STELLAR CONVECTION

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The most important function of a convection theory for stellar model building is to determine the temperature stratification in terms of the heat flux. Apart from in some recent work by Latour et al. (1), mixing length theory, in one or other of its guises, still provides the only prescription that is used. Unfortunately this is not a reliable procedure because of the crude way in which the dynamics is treated. Furthermore, the resulting formulae depend on the mixing length ℓ which occurs in the theory as an undetermined function. Work on convection that may one day lead to a more satisfactory theory is taking place, but none of it has yet reached the point to warrant displacing the methods currently practised in stellar evolution computations. The reader is referred to the reviews by Spiegel for a discussion of the astrophysically relevant work on convection up to 1972 (2,3).

A. ATTEMPTS TO MODEL THERMAL CONVECTION

One of the principal factors inhibiting progress in stellar convection theory is that conditions in stars are very different from those in the laboratory. Stellar convection is characterized by high values (10^{20}) of the Rayleigh number R, which is a dimensionless measure of the temperature gradient, and low values (10^{-9}) of the Prandtl number σ, which is the ratio of kinematic viscosity to thermal diffusivity. On the other hand in the laboratory R is quite low ($< 10^{11}$) by astrophysical standards, and σ is of the order or greater than unity. Moreover, stellar convection zones extend typically over many scale heights of pressure and density, leading to compressible motions, whereas in the laboratory the depth of a convecting layer is always a minute fraction of a scale height: the motion is essentially incompressible and can be described by the Boussinesq approximation (4).

Most of the theoretical work is aimed at mimicking laboratory conditions. A thin layer of fluid bounded by two isothermal planes, the lower boundary being at the higher temperature, is usually considered. The equations of motion are solved, usually in the Boussinesq approximation, either numerically at moderate R and σ (5-10) or analytically at low R close to the critical value R_c at which a static fluid layer becomes unstable to convection (11). At present the computational difficulties are too severe to extend these calculations to values of R and σ of astrophysical interest. The main objective

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is to determine the dependence of the Nusselt number N, a dimensionless measure of the heat flux, on σ and R. A plausible extrapolation procedure might then lead to a better prescription for stellar convection.

The most obvious procedure, one might think, would be to apply mixing-length theory to laboratory convection and attempt to determine ℓ experimentally. However it is the astronomer's belief that this is of no use, for whereas in the laboratory eddies extend across the whole of the convecting layer, in a compressible fluid many scale heights deep the shear produced by differential expansion and contraction of vertically moving fluid is thought to disrupt the convective motion in about a scale height (Schwarzschild, (12)). This is a nonlinear argument, and so is not contradicted by the fact that the most unstable linear modes extend across the entire convectively unstable region (Spiegel, Böhm, (13-16)). Accordingly, ℓ is presumed to be proportional to a density or pressure scale height (Öpik, Vitense, 17,18)), usually the latter, the constant of proportionality being determined astronomically, and contact with terrestrial experience is lost.

In its most usual form the mixing-length theory provides a local relationship between the heat flux F and the superadiabatic temperature gradient $\mathcal{D} = \nabla - \nabla_{ad}$. There are many uncertainties in the theory, and consequently there is opportunity to incorporate into it several adjustable parameters, though only two are of immediate interest (19): $\alpha \equiv \ell/H$, where H is an appropriate scale height, and a measure γ of the radiative losses. The theories are calibrated either by constructing a solar model and adjusting it to have the correct luminosity and effective temperature at an age of about 4.7×10^9 yr (Schwarzschild et al; Sears (20,21)), or by fitting a theoretical sequence to a young cluster diagram (Demarque and Larson; Copeland, Jensen and Jørgensen (22,23)). Both methods yield $\alpha \simeq 1$, the precise value depending on the details of the theory adopted, but leave γ undetermined. Some comfort is derived from the observation that this implies an eddy size at the top of the solar convection zone comparable with the length scale of the granulation (Schwarzschild (12)). The gross structure of a main sequence stellar model is insensitive to γ, which matters only near the outer edge of the envelope convection zone. The parameter γ does affect the convective envelopes of red giants, however, which have large nonadiabatic regions (Henyey et al (24); Schwarzschild (25)). Red giant models are probably more sensitive to other details of how convection is treated, too, especially in the surface layers where fluctuations are large in magnitude and horizontal extent.

It should be noticed that the mixing-length formalisms used in stellar structure computations are based on the Boussinesq approximation to incompressible flow. One would have expected (4) this to have been valid had ℓ been much less than H, but indications are that it is not a good approximation otherwise (Graham (26), Deupree (27)). Thus the calibration $\ell \simeq H$ exposes an inconsistency in the theory. However, the evidence for the functional dependence $\ell \propto H$ is hardly overwhelming, and adopting it no doubt introduces errors that are just as great. The apparent success of the mixing-

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length formalism lies in the fact that the gross structure of a main sequence stellar model is almost independent of the functional form of the relationship between F and $\mathcal{D}$ (Gough and Weiss (19)).

Stellar mixing-length theory ignores viscosity. This sounds plausible since the Reynolds number of the heat-transporting flow is large and σ is small. It implies that N depends on σ and R only in the combination σR which is independent of viscosity. Furthermore, since N increases with R at fixed σ, it must therefore increase with σ at fixed R, provided σ remains small.

The beginnings of an attempt to bridge the gap between laboratory and stellar conditions, using a truncated modal expansion, has recently been reported (Gough, Spiegel, Toomre (28,29)). Although the analysis is in the Boussinesq approximation, it can treat with the same assumptions the extreme values of R and σ typical of stars and the more moderate values encountered in the laboratory. The procedure can be made to reproduce some of the features of laboratory convection, but its most obvious drawback is that it contains several undetermined parameters. In its simplest form there are just two such parameters, characterizing the horizontal scale and shape of the convective eddies. Although this is perhaps an improvement over mixing-length theory, which depends on an undetermined function ℓ, an unambiguous calibration by comparison with laboratory convection has not been possible. The theory has the property that for $\sigma << 1$, N is a function of σR, provided the convection is three-dimensional, which accords with astronomers' prejudices.

It should be pointed out, however, that the σ dependence of N is not universally believed. This arises partly because almost all laboratory experience is with fluids that have $\sigma \gtrsim 1$, and for these both theory and experiment show that N is almost independent of σ at fixed R. Furthermore, numerical solutions of the Boussinesq equations at moderate R, which until recently have always constrained the flow to be two-dimensional, have predicted almost no σ dependence, and even a slight increase of N as σ is decreased below unity (Veronis (5); Quon (8); Moore and Weiss (9)), though it has been argued that this may be a result of constraining the horizontal length scale of the motion (Lipps and Somerville (6); Willis, Deardorff and Somerville (7)).The simplest modal analysis predicts that N is independent of σ when the motion is two dimensional (28). Analytical expansions of the full Boussinesq equations for R near R_c reveal only a weak dependence on σ in that case too (Schlüter, Lortz and Busse (30)) but, like the modal results, suggest a strong decrease in N at low σ when the motion is three-dimensional. This led Jones, Moore and Weiss (31) to investigate numerically axisymmetrical convection in a cylinder which, though mathematically dependent on only two space variables, is geometrically three-dimensional. They reproduced the analytical results for R just above R_c, but showed that at moderate R the flow readjusted itself to resemble the two-dimensional flows, and produced an N that is independent of σ at high and low σ, and slightly decreasing with σ in the neighbourhood of $\sigma = 1$.

The issue is unresolved. Jones et al. suggest that their flow is unstable and that at sufficiently high R it would become turbulent with N independent of viscosity at low σ. Moreover, three-dimensional calculations reported recently by Veltishchev and Želnin (10) at $\sigma = 0.7$ and $\sigma = 1$ suggest that the flow does not adjust itself to the kind of structure that is preferred when axisymmetry is imposed, and that N is less when $\sigma = 0.7$ than it is when $\sigma = 1$. This concurs with the evidence from laboratory experiments (29), though this is admittedly weak. Finally, if a convection theory based on eddies of scale $\ell \propto H$ and governed by dynamics similar to that exhibited by the two-dimensional and axisymmetrical Boussinesq computations (and therefore implying N is independent of σ) were subject to the usual astronomical calibration, the result would be that ℓ would be but a very small faction of H, which most astronomers would find unpalatable.

Little study has been made of fully developed convection in a layer of compressible fluid many scale heights deep. Graham (26) has made some two-dimensional computations for a perfect gas at moderate R and σ. The property exhibited by similar Boussinesq calculations that N is a decreasing function of σ when $\sigma \simeq 1$ is accentuated as the layer depth, and the effects of compressibility, are increased. Moreover, no tendency for eddies to break up on a scale of H was found. Graham's more recent three-dimensional compressible calculations yield similar results (32). Compressible modal calculations have also been performed in the anelastic approximation; by van der Borght (33) with $\sigma = 1$ and by Latour et al. (1) under more realistic stellar conditions modelling an A star envelope. As with the Boussinesq calculations there are undetermined parameters which can be chosen to produce plausible results. Once again, time-dependent calculations (1) show no tendency for the motion to break up into eddies on the scale of H.

B. PENETRATION AND OVERSHOOTING

The edges of stellar convection zones are not rigid inpenetrable boundaries as they are in most laboratory and theoretical investigations. The density stratification changes from being convectively unstable to convectively stable, from the point of view of linear stability analysis, and fluid accelerated in the convectively unstable region can penetrate, or overshoot, into the adjacent stable regions.

This phenomenon has been of interest particularly to meteorologists interested in mixing at the atmospheric inversion (34). D.W. Moore (35) gives a brief account of the relevant physics. A convective element, or thermal, on reaching the top (or bottom) of the convectively unstable region, still has a temperature excess (or deficiency) relative to its immediate surroundings and continues to experience a buoyancy force. If the element were to maintain its identity and move adiabatically in pressure equilibrium with its surroundings, buoyancy would not disappear until the level $z = z_s$ were

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reached at which the specific entropy were the same as at the level at which the element originated. This point does not necessarily mark the edge of the zone of penetration, however, because the element still has momentum to carry it on yet further. Entrainment of stable fluid, on the other hand, retards the motion of the element. Thus $z = z_\delta$ may either overestimate or underestimate the extent of penetration. Observations of the motions of cloud tops suggest that it is usually an overestimate though in some circumstances, such as in tropical storms, large plume - like structures penetrate well above the tropopause (36). An additional complication, usually ignored by meteorologists in this context, is radiative diffusion, which tends to reduce both the buoyant acceleration and retardation by reducing temperature fluctuations. Buoyant thermals penetrating into the stable layer advect heat upwards, though this is offset by the induced return flow. Near the outer edge of the penetrated region both upward and downward moving fluid presumably transport heat counter to the net flux.

Some aspects of the situation can be modelled with the ice-water experiment. This consists of a layer of water cooled from below at 0°C and with its upper boundary maintained above 4°C, the temperature of the density maximum. Laboratory experiments show that the unstable layer extends beyond the limits it would have occupied had there been no motion (Townsend (37), Adrian (38)), and in addition plume-like motions in the unstable region penetrate into the stable layer above. Adjacent layers of convectively stable and unstable fluid have also been created by inducing spatially varying temperature gradients in water near room temperature, either by imposing time varying boundary conditions (Krishnamurti (39); Deardorff, Willis and Lilly (40)) or by internal heating or cooling (Faller and Kaylor (41); Whitehead and Chen (42)). The nature of the motions in the stable layer is not entirely clear, but the temperature fluctuations observed by Townsend (37) seem to be the product of trapped gravity waves. Theoretical numerical experiments in two-dimensions by Moore and Weiss (43) also exhibit the encroachment of the unstable region into the region that would have been stable in the absence of motion, and the excitation of gravity waves. They also predict weak viscously driven countercells which are not seen in the laboratory, and little evidence of plumes. Earlier steady one-mode mean-field calculations, which in some sense represent two-dimensional motion, yielded similar results, without the gravity waves (Musman (44)). Thus some of the observed features of laboratory experiments are reproduced theoretically; the differences, as Spiegel (3) has pointed out, might result from the two-dimensional constraint imposed on the numerical computations.

Although the ice-water experiment sheds some light on the mechanism of penetration it seems difficult to generalize to stellar conditions. There is some evidence from the two-dimensional numerical experiments that penetration increases as Prandtl number decreases (D.R. Moore (45)). Modal calculations by Latour (46) et al.(1) modelling three-dimensional convection in A star envelopes predict greater penetration by the almost plume-like columns in the centres of hexagonal cells than by two-dimensional rolls. However, this analysis has not been applied to the ice-water problem; there is yet no bridge between stars and laboratory experience.

The various theoretical prescriptions that are usually employed to describe overshooting from stellar convection zones are all essentially based on mixing-length theory (Spiegel (47), Parsons (48), Ulrich (49), Scalo (50), Shaviv and Salpeter (51)). Of necessity they are nonlocal theories, though they all rely on the Boussinesq approximation. They have been used, in particular, to model the solar atmosphere which is perhaps the most sensitive astrophysical testing ground at present, because quite detailed comparison of theoretical predictions with observations can in principle be made. It is not easy to deduce the height dependence of the solar atmospheric velocity fluctuations, nor is it easy to disentangle convective motion from waves, though an attempt has been made (Frazier (52)). It seems likely, however, that velocities of about 2 km s^{-1} extend well above the photosphere (de Jager (53)), which agrees with a model computed by Ulrich (54), though the theory does appear to overestimate the overshoot. Travis and Matsushima (55), using a theory of Spiegel (47), compare their models with limb darkening measurements and conclude also that too great an overshoot is predicted if a mixing length to scale height ratio α of about unity is adopted; they favour $\alpha \lesssim 0.35$, in contradiction to the usual calibration. A subsequent investigation by Travis and Matsushima (56) of the colours of cool main sequence stars and metal-deficient subdwarfs also suggested a low value for α. Nordlund (57), using Ulrich's approach, found overshoot to a lesser degree for a given α, and produced a model in better agreement with the Harvard-Smithsonian Reference Atmosphere (58) and similar to an earlier model built by Parsons (48) using a convective heat flux calculated from a nonlocal estimate of vertical velocity and a local estimate of temperature fluctuations. In contradiction, Edmonds's analysis (59) of the photospheric velocity and brightness fluctuations favours a greater degree of overshoot, so the matter seems unresolved. One thing that does seem clear is that at their present stage of sophistication nonlocal convection theories should not be relied upon to explain fine details, especially in regions in which the assumptions on which they are based are not satisfied. All the theories have adjustable parameters and can no doubt be tuned to rationalize the limb darkening; adjusting the radiative loss coefficient in Spiegel's theory, for example, could probably lead to an atmosphere hardly distinguishable from Nordlund's with an α consistent with the evolutionary calibration. Indeed Spruit (60) has produced a model with the correct centre to limb flux variation using a local mixing-length theory with no overshoot at all, though presumably this does not reproduce the fluctuation measurements discussed by Edmonds (59). Although there seems to be too much uncertainty in the theories at present to apply such subtle tests, detailed analyses of inhomogeneous atmospheres must eventually be undertaken both for theoretical model building and for analysing observational data. Horizontal temperature fluctuations increase the horizontally averaged opacity, for example, since opacity is a steeply increasing function of temperature, which leads to an increase in the actual mean temperature gradient. Furthermore, since the magnitude of the fluctuations decrease with height, the temperature gradient currently inferred from limb darkening observations (55,57) is overestimated when fluctuations are ignored. Abundance measurements may be affected. Turbulent Reynolds stresses generated by both the convection and the gravity waves in the photospheric regions also influence the stratification.

Another important consequence of overshooting is material mixing, particularly at the edges of convective cores. Early estimates (Roxburgh (61); Saslaw and Schwarzschild (62)) which ignored the influence of the convective energy flux on the temperature stratification, implied negligible mixing rates. But recently Shaviv and Salpeter (51) pointed out that the modification to the stratification increases the penetration of the motion into the stable envelope, just as in the case of the ice-water experiment. Maeder (63) and Cogan (64) independently confirmed this conclusion with more detailed calculations. The influence of the overshooting on colour-magnitude diagrams for old open clusters was subsequently investigated. Of particular interest is the position and magnitude of the gap at the top of the main sequence, which can be more accurately reproduced theoretically if an appropriate degree of mixing at the core boundary is assumed (Maeder (65), Prather and Demarque (66)). Using Shaviv and Salpeter's prescription for overshooting, Maeder (67) found once again that a value of α somewhat less than unity gives the best results. This too should not be regarded as contradicting the usual calibration, partly because the chemical composition adopted for the models may not have been appropriate, partly because there lies buried in the mixing-length formalism an undetermined parameter in the relation between velocity and temperature fluctuations that does not appear in the formula for the heat flux (19), partly because the geometry of the core has not been taken into account, and partly because the ratio of the mixing length to pressure scale height can hardly be a universal constant.

Calculations by Sugimoto and Nomoto (68) and Iben (69) suggest that theoretical predictions of nucleosynthesis in post main sequence stars would be significantly affected by overshooting beneath convective envelopes. It would also have some bearing on the observed lithium abundance in the sun (Spiegel (70)).

C. SUBCRITICAL CONVECTION

In the relatively straightforward case of ordinary convection discussed in §A, N is an increasing function of R at fixed σ. That is not necessarily the case when agents such as rotation, magnetic fields or nonuniformities in composition are present to inhibit the motion. The minimum Rayleigh number R_0 above which convection can exist is modified by the presence of the stabilizing agent, but it is not always possible to determine its value by linear stability analysis. It is often the case that direct convective motion of finite amplitude can adjust itself to reduce the efficacy of the stabilizing forces, and so exist at a Rayleigh number below the

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value R_c predicted by linear theory (Veronis (71)). This is called subcritical convection. Of course such a state can be achieved only if it were approached by lowering R from a value greater than R_c or if a metastable state, with $R_o < R < R_c$, were appropriately perturbed by a finite amount. It seems that the latter is often achieved spontaneously because in many circumstances there is a range of R below R_c within which the fluid is overstable, that is to say unstable to infinitesimal oscillations (Veronis (71), Weiss (72)). Weak experimental evidence exists to support the idea that such motion might grow to an amplitude great enough to trigger subcritical convection (Turner (73), Shirtcliffe (74), Rossby (75)).

The most widely studied problem of this type, and perhaps the easiest to understand, is thermohaline convection. Turner (34) summarizes well the present state of knowledge. The case of interest here is when salt stabilizes a layer of water heated from below. Veronis (71 , 76) studied the overstability and subcritical direct convection and gave a simple physical explanation of why they should occur (71). Laboratory experiments reveal that instability does occur first as a growing oscillation (74), and that convection subsequently organizes itself into a series of superposed shallow layers separated by diffusive interfaces (Turner and Stommel (77)), a configuration that has been observed to occur naturally (Hoare (78), Neal, Neshyba and Denner (79)). The fluxes of heat and salt appear to be controlled by the diffusive interfaces, and Turner (80) has observed that their ratio Φ, when measured in units of the fluxes that would have occured had motion been absent, appears to be independent of the ratio λ of the jumps in salinity and temperature across the layer, over a wide range of λ. Indeed, it has been suggested (Turner (34)) that this value of Φ depends only on the diffusion coefficients of the fluid, though recent experimental work indicates that it depends also on R (Marmorino and Caldwell (81)).

The astrophysical relevance of thermohaline convection is to the edges of convective cores of stars (Spiegel (82)),where the products of nuclear reactions, usually helium, take place of salt. When the usual criterion for convective instability is employed in a massive stellar model evolving off the main sequence, it is found that once a sufficient, stable discontinuity of composition is built up at the edge of the convective core, the envelope immediately outside it is also convectively unstable. This has been considered unacceptable by many astrophysicists, and it is assumed that the discontinuity is somehow smoothed out, usually to precisely the degree that results in no more than marginal stability immediately beyond the truly convective core (Ledoux (83), Tayler (84), Schwarzschild and Härm (85)), though other amounts of mixing have been proposed (Gabriel (86), Saio (87)). Different criteria are used to define marginal stability, which lead to rather different results, but it does not seem possible to choose between them by astronomical means (e.g. Chiosi and Summa (88) ; Robertson (89); Sweigart and Demarque (90); Ziółkowski (91); Varshavskii (92); Sreenivasan and Ziebarth (93); Stothers and Chin (94)).

This situation has some similarities to thermohaline convection set up by heating from

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below an initially isothermal layer of water stably stratified with salt, but the analogy is not perfect. Gabriel (86) has argued against generalizing from laboratory experience in this case. However, the idea that at least one shell of ordinary convection is created outside the convective core does not seem implausible, though it is not obvious whether the interface separating the two convecting regions would be stable enough to survive the disrupting forces of the turbulence. Such a possibility has been pointed out by Tayler (84, 95) as a mathematically consistent alternative to the conventional procedure. To determine the structure of the region an understanding of the diffusive interfaces is required. Had Φ been independent of λ and R one might have had some confidence in extrapolating laboratory measurements, especially since, if thin convective layers are formed, this is one place where the Boussinesq approximation might be valid ; but it appears that the answer is still out of reach.

Subcritical convection may also be relevant to solar type stars. It has been pointed out that the stability characteristics of the solar core are potentially similar to those of the thermohaline situation : overstable to infinitesimal perturbations and able to sustain direct convection of finite amplitude (Dilke and Gough (96)). An important difference, however, is that whereas the usual saline layer derives its energy from an externally imposed heat source and will convect so long as that source is maintained, the sun must derive its extra energy from burning a supply of ^{3}He which is mixed from the edge of the core. The amount of ^{3}He available is finite and after it is burnt convection is presumed to cease, and the solar core becomes quiescent again until a new supply of fuel has accumulated near its edge. If it occurs, this process may have some bearing on the solar neutrino problem and the occurrence of terrestrial ice ages. Subsequent more detailed analysis has supported the overstability postulate (e.g. Noels et al., (98) Unno (99, 97, 100)), though some computations have cast doubt on it (Christensen-Dalsgaard and Gough (101)). The likelihood of subcritical convection is questionable too (Ezer and Cameron (102) ; Ulrich (103)), though some evidence for it has been found (Rood (104)).

D. ROTATION AND MAGNETIC FIELDS

Uniform rotation inhibits convective motion and so increases the critical Rayleigh number above which convection can take place. At finite amplitude the motion can redistribute the angular momentum so that subcritical convection can occur (Veronis (105)). Typically the constraint cannot be cancelled entirely and the rotation reduces the heat flux. This is not always true, however. Rossby found in the laboratory that rotation sometimes increases N at fixed R, a behaviour seen also in three-dimensional numerical experiments (Somerville and Lipps (106)) and a modal analysis (Baker and Spiegel (107). It gives fair warning to those who argue that factors inhibiting linear instability necessarily inhibit subsequent nonlinear development.

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Astrophysical interest in the interaction between convection and rotation has been concerned in recent years with the structure of the convection zone in the solar envelope and the maintenance of the differential rotation. Of particular interest are the numerical experiments by Gilman (108). The subject has been reviewed recently by Gilman (109), Durney (110) and Weiss (111). Tayler (112) has discussed convection in rotating stellar cores.

The solar convection zone will not be understood until it is known how convection interacts with magnetic fields. It is hard to infer the field strengths beneath the surface, especially since the topology of the convective motion is such as to submerge the field (Drobyshevski and Yuferev (113)). The formation and decay of magnetic field concentrations are of obvious interest, and are reviewed in the proceedings of IAU Symposium n° 71. Like uniform rotation, a uniform magnetic field tends to inhibit convective motion. The linear stability characteristics of a plane Boussinesq fluid layer heated from below are similar to the rotating case with no magnetic field. But Weiss (114) has pointed out that there are fundamental differences between Lorentz and Coriolis forces and that care must be taken when comparing the two cases. Weiss found that the nonlinear development of both overstable and direct infinitesimal motions can be oscillatory, provided the magnetic field is not too weak. The final state is not necessarily one in which there is equipartition between kinetic and magnetic energies (Peckover and Weiss (115)). Modal calculations (van der Borght, Murphy and Spiegel (116)) have revealed only a decrease in N at fixed R as the magnetic field increases, but a magnetic field appears to be able to interact with a rotating fluid in such a way that the resulting Nusselt number is greater than it would have been in the absence of the field (van der Borght and Murphy (117)).

Of interest recently has been the question of whether convection can be the source of dynamo action. Childress and Soward (118) demonstrated that the kind of flow encountered in a rotating convecting fluid is suitable for amplifying magnetic fields, as has been noticed also by Spiegel (3). Perturbation expansions about the marginal state (Soward (119) ; Roberts and Stewartson (120)) and a modal analysis (Baker (121)), both of which incorporate the forces on the fluid arising from the perturbed magnetic field, indicate that a convecting fluid can indeed sustain a magnetic field by induction.

E. TIME-DEPENDENT CONVECTION

New difficulties are encountered when a star is varying globally on a time-scale comparable with the convective turnover time. This may occur when the star is not in hydrostatic equilibrium : during gravitational collapse, a nova or supernova explosion, a flare or envelope ejection, or whenever a star pulsates. It is perhaps for pulsating stars that an understanding of the time dependence of convection is most urgently needed because both theory and observations have progressed further than in the studies of other classes of intrinsically variable stars. Many of the gross features of the observations have been explained, but the position of the red edge of the Cepheid strip, for example, remains unsolved. This can probably be blamed on an inadequate treatment of convection in the theoretical models.

Most computations of stellar pulsations have either ignored convection entirely, ignored perturbations (either Lagrangian or Eulerian) in the convective heat flux induced by the pulsations, assumed the convection to adjust instantaneously to its changing environment, or assumed it to relax towards the state given by the usual mixing-length formulae at a rate proportional to the amount by which it deviates from that state and inversely proportional to the eddy lifetime. The last of these prescriptions is perhaps the most credible, and was first used by Cox et al. (122) to compute model Cepheids. However, its obvious deficiency is that it contains a free parameter : a constant of proportionality that determines the rate at which convection readjusts to the pulsation. This in turn determines the phase difference between the convection and pulsation, upon which the pulsational stability of the star directly depends.

Attempts have been made by Gough (123) and Unno (124) to generalize the mixing-length theory. Unfortunately there are different ways of formulating the fundamental postulates. The resulting formulae are essentially the same for hydrostatic stars but differ when the star is presumed to pulsate. Moreover, there appear to be no relevant laboratory experiments with which to compare the various possibilities. Despite these uncertainties it would be interesting to know how sensitive pulsating stellar models are to the assumptions behind the convection formalism, and whether a possible choice of the uncertain parameters exists that rationalizes the observations. Computations in the quasiadiabatic approximation suggest this may be so, but apart from misrepresenting nonadiabatic effects these computations are deficient in an important respect : they do not take due account of the turbulent Reynolds stress.

It is usual to ignore the Reynolds stress when computing stellar models, partly because *a posteriori* mixing-length estimates are less than the gas pressure gradient in all but a thin region at the top of the hydrogen ionization zone. Attempts to include this stress in nonpulsating stars have been made, notably by Henyey et al. (24) and Parsons (48) but the formulation adopted is not entirely consistent. The mixing-length formula for the Reynolds stress adds second derivatives of temperature and pressure to the hydrostatic equation, raising its order and introducing singular points at the edges of the convection zones. This has led to numerical instabilities (24) which have been removed by judiciously ignoring high derivatives. It is claimed that this should not alter the results substantially. A consistent computation should be done to check.

It is even more important to include the Reynolds stress in pulsating models. The motion of most of the star is almost adiabatic ; density and pressure perturbations are almost in phase and the work done in a single cycle is much less than the energy exchanged between thermal, gravitational and kinetic forms. There is no reason to suppose that the turbulent stress is in phase with the density, however, so even though its magnitude may be much less than the gas pressure gradient the work it does might not be negligible.

The modal approach adopted by Latour et al. (1) includes the Reynolds stress. Because viscosity is include the equations are not singular, but the low Prandtl number gives such severe numerical trouble that it has not yet been possible to compute stellar models with deep convection zones. In principle this method can be used for pulsating stars, but certain aspects of the turbulent energy transfer are lost in the modal truncation and once again the results would have to be treated with some caution.

A recently discovered pulsating star of some interest is the sun (Hill and Stebbins (125) ; Fossat and Ricort (126) ; Severny, Kotov and Tsap (127) ; Brookes, Isaak and van der Raay (128)), which is pulsating in many modes simultaneously. The pulsations are of too low an amplitude to have a noticeable influence on the structure of the star, but they could provide a powerful diagnostic tool. The oscillation periods are in satisfactory agreement with theoretical estimates (Christensen-Dalsgaard and Gough (130), Scuflaire, Gabriel, Noels and Boury (129)), but how the oscillations are driven is not yet known. It is unlikely that convection is unimportant. Differences between linear analyses which have either ignored convective flux perturbations (e.g. (97) Shibahashi et al. (100, 101)) or have taken them into account (Noels (98, 131) et al.) using Unno's (124) approach suggest that the stability of the modes of oscillations are rather sensitive to the assumptions adopted. In the light of experience with stars in hydrostatic equilibrium (Gough and Weiss (19)) perhaps it is too optimistic to hope for an unambiguous solar calibration of time-dependent convection in the near future.

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