{\infty} \psi\left(\frac{x - a_i}{|a_i|}\right) (\varphi_1(a_i) + \epsilon_i|x - a_i|^2), \quad \epsilon_i > 0,$$

$$\eta_k(x) = \sum_{i=1}^{\infty} \psi\left(\frac{x - a_i}{|a_i|}\right) (\varphi_k(a_i) - \lambda_i^{(k)}(x - a_i)), \quad k = 2, \ldots, p.$$

Observe that

(α) If we choose $\epsilon_i > 0$ such that for each $s \in \mathbf{N}$, $\epsilon_i = o(|a_i|^s)$ then $\eta = (\eta_1, \ldots, \eta_p)$ is of class C^∞;

(β) η is (infinitely) flat at the origin;

(γ) For each $i \in \mathbf{N}$, $(\varphi - \eta)(a_i) = 0$.

Now put $\tilde{\varphi} = \varphi - \eta$. We shall show that ϵ_i, $(\epsilon_i = o(|a_i|^s))$, can be chosen in such a way that near each a_i, $\varphi^{-1}(0)$ is not a topological manifold.

By condition (iii) in the Lemma, $\operatorname{grad} \varphi_1(a_i)$ is a linear combination of $\operatorname{grad} \varphi_k(a_i) + \lambda_i^{(k)}$, $k \geqq 2$, say

$$\operatorname{grad} \varphi_1(a_i) = \sum_{k=2}^{p} \xi_{ki}(\operatorname{grad} \varphi_k(a_i) + \lambda_i^{(k)}), \quad \xi_{ki} \in \mathbf{R}.$$

Choose $\epsilon_i = o(|a_i|^s)$ such that each a_i is a non-degenerate critical point of

$$\rho(x) = \varphi_1(x) - \eta_1(x) + \sum_{k=2}^{p} \xi_{ki}(\eta_k(x) - \varphi_k(x)).$$

In a neighborhood of any a_i, for fixed i,

$$\begin{aligned}\tilde{\varphi}^{-1}(0) &= \{x \in \mathbf{R}^n : \varphi_1(x) - \eta_1(x) = \ldots = \varphi_p(x) - \eta_p(x) = 0\} \\ &= \{x \in \mathbf{R}^n : \rho(x) = \varphi_2(x) - \eta_2(x) = \ldots = \varphi_p(x) - \eta_p(x) = 0\}.\end{aligned}$$

Hence, near a_i, $\tilde{\varphi}^{-1}(0)$ is homeomorphic to the intersection of the locus of a non-degenerate quadratic form $\rho^{-1}(0)$ (Morse Lemma) with the $(p-1)$-codimensional differentiable submanifold of $\mathbf{R}^n$, defined by $\varphi_2(x) - \eta_2(x) = \ldots = \varphi_p(x) - \eta_p(x) = 0$; thus $\tilde{\varphi}^{-1}(0)$ is not a manifold near a_i.

(c) $\Leftrightarrow$ (d). This follows easily from the following theorem.

THEOREM 2 (Tougeron-Merrien). *An ideal I of $\mathscr{E}$ is elliptic if and only if I is finitely generated and $\mathscr{M}^\infty \subset I$.*

Recall (compare [**13**]) that I is called *elliptic* if it contains an element f having the property that $|f(x)| \geqq c|x|^\alpha$ in a neighborhood of $0 \in \mathbf{R}^n$, where α and c are positive constants. Such a function f is also called elliptic.

It is easy to see that if I is elliptic and generated by $f_1, \ldots, f_q$ then the element $f_1^2 + \ldots + f_q^2$ is elliptic. Hence (c) $\Leftrightarrow$ (d) follows from Theorem 2, because Z_φ is the sum of squares of generators of $J(\varphi)$.

We now prove Theorem 2. Let f be an elliptic element of I. Let $\psi \in \mathscr{M}^\infty$ be any element; then $\eta(x) = \psi(x)/f(x)$, $\eta(0) = 0$, is a germ of a C^∞ function. Hence $\psi = \eta f \in I$ and $\mathscr{M}^\infty \subset I$. To show that I is finitely generated choose C^∞ functions $\varphi_1, \ldots, \varphi_k$ so that their formal Taylor's expansions $j^\infty(\varphi_1), \ldots, j^\infty(\varphi_k)$ generate the ideal $j^\infty(I)$ in the ring of formal power series. Here $j^\infty(I)$ consists of all formal Taylor's expansions of elements of I. Then $\{\varphi_1, \ldots, \varphi_k, f\}$ is clearly a set of generators of I.

Conversely, suppose I is generated by $f_1, \ldots, f_q$ and $\mathscr{M}^\infty \subset I$. Choose an open neighborhood W of 0 at $\mathbf{R}^n$ and representations $\tilde{f}_i \in C^\infty(W)$ of f_i such that for any $h \in C^\infty(W)$ with $D^\alpha h(0)$, $|\alpha| = 0, 1, 2, \ldots,$ there exist $g_1, \ldots, g_q \in C^\infty(W)$ for which $h = \sum_{i=1}^q \tilde{f}_i g_i$.

This choice is possible. We can certainly choose $\tilde{f}_i$ defined in a neighborhood W of 0, such that $\bigcap_{i=1}^q \tilde{f}_i^{-1}(0) = \{0\}$. Then by applying a partition of unity, it is easy to fulfill the above requirement.

Hence, by construction, the ideal $\tilde{I}$ of $C^\infty(W)$ generated by $\tilde{f}_i$, $1 \leqq i \leqq q$, of which the zero set reduces to $\{0\}$, contains all functions in $C^\infty(W)$ which are infinitely flat at 0. Now by [**10**, Proposition 1],

$$\sum_{i=1}^{q} (\tilde{f}_i(x))^2 \geqq c|x|^\alpha$$

near 0 for some c, α positive.

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Remark. Theorem 2 has been communicated to the first author by J. Mather who has also given a slightly different proof.

(d) $\Rightarrow$ (a). This has been proved by Tougeron [**13**, p. 220]. For any subset I in $\mathscr{E}$ and $\mu \in \mathbf{N}$, let $\mathscr{E}_{[\mu]}(I)$ denote the ideal generated by I in $\mathscr{E}_{[\mu]}$. Now $\mathscr{M}^{\infty} \subset J(\varphi)$ implies that for any $\mu \in \mathbf{N}$, there exists $s \in \mathbf{N}$ such that $\mathscr{M}^{s} \subset \mathscr{E}_{[\mu+1]}(J(\varphi))$. Indeed, since Z_{φ} is elliptic, $x^{\alpha}/Z_{\varphi}(x)$ is of class $C^{\mu+1}$, where $\alpha = (\alpha_1, \ldots, \alpha_n)$ is any multi-index with $|\alpha| = s$, s large. Hence $x^{\alpha} \in \mathscr{E}_{[\mu+1]}(J(\varphi))$ and $\mathscr{M}^{s} \subset \mathscr{E}_{[\mu+1]}(J(\varphi))$.

We now show that $j^{2s}(\varphi)$ is C^{μ}-rigid in $\mathscr{E}^{p}$. Let $\psi \in \mathscr{E}^{p}$ be any element with $j^{2s}(\psi) = j^{2s}(\varphi)$. Then $\mathscr{E}(\varphi - \psi) \subset \mathscr{M}^{2s+1}$, hence

$$\mathscr{E}(\varphi - \psi) \subset [\mathscr{E}_{[\mu+1]}(\mathscr{M})][\mathscr{E}_{[\mu+1]}(J(\varphi))]^2.$$

Now applying Tougeron's theorem [**4**, Theorem 1(b)], the proof is complete.

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