Astin Bulletin (2023), **53**, pp. 332–350
doi:10.1017/asb.2023.16

International Actuarial Association
Association Actuarielle Internationale

RESEARCH ARTICLE

Shortcuts for the construction of sub-annual life tables

Jose M. Pavía and Josep Lledó

Universitat de Valencia, Valencia, Spain
Corresponding author: Jose Pavía; Email: pavia@uv.es

Received: 9 October 2022; **Revised:** 15 January 2023; **Accepted:** 22 March 2023; **First published online:** 24 April 2023

Keywords: Quarterly life tables; seasonal-ageing indexes; competitive insurance markets; death rates; life-risk insurances.

Abstract

Fuelled by the big data explosion, a new methodology to estimate sub-annual death probabilities has recently been proposed, opening new insurance business opportunities. This new approach exploits all the detailed information available from millions of microdata records to develop seasonal-ageing indexes (SAIs) from which sub-annual (quarterly) life tables can be derived from annual tables. In this paper, we explore whether a shortcut could be taken in the estimation of SAIs and (life insurance) sub-annual death rates. We propose three different approximations, in which estimates are attained by using just a small bunch of thousands of data records and assess their impact on several competitive markets defined from an actual portfolio of life insurance policies. Our analyses clearly point to the shortcuts as good practical alternatives that can be used in real-life insurance markets. Noticeably, we see that embracing the new quarterly based approach, even using only an approximation (shortcut), is economically preferable to using the associated annual table, offering a significant competitive advantage to the company adopting this innovation.

1. Introduction

In economic terms, mortality plays a pertinent role in the development of countries and regions (Rocco *et al*., 2021), with life tables constituting the fulcrum on which the life insurance business rests. Except for certain events, such as a war or a pandemic, mortality risks evolve smoothly and continuously over time. Indeed, as a rule, annual death rates have been decreasing steadily all over the world since the end of World War II. Many studies have analysed the phenomenon from a dynamic approach (e.g., Lee and Carter, 1992; Cairns *et al*., 2011; Haberman and Renshaw, 2012; Börger *et al*., 2014; Enchev *et al*., 2016; Dong *et al*., 2020), while others have focussed on trying to improve the estimation of life tables from a static perspective (e.g., INE, 2016; ONS, 2012, Pavía *et al*., 2012; Lledó *et al*., 2019). But, whatever the approach, they perform their calculations over a biometric variable measured on an annual basis: the probability that a person of integer age x will not reach age $x+1$, q_x. Mortality and death rates, however, show intra-annual fluctuations.

From the actuarial perspective, the analysis of probabilities of surviving from fractional ages or for fractional durations has been based almost exclusively on the analysis of the well-known three fractional age assumptions (FAAs): a uniform distribution of deaths, a constant (intra-age) force of mortality or the hyperbolic (Balducci) assumption (see Jones and Mereu, 2000, 2002). Extensions of these hypotheses have been developed by Hossain (2011), who proposes a quadratic fractional age hypothesis. All these assumptions, however, can be misleading as deaths and death rates, in addition to intra-age patterns, also show calendar (seasonal) patterns (Rau *et al*., 2018) that interact with age. Intra-annual death rate fluctuations mainly occur due to seasonal variations in temperatures and diseases. For instance, the data show that in Spain the probability of dying (i) tends to be higher during winter, (ii) is not the same for all persons with the same age at last birthday and (iii) depends on both the season and age-quarter and their interaction. For example, In the case of Spain, the probability of dying for a 68-year-old man during winter is slightly higher than the probability of dying for a 68, 68.25, 68.50 or 68.75-year-old equivalent man in either spring, summer or autumn, but the latter is significantly smaller than the probability of dying for a 68.75-year-old man during winter (see details in Table S1, Section S2, of the Supplementary Material). Hence, very recently, several authors (Parks *et al*., 2018; Richards *et al*., 2020; Ledberg, 2020) have begun to study, via modelling and using individual data, how seasonal variations impact on death rates, including their interaction with income and causes of death.

Pavía and Lledó (2022a) go a step further and develop a new hypothesis-free estimator for computing sub-annual death rates that concurrently account for ageing and calendar fluctuations of demographic events. Their estimator, however, relies on large and detailed volumes of data (not always available) to be computed. Indeed, in their application, they use a database of Spain composed of more than 186 million records related to population, deaths, migrations and births, of which 1.5 million are related to deaths. Fortunately, they also show how specific seasonal-ageing indexes (SAIs) can be attained and explain how they can be employed for constructing, from annual tables, new sub-annual tables. They propose estimating SAIs by modelling the intra-annual variations of death rates and encourage them to be used by insurance companies, stating that "*[t]he computed indexes are not only valid for Spain*", but they "*could be used in other areas or countries with similar climatic (and socio-economic) conditions*" (Pavía and Lledó, 2022a:491). But, what happens when these conditions are not met? Furthermore, given that sub-annual death risks are not time stationary (Richards *et al*., 2020), what should we do to update SAIs? Is it necessary to incur again the great data wrangling and computational costs that their approach entails? What is more, what can we do when such detailed data is not available? The aim of this paper is to study whether a shortcut can be taken in the estimation of sub-annual death rates and SAIs.

The estimator proposed by Pavía and Lledó (2022a) has two components: the number of deaths and the total time of exposure to risk. The first component shows evident intra-age and seasonal fluctuations while the second one shows for all ages, excluding extreme ages (new-borns and centenarians), a relatively greater sub-annual stability (in both dimensions, age and calendar time). This begs the following question: for the construction of these SAIs, is the detailed treatment of all the microdata necessary? In other words, would it be enough to have some degree of detailed information just about the deaths?

Using quarterly tables as a case study, this paper answers the previous questions through a two-stage process. On the one hand, it introduces several methodological approximations that allow quarterly SAIs to be obtained just by simply using aggregate quarterly data. On the other hand, it evaluates the new approaches by assessing their impact on several simulated life-insurance markets of rational consumers. In these markets, three hypothetical companies (Jahr, Viertel and Ungefahr) compete selling annual life-risk renewable term policies, with the premiums set by each company being calculated using, respectively, annual tables, full-information SAI-quarterly tables and quarterly tables with approximate SAIs.

The rest of the paper is structured as follows. Section 2 empirically analyses the viability of using shortcuts and proposes three alternative methodological proposals to approximate SAIs and construct quarterly tables. Section 3 details how basic risk premiums are computed using annual and quarterly life tables and defines the market. Section 4 studies the impact of using each of the shortcuts in a scenario without loading the death probabilities, and Section 5 extends the analysis by introducing loadings. Section 6 discusses and concludes.

2. Methodology

Mortality risks are summarised into the so-called life tables, which are composed of multiple biometric variables that depend on age, x (and the reference year, a). A life table (of annual frequency, by default) is built using as seeds death probabilities or rates, q_x or m_x; with other variables, such as the probability that a person of age x reaches age $x+1$, $p_x = 1 - q_x$, being derived from them. Life expectancy at birth, e_0^0, is perhaps the best-known biometric variable, with e_x^0 denoting the (expected) average number of years left to live for a person of age x.

https://doi.org/10.1017/asb.2023.16 Published online by Cambridge University Press

The construction of a (general population) life table depends on the kind of estimator used. To estimate the probability of death, q_x, or the death rate, m_x, one can use either a cohort-based estimator (Pavía *et al*., 2012, Lledó *et al*., 2017) or a period-based estimator (Lledó *et al*., 2019; Wilmoth *et al*., 2020), currently more popular. On an annual basis, the (central) death rate for age x in year a, m_x^a, is obtained as the quotient between the number of people who die at that age, D_x^a, and the central population exposed to risk, L_x^a (e.g., Enchev *et al*., 2016), see Equation (2.1).

$$m_x^a = \frac{D_x^a}{L_x^a} \tag{2.1}$$

From a sub-annual perspective, the study of mortality requires estimating f^2 different rates for each age, f being the number of annual sub-periods that make up a year. In a quarterly frequency ($f = 4$), death rates are computed taking into account both the age-quarter r and the calendar-quarter s when the death occurs. Using notation in Pavía and Lledó (2022a), the mortality rate for a person of age x during her/his quarter of age r and calendar s is defined, accounting for both ageing and seasonal effects, as

$$ {}_s^r m_x^a = \frac{{}_s^r D_x^a}{{}_s^r L_x^a} \tag{2.2}$$

where ${}_s^r L_x^a$ and ${}_s^r D_x^a$ denote, respectively, the number of person-years lived and the number of deaths last birthday x recorded in age-quarter r (i.e., between exact ages $x + \frac{r-1}{4}$ and $x + \frac{r}{4}$) and season-quarter s of year a. The exact calculation of the total time of exposure to the risk of dying, ${}_s^r L_x^a$, requires detailed statistics for the whole population (including exact dates of births) and of immigrants, emigrants and deaths (including the exact dates of the events and their births). For $x = 0$, detailed statistics of births (including exact dates of births) are also required.

Once estimates of annual death rates, $\widehat{m}_x^a$, and quarterly death rates, ${}_s^r\widehat{m}_x^a$, are available for each age-quarter r and seasonal-quarter s in a set of A years, the SAIs of each age x and pair (r, s), $\gamma_{rs}^{(x)}$, are estimated using model (2.3)—where ${}_s^r\varepsilon_x^a$ are random disturbances—, and from them (general population and life insurance) quarterly life tables can derived after performing a number of other statistical computations. In particular, (r, s) quarterly death rates at each age x, and quarterly tables, are obtained after applying to the reference annual death rates the estimated smoothed SAIs, ${}_s^r m_x = \tilde{\gamma}_{rs}^{(x)} m_x$, where m_x is the death rate at age x in the annual table of reference (see Pavía and Lledó, 2022a, for details).

$$\log\left(\frac{{}_s^r\hat{m}_x^a}{\hat{m}_x^a}\right) = \log\left(\gamma_{rs}^{(x)}\right) + {}_s^r\varepsilon_x^a \quad \textit{for} \quad a = 1, 2, \ldots, A \tag{2.3}$$

The procedure described entails individually handling millions of microdata records related to population, deaths, migrations and births. The issue is whether we can reduce the amount of required information for estimating the $\gamma_{rs}^{(x)}$ (SAIs) coefficients and what would be its impact.

To explore this, as stated in the introduction, we exploit the fact that almost all the intra-annual (intra-age and seasonal) fluctuations observed in the death quarterly rates, ${}_s^r m_x^a$, come from the numerators (the number of deaths, ${}_s^r D_x^a$), as the denominators (the average number of exposed-to-risk, ${}_s^r L_x^a$) are relatively much more stable. This fact is clearly visible in Figure 1, where we present for each (r, s) quarter the average of relative differences for deaths (left panel) and exposed-to-risk (right panel) between the actual statistics corresponding to the data employed in this research and their corresponding expected values under the hypothesis of no ageing and no seasonal fluctuations.

In particular, under the hypothesis of no intra-annual fluctuations, we would expect that ${}_s^r D_x^a \approx \frac{D_x^a}{16}$ and ${}_s^r L_x^a \approx \frac{L_x^a}{16}$. Figure 1 shows the results of computing for each (r, s) pair $\Delta_{rs}(L) = \frac{1}{A}\sum_{a=1}^{A}\frac{1}{\pi_2-\pi_1+1}\sum_{x=\pi_1}^{\pi_2}\left|16{}_s^r L_x^a - L_x^a\right|/L_x^a$ and $\Delta_{rs}(D) = \frac{1}{A}\sum_{a=1}^{A}\frac{1}{\pi_2-\pi_1+1}\sum_{x=\pi_1}^{\pi_2}\left|16{}_s^r D_x^a - D_x^a\right|/D_x^a$ using the demographic statistics of Spain for the years 2005, 2006, 2007 and 2008 and for the range of ages from $\pi_1 = 18$ to $\pi_2 = 80$ years old. This range embraces the range of ages in which the risk-life insurance market usually focuses. The differences (in both panels) for the whole range of ages (from 0 to 100) are even more evident (see Figure S1 in Section S1 of the Supplementary Material).

https://doi.org/10.1017/asb.2023.16 Published online by Cambridge University Press

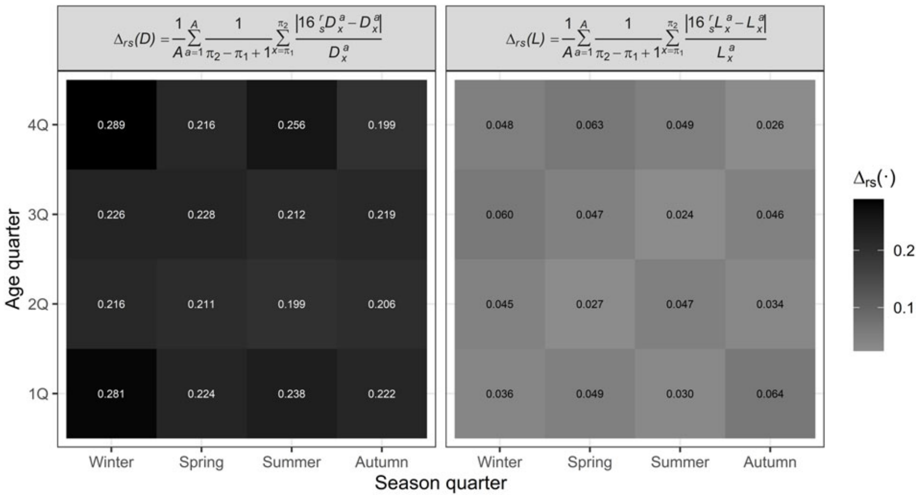

Figure 1. *Average of relative differences for deaths (left panel) and exposed-to-risk (right panel) between actual statistics and their corresponding expected values under the hypothesis of a uniform distribution of deaths and exposed-to-risk among ageing-seasonal quarters. The range of ages covers from $\pi_1 = 18$ to $\pi_2 = 80$. The results have been computed for each age quarter (1Q, 2Q, 3Q and 4Q) and season quarter: Winter (January, February and March), Spring (April, May and June), Summer (July, August and September) and Autumn (October, November and December) using detailed demographic data from Spain for the years 2005, 2006, 2007 and 2008 ($A = 4$); details in the data availability statement.*

2.1. Shortcut 1

As can be seen in Figure 1, the averages of the absolute values of the relative discrepancies of quarterly fluctuations in terms of number of person-years exposed-to-risk (right panel) are significantly less relevant than those observed for deaths (left panel). The latter are around 3.4%, a number that represents just a fifth of the discrepancies seen for deaths. So, as the simplest and first shortcut to estimate SAIs, we propose the assumption that ${}_s^rL_x^a \approx \frac{L_x^a}{16}$, which, after incorporating it into Equation (2.2), leads to the approximation for estimating quarterly death rates given by Equation (2.4).

$$ {}_s^rm_x^a = \frac{{}_s^rD_x^a}{{}_s^rL_x^a} \approx \frac{16\,{}_s^rD_x^a}{L_x^a} \tag{2.4} $$

This approximation allows new relationships to be derived between quarterly death rates and annual death rates for each (r, s)-quarter and age x independent of the total time of exposure to the risk of death. As can be seen in Equation (2.5), these just depend on the total number of deaths recorded in the year, D_x^a, and the quarterly number of deaths recorded in the age-quarter r and season-quarter s, ${}_s^rD_x^a$. In this scenario (shortcut 1), SAIs are estimated using just statistics of deaths:

$$ \frac{{}_s^rm_x^a}{m_x^a} \approx \frac{{}_s^rD_x^a \cdot 16/L_x^a}{D_x^a/L_x^a} = \frac{16\,{}_s^rD_x^a}{D_x^a} \tag{2.5} $$

In particular, starting with Equation (2.6), which is obtained by inserting in Equation (2.3) the approximation (2.5), quarterly life tables are attained from initial estimates of the $\hat{\gamma}_{rs}^{(x)}$ coefficients following a three-step process that involves (i) linearizing the $\hat{\gamma}_{rs}^{(x)}$ estimates, (ii) estimating quarterly death rates, ${}_s^r\tilde{m}_x = m_x \cdot \tilde{\gamma}_{rs}^{(x)}$, where $\tilde{\gamma}_{rs}^{(x)}$ denotes the linearized version of $\hat{\gamma}_{rs}^{(x)}$, and (iii) computing the corresponding quarterly death probabilities, ${}_{\frac{1}{4}}\tilde{q}^{(s)}_{x+\frac{(r-1)}{4}} = \frac{{}_s^r\tilde{m}_x}{4+\frac{1}{2}{}_s^r\tilde{m}_x}$.

https://doi.org/10.1017/asb.2023.16 Published online by Cambridge University Press

$$\log\left(\frac{16\,{}_{s}^{r}D_{x}^{a}}{D_{x}^{a}}\right)=\log\left(\gamma_{rs}^{(x)}\right)+{}_{s}^{r}\varepsilon_{x}^{a}\quad \text{for}\quad a=1,\ 2,\ldots,\ A \tag{2.6}$$

2.2. Shortcuts 2 and 3

Shortcut 1 is simple and scarcely data-demanding. With it, quarterly life tables are derived from annual life tables using as collateral information just the jointly intra-age and intra-calendar distributions of deaths for each age. To reach it, however, we have assumed the hypothesis of uniformity of the intra-annual distributions of demographic events when computing the ${}_{s}^{r}L_{x}^{a}$ quantities; an assumption that has been pointed out as inaccurate in several studies (e.g., Lledó *et al*. 2017, 2019).

In this subsection, we suggest two new shortcuts (shortcuts 2 and 3) by theoretically improving the estimation of the ${}_{s}^{r}L_{x}^{a}$ figures. Keeping the extra amount of information required to perform the computations under control, two alternatives are introduced by relaxing the uniform hypotheses.

On the one hand, in order to improve the estimation of the ${}_{s}^{r}L_{x}^{a}$ statistics, we propose to observe the data in the Lexis scheme (e.g., Lledó *et al*., 2017) and, in addition to deaths, to also consider stocks of populations, but assuming that ins and outs of the population cancel out. In particular, in shortcut 2, we suggest to approximate the ${}_{s}^{r}L_{x}^{a}$ statistics through Equation (2.7):

$$\begin{aligned}{}_{s}^{r}L_{x}^{a}\approx\frac{1}{4}\Bigg[&\frac{1}{8}S_{x-I(s>r)}^{a}-\frac{1}{2}I\left(s>1\right)\sum_{k=1}^{s-1}{}_{\sigma(s-k)}^{\sigma(r-k)}D_{x-I(r-k<1)}^{a}+\\&+\frac{1}{8}S_{x+I(r>s)}^{a+1}+\frac{1}{2}I\left(s<4\right)\sum_{k=1}^{4-s}{}_{\sigma(s+k)}^{\sigma(r+k)}D_{x+I(r+k>4)}^{a}\Bigg]\end{aligned} \tag{2.7}$$

where $I\left(\cdot\right)$ is the indicator function, which takes the value 1 if the condition is met and 0 otherwise; S_{x}^{a} denotes the stock of population aged x last birthday on January 1st of year a; and $\sigma\ \left(\cdot\right)$ is the (permutation) function defined by Equation (2.8), which given any integer number returns an integer number between 1 and 4.

$$\sigma\ (l)=\begin{cases} l\,mod\,4 & if\ l\,mod\,4\neq 0\\ 4 & if\ l\,mod\,4=0\end{cases} \tag{2.8}$$

On the other hand, in shortcut 3, we propose to explicitly consider in and out quarterly flows and approximate the ${}_{s}^{r}L_{x}^{a}$ figures through Equation (2.9):

$$\begin{aligned}{}_{s}^{r}L_{x}^{a}\approx\frac{1}{4}\Bigg\{&\frac{1}{2}\left({}^{\sigma(r-(s-1))}S_{x-I(s>r)}^{a}+{}^{\sigma(r-s)}S_{x+I(r>s)}^{a+1}\right)\\&-\frac{1}{2}I\left(s>1\right)\left[\sum_{k=1}^{s-1}{}_{\sigma(s-k)}^{\sigma(r-k)}D_{x-I(r-k<1)}^{a}-\sum_{k=1}^{s-1}{}_{\sigma(s-k)}^{\sigma(r-k)}I_{x-I(r-k<1)}^{a}+\right.\\&\left.+\sum_{k=1}^{s-1}{}_{\sigma(s-k)}^{\sigma(r-k)}O_{x-I(r-k<1)}^{a}\right]+I\left(s<4\right)\left[\sum_{k=1}^{4-s}{}_{\sigma(s+k)}^{\sigma(r+k)}D_{x+I(r+k>4)}^{a}-\right.\\&\left.\sum_{k=1}^{4-s}{}_{\sigma(s+k)}^{\sigma(r+k)}I_{x+I(r+k>4)}^{a}+\sum_{k=1}^{4-s}{}_{\sigma(s+k)}^{\sigma(r+k)}O_{x+I(r+k>4)}^{a}\right]\Bigg\}\end{aligned} \tag{2.9}$$

where ${}_{s}^{r}I_{x}^{a}$ and ${}_{s}^{r}O_{x}^{a}$ denote the quarterly number of entries and exits registered in the population during the season-quarter s with an age between $x+\frac{r-1}{4}$ and $x+\frac{r}{4}$.

In Equations (2.7) and (2.9), the approximations to the total time exposed at risk during the (r,s)-quarter, ${}_{s}^{r}L_{x}^{a}$, of the whole population is (theoretically) improved by (i) including, within each quarter,

https://doi.org/10.1017/asb.2023.16 Published online by Cambridge University Press

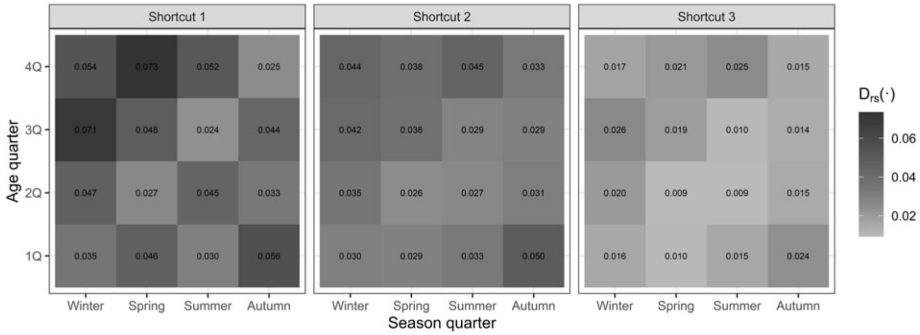

Figure 2. *Average of relative differences for exposed-to-risk between actual statistics and corresponding estimated figures obtained using, respectively, uniform hypothesis (left panel), Equation (2.7) (middle panel) and Equation (2.9) (right panel). The results have been computed for each age-quarter (1Q, 2Q, 3Q and 4Q) and season-quarter (Winter, Spring, Summer and Autumn) using detailed demographic data from Spain for years 2005, 2006, 2007 and 2008 ($A = 4$) and for the range of ages from $\pi_1 = 18$ to $\pi_2 = 80$. Please see the data availability statement for details about the data and its sources.*

the time of exposure linked to the demographic events—deaths in Equation (2.7) and both deaths and migrants in Equation (2.9)—after assuming intra-quarter uniform distributions and (ii) using the stocks of populations available at the beginning and at the end of the reference year. Under an independent uniform distribution of demographic events within each quarter, each death or migrant recorded in the quarter is exposed to risk, on average, $\frac{1}{8}$ of year (or a half quarter). The successive formulae to estimate ${}_s^rL_x^a$ represent, therefore, progressive theoretical refinements—Equation (2.9) relaxes the hypothesis of equality of entries and exits in each quarter assumed in Equation (2.7) and so is expected to perform better—that, at the cost of including more data (being slightly more data-demanding), aim to improve the accuracy of the estimates. This is clearly shown in Figure 2, where we present, using detailed demographic data from Spain corresponding to the years 2005, 2006, 2007 and 2008, the relative average distances between estimated and actual values for the exposed-to-risk in each ageing-calendar quarter for the range of ages from 18 to 80 years old (the interested reader can consult Figure S2 in the Supplementary Material for the full range: 0–100). In the database analysed, the average discrepancies reduce from 0.0339 in shortcut 1 to 0.0100 in shortcut 3, being 0.0298 in shortcut 2. The distances shown in Figure 2 have been calculated with the expression $D_{rs}\left(\hat{L}, L\right) = \frac{1}{A}\sum_{a=1}^{A}\frac{1}{\pi_2-\pi_1+1}\sum_{x=\pi_1}^{\pi_2}\left|{}_s^rL_x^a - \widehat{{}_s^rL_x^a}\right| / {}_s^rL_x^a$, where $\widehat{{}_s^rL_x^a}$ is equal to $\frac{L_x^a}{16}$ in shortcut 1 and is computed via Equations (2.7) and (2.9) in, respectively, shortcuts 2 and 3.

3. Assessing the impact of the shortcuts in a competitive market

In any market, the introduction by a company of an innovation (e.g., a new pricing methodology) has an impact on competitors (Porter, 1980). The life insurance market is no exception. In this market, there are two basic products: a product whose main coverage is survival, life savings, and a product whose main coverage is death, life-risk. The prices set by an insurer for these kinds of products are a function of both extrinsic and intrinsic factors (Berry-Stölzle *et al*., 2010), with intrinsic factors including (i) the pure (or risk) premium, (ii) operating and investment expenses and desired profitability, usually referred to as internal expenses, (iii) commercial expenses, referred to as external expenses, and (iv) legally imposed taxes. The last three components depend on issues such as the effectiveness of the company internal management processes, the sales channel used and the country where the product is marketed.

https://doi.org/10.1017/asb.2023.16 Published online by Cambridge University Press

The pure premium is comprised of the death risk, q_x, plus a safety load premium (Sandström, 2011), δ_x, introduced to compensate for possible unfavourable deviations in the associated stochastic processes (Pavía *et al*., 2019). This premium represents the (minimum) monetary amount required by the insurer to cover (possible) future claims. A correct measurement and selection of risks help ensure unlikely financial losses.

In the pricing of (annually renewable) life-risk products, death probabilities directly impact on the premium of the policies. Omitting the non-biometric components (such as (ii), (iii) and (iv)), the classical (using annual tables) calculation of an annual risk premium, P, involves the probability that a person of age x does not reach age $x+1$, q_x, the capital at risk (or sum insured), C, and the security loading applied, δ_x: $P = C \cdot q_x \cdot (1+\delta_x)$.

When working with quarterly probabilities, the complexity of the formula grows. The premium corresponding to an insured of age $x+\frac{r-1}{4}$ buying the product in quarter s for the 'equivalent' (when $x+\frac{r-1}{4} < x+\frac{1}{8}$) annual premium is computed using Equation (3.1). More details, with a more general example developed without loadings, are presented in Section S4 of the Supplementary Material. There, the interested reader can also find an example for the calculus of a multi-year life-risk insurance.

$$P^{(s)}_{x+\frac{r-1}{4}} = C \cdot \Bigg[{}_{\frac{1}{4}}q^{(\sigma(s))}_{x+\frac{r-1}{4}} + \left(1 - {}_{\frac{1}{4}}q^{(\sigma(s))}_{x+\frac{r-1}{4}}\right) \cdot {}_{\frac{1}{4}}q^{(\sigma(s+1))}_{x+\frac{r}{4}} + \left(1 - {}_{\frac{1}{4}}q^{(\sigma(s))}_{x+\frac{r-1}{4}}\right) \cdot \left(1 - {}_{\frac{1}{4}}q^{(\sigma(s+1))}_{x+\frac{r}{4}}\right) \cdot {}_{\frac{1}{4}}q^{(\sigma(s+2))}_{x+\frac{r+1}{4}} + \left(1 - {}_{\frac{1}{4}}q^{(\sigma(s))}_{x+\frac{r-1}{4}}\right) \cdot \left(1 - {}_{\frac{1}{4}}q^{(\sigma(s+1))}_{x+\frac{r}{4}}\right) \left(1 - {}_{\frac{1}{4}}q^{(\sigma(s+2))}_{x+\frac{r+1}{4}}\right) {}_{\frac{1}{4}}q^{(\sigma(s+3))}_{x+\frac{r+2}{4}} \Bigg] \cdot (1+\delta_x) \tag{3.1}$$

Notice that (3.1) could be (theoretically) improved at the cost of adding more complexity considering different loadings for each (r, s)-quarter. These technicalities are not discussed further in this paper as this would detract from our main focus.

In both annual- and quarterly-based premiums, the actuarial ages of the insured are calculated by approximating their exact ages to either the closest integer with annual tables or the closest quarter with quarterly tables. For instance, the actuarial age for a person of exact age 55.40 years old would be 55 if we use an annual table and 55.50 if we use a quarterly table.

Having reviewed how risk premiums are computed, we consider a perfect competitive market composed of a large number of rational buyers and three life insurance companies (Jahr, Viertel and Ungefahr), with all three of them using as reference the same (life-insurance) annual tables and selling homogeneous life-risk products (annual renewable term (ART) insurances) that only differ in their prices, composed of just pure premiums. Although the three companies assume the same death risk averages by age (and sex), each company applies a different formula to calculate their premiums. Jahr directly uses the annual tables for pricing. Given an insured of (actuarial) age x, Jahr sells its annual (life-risk) renewable term (ART) insurance at a price $P_{x:J}$ regardless of the season of selling. Viertel uses quarterly tables derived from the annual tables by employing the methodology developed by Pavía and Lledó (2022a). It offers its ART policies in season s to an insured of (actuarial) age $x+\frac{r-1}{4}$ at a price $P^{r}_{x,s:V}$. Likewise, Ungefahr also uses quarterly tables but derived from SAIs computed using one of the three shortcuts introduced in the previous section. Depending on which shortcut Ungefahr employs, it sells its ART insurances for an insured of (actuarial) age $x+\frac{r-1}{4}$ in season s at a different price: $P^{r,h}_{x,s:U}$, where h $(=1, 2, 3)$ indexes the shortcut. In our comparative analysis, Ungefahr applies only one of the three possible approaches at a time.

In our market, prices are known to all the agents (policyholders and companies) and buyers make rational decisions. For the same product, they would choose to buy the cheapest one. Companies play with asymmetric information. Jahr, which prices using annual tables, does not know how its competitors compute their premiums. On the other hand, Viertel and Ungefahr do know how Jahr computes them. In any case, the three companies are confident about the methodology they use to set their prices and they compete to sell as many policies as they can.

https://doi.org/10.1017/asb.2023.16 Published online by Cambridge University Press

In economic terms, the premium income is the revenues, R_c (where c stands for J, V or U), that the company receives and the technical profits, $B_c = R_c - L_c$, are obtained after deducting losses by claims, L_c. The claims (losses) are random payment flows that, in actuarial terms, summarize the incidence of mortality that affects the portfolio of the company in a given moment of time. The expected value of the annual losses of a particular policy is computed as a product of its capital at risk, C, and the actual annual probability of death of the insured and it coincides with the pure premium without security loading.

In order to assess the different shortcuts in our competitive market, in the next two sections, we compute the expected values of the technical profits, $E(B_c) = R_c - E(L_c)$, that Jahr, Viertel and Ungefahr would gain competing in a hypothetical market of buyers made up of the insured of the actual portfolio for the year 2009 of a Spanish life insurance company (Lledó and Pavía, 2022). We calculate their market shares and their (relative) expected profits in 12 different scenarios. The scenarios differ in the companies that compete (with the inclusion, or not, of Viertel), the shortcut that Ungefahr employs and the insured composition and capitals at risk of the portfolio. To reduce the complexity of the initial comparisons, we assume in Section 4 that premiums are priced with $\delta_x = 0$. The analysis of the impact of loadings is deferred until Section 5. In all the cases, however, we assume that the actual probabilities of death are the ones obtained after computing SAIs exploiting all the microdata, that is, that they correspond to the ones employed by Viertel. This entails that without including security loadings Viertel has (by definition) null expected values for technical profits.

The comparisons we make in the next two sections are of a realistic nature, since they assess how the different companies/approaches perform in a real portfolio. However, they are to some extent dependent on the age (sex-age) composition of the portfolio. Hence, to better understand how the new approaches differ among themselves and with the classical age fractional methods, we have extended the analysis and included this information in the Supplementary Material. The interested reader can find in Sections S2 and S3 of the Supplementary Material the estimated quarterly death tables (see Tables S1 and S2) that are employed in the next sections as well as an assessment of these by age (see Tables S3 and S4). The assessment is made using as discrepancy measure the averages of the relative differences in absolute values between the premiums to be paid for ARTs calculated employing either the shortcuts or the classical fractional age tables with regard to the premiums to be paid employing the full-information SAIs tables. This shows that, on average, shortcut 1 and FAAs tables incrementally differ from the full information tables as people get older—when the hypothesis of uniform intra-annual distribution of deaths is more frequently rejected (Lledó *et al*., 2017, 2019)—, whereas shortcut 2 and 3 tables are closer to full information tables.

4. Empirical results with null loadings

Section 2 introduced three different shortcuts to derive quarterly tables, and Section 3 detailed a strategy to evaluate them in a competitive market. In this section, we analyse, assuming null loadings and using real data, how the use of the proposed shortcuts would impact two different markets. One is a market where only Jahr (which employs annual tables) and Ungefahr (which employs quarterly tables with approximate SAIs) compete, and the other is a market where Viertel (which employs full-information SAI-quarterly tables) is also a player.

As an annual reference life table from which quarterly tables have been derived, we have used the so-called PASEM2019_second_order table (BOE, 2020). According to the Spanish insurance regulator, this annual table is the (unloaded) risk table to be used as reference for reserving risk-life insurance products by Spanish insurance companies. This table therefore should capture the underlying risk of death of Spanish insured population. We have not used the so-called first order table, recommended for pricing, because this is already a risk-loaded table, and the effect of loadings is analysed in the next section.

SAIs and approximate SAIs have been computed using the database for Spain handled in Pavía and Lledó (2022a). The database, composed of more than 186 million microdata records, consists of detailed

https://doi.org/10.1017/asb.2023.16 Published online by Cambridge University Press

statistics of births, deaths, emigrations and immigrations recorded in Spain during the years 2005–2008 as well as microdata of stocks of population residing in Spain as of January 1st for each of these years. Population and death microdata were provided by the Spanish National Institute of Statistics (INE) and are subject to confidentiality constraints. Microdata of births and migration flows are freely available on the INE website. More details about these data and how they have been handled can be found in the data availability statement accompanying this paper and in Pavía and Lledó (2022a).

The insurance portfolio data comes from a Spanish insurance company whose main business is life-risk insurance. It predominantly sells annual renewable temporary policies with death as main coverage. The portfolio corresponds to the year 2009 and includes insured with ages $x \in [18, 79]$. The data set of the portfolio contains sex and dates of birth of the insured as well as capitals at risk and dates of policy renewal. The total size of the portfolio, including both sexes, contains 76,102 policies, being the third season (summer) the quarter with less sales. The data set and more details about it are available in Lledó and Pavía (2022).

All the calculations have been performed using ad-hoc scripts in the statistical software R, version 4.1.0 (R Core Team, 2021) and dealing separately with men and women. Although, following the Test-Achats case, sex cannot be used to discriminate premiums and benefits under insurance contracts in the EU, the calculations have been done differentiating by sex in order to measure the real impact of the shortcuts.

Table 1 presents a summary of the distribution of the markets in terms of shares of the portfolio and of main components of income statements, assuming $\delta_x = 0$, in the two-company markets composed only of Jahr and Ungefahr: Jahr a company that uses annual tables and Ungefahr a company that is aware of the quarterly fluctuations of annual death risks and employs a shortcut (maybe because it does not have access to all the detailed data) to approximate quarterly death risks. Table 2 presents the same information but in the markets in which Viertel also competes.

Before analysing the numbers in Tables 1 and 2, we should note that some of the differences are not a consequence of the pricing strategies, but a consequence of differences in (i) capitals at risk among the policies, (ii) the particular age distribution of the portfolio, (iii) the uneven impacts of rounding when calculating the actuarial ages, and (iv) the unequal distribution of sales among quarters. Indeed, in the portfolio considered, almost a third of the policies are renewed during the last quarter of the calendar year, almost two-thirds of the insured are men and the majority of policyholders fall between the ages of 40 to 60 (Lledó and Pavía, 2022). Thus, to add robustness to the conclusions, we also study how points (i)–(iv) impact on the results after controlling for their variability, that is by keeping them constant.

First, in order to control the possible noising effect of the heterogeneous composition of the portfolio in terms of both capitals at risk and (r, s) quarterly distribution of insured (points (i) and (iv)), we have also computed the main summary income statements in a 'standardised' portfolio. In our standardised portfolio, all the capitals at risk are fixed at 100,000€ and the same population of age-integer insured (as defined by Jahr) is considered for each of the four quarters of the calendar year. In other words, we assume that for each insured of (annual) actuarial age x in the actual portfolio the standardised portfolio has four replies, but each one of them is buying insurance in a different calendar quarter. Tables S7 and S8 in Section S5 of the Supplementary Material present, respectively, the same measures as Tables 1 and 2 in the standardised portfolio.

Second, in order to disentangle the impact on income statements and market shares of the age distribution of the portfolio, point (ii), Tables S9–S16 in Section S6 of the Supplementary Material offer by age groups (and aggregated by sex) the same summaries presented by sex in Tables 1 and 2. Tables S9–S16 summarise how the portfolio is distributed in four age groups: [18–34], [35–54], [55–64] and [65–79].

Third, given that the rounding can have an uneven impact by company and that it would make Jahr a weak competitor, to study the effect of point (iii) we have simulated a market in which Jahr, in the same fashion as the other companies, also rounds to quarterly ages and prices with a quarterly table, but built under a classical fractional age hypothesis. Despite the fact that, in Spain, decimal ages are rounded to

https://doi.org/10.1017/asb.2023.16 Published online by Cambridge University Press

Table 1. *Summary of the main components of income statements by shortcut in the two competitors' markets in the actual portfolio, assuming* $\delta_x = 0$.

	Shortcut 1		Shortcut 2		Shortcut 3	
Men	Jahr	Ungefahr	Jahr	Ungefahr	Jahr	Ungefahr
Number of Policies	23,948	23,704	23,418	24,234	22,682	24,970
Average capital al risk	93,285€	94,224€	93,583€	93,916€	94,262€	93,288€
Insurance Premiums	3,565,937€	3,864,414€	3,610,586€	3,836,024€	3,609,467€	3,824,392€
Claims	3,694,734€	3,873,150€	3,730,985€	3,836,899€	3,747,641€	3,820,243€
Income Statement	−128,796.90€	−8,736€	−120,398.64€	−875€	−138,173.28€	4,148€
Income Statement %	−3.61%	−0.23%	−3.33%	−0.02%	−3.83%	0.11%
	Shortcut 1		Shortcut 2		Shortcut 3	
Women	Jahr	Ungefahr	Jahr	Ungefahr	Jahr	Ungefahr
Number of Policies	14,524	13,926	14,343	14,107	14,556	13,894
Average capital al risk	88,208€	86,965€	88,611€	86,572€	88,590€	86,563€
Insurance Premiums	868,987€	896,016€	852,983€	915,141€	862,477€	902,445€
Claims	901,167€	895,839€	884,066€	912,939€	895,459€	901,546€
Income Statement	−32,180.11€	177€	−31,082.72€	2,202€	−32,981.79€	899€
Income Statement %	−3.70%	0.02%	−3.64%	0.24%	−3.82%	0.10%

Source: compiled by the authors.

Table 2. *Summary of the main components of income statements by shorcut in the three competitors' markets in the actual portfolio, assuming $\delta_x = 0$.*

	Shorcut 1			Shorcut 2			Shorcut 3		
Men	Jahr	Viertel	Ungefahr	Jahr	Viertel	Ungefahr	Jahr	Viertel	Ungefahr
Number of Policies	19,977	15,527	12,148	19,240	14,592	13,820	20,949	11,353	15,350
Average capital at risk	94,392€	91,729€	95,286€	94,794€	91,289€	94,903€	94,051€	93,447€	93,569€
Insurance Premiums	3,053,574€	2,714,935€	1,629,859€	2,866,443€	2,586,365€	1,942,352€	3,390,905€	1,739,834€	2,276,622€
Claims	3,186,287€	2,714,935€	1,666,662€	2,998,121€	2,586,365€	1,983,398€	3,530,657€	1,739,834€	2,297,392€
Income Statement	−132,713€	0.00€	−36,803€	−131,678€	0.00€	−41,046€	−139,753€	0.00€	−20,770€
Income Statement %	−4.35%	0.00%	−2.26%	−4.59%	0.00%	−2.11%	−4.12%	0.00%	−0.91%
	Shorcut 1			Shorcut 2			Shorcut 3		
Women	Jahr	Viertel	Ungefahr	Jahr	Viertel	Ungefahr	Jahr	Viertel	Ungefahr
Number of Policies	12,557	8,529	7,364	12,641	8,178	7,631	13,279	7,551	7,620
Average capital at risk	88,723€	85,229€	88,431€	88,656€	85,195€	88,428€	88,615€	87,453€	85,977€
Insurance Premiums	782,492€	530,644€	442,891€	756,376€	502,856€	496,344€	809,918€	435,036€	512,925€
Claims	815,512€	530,644€	450,850€	789,072€	502,856€	505,077€	843,347€	435,036€	518,623€
Income Statement	−33,020€	0.00€	−7,959€	−32,696€	0.00€	−8,733€	−33,429€	0.00€	−5,698€
Income Statement %	−4.22%	0.00%	−1.80%	−4.32%	0.00%	−1.76%	−4.13%	0.00%	−1.11%

Source: compiled by the authors.

integer ages when annual tables are used (as in Jahr's case, by definition), a quarterly rounding approach is more realistic with the practice in other markets where fractional ages and adjusted mortality rates are utilised. As the results do not depend on the FAA tables employed, we simply consider the case in which Jahr employs the quarterly tables constructed using the intra-age uniform distribution of deaths (UDD). Tables S17 and S18 in Section S7 of the Supplementary Material are for these markets the equivalents to Tables 1 and 2, respectively. Tables S19–S26 offer the results by sex-age groups.

Focusing first on the two competitors' market (Table 1), we clearly see that adopting the new quarterly-based approach, even when only using an approximation, offers a significantly competitive advantage for Ungefahr in the markets where the competitor does not adopt the innovation, with the advantage tending to be even higher the better the approximation, also in terms of market share (number of policies), which improves after adopting shortcut 3. This issue will become important in the next section, where we expand the analysis by considering scenarios where $\delta_x > 0$.

Switching the analysis to the three competitors' markets (Table 2), we see that, as expected, the shortcuts lose (part of) their attractiveness as soon as a company with a strategy such as that of Viertel is also included in the market. When $\delta_x = 0$, we only observe negative expected values of technical profits for Jahr and Ungerfahr (see Table 2), but significantly smaller for Ungerfahr. What is apparent, however, is the significant share of market that Jahr and Ungerfahr would still gain. This will reveal itself to be key in the next section where we analyse the impact on the technical profits of including loadings when computing pure premiums.

The above results are, moreover, quite robust, albeit with some remarkable deviations that involve (directly or indirectly) age. As we will see, the relative superiority in the whole portfolio of the new approaches reduces/expands in other versions/subparts of the portfolio depending on the suitability of the assumption of uniform intra-annual distribution of deaths, whose age range of validity is different by sex. This can be seen in the case of the two competitors' markets, comparing the relative income statements (a proxy of crude profitability) gathered in Tables S7, S9–S12, S17, and S19–S22 of the Supplementary Material with the equivalent figures presented in Table 1 and, in the case of the three competitors' markets, comparing the relative income statements displayed in Tables S8, S13–S16, S18, and S23–S26 in the Supplementary Material with the corresponding figures shown in Table 2. In what follows, we just focus on the three competitors' markets, since the same remarks apply to the two competitors' markets. In the analysed portfolio, the impact of (i)–(iv) on the robustness of the results is similar in both the two and three competitors' markets.

Points (i) and (iv) have no impact on how the portfolio is distributed among companies and on their profitability. Relative income statements remain (almost) the same as those recorded for the actual portfolio when, instead of the actual portfolio, a similar artificial portfolio, homogeneous in terms of both capitals at risk and intra-annual (quarterly) distribution of subscriptions, is employed (compare Tables 2 and S8).

Some differences emerge decomposing the markets by age groups, point (ii). Despite the general picture attained being the same when we study how profitability is distributed among the different age groups (compare Table 2 with Tables S13–S16), some relevant divergences arise comparing groups among themselves (see Tables S13–S16). Indeed, although it is found that Ungerfahr's profitability is preferable to that of Jahr for the four age groups and in the three shortcuts (with the same order of preference among them), the smallest differences are recorded in the youngest part of the portfolio. Ungefahr's relative income statements and more notably Jahr's ones tend to worsen the older the portfolio, most likely due to a breach of the hypothesis of uniform distribution of deaths present at adult ages (Lledó *et al.*, 2017). In terms of market share the picture by age groups (almost) remains unchanged.

A change in rounding and pricing strategy by Jahr does have an impact, although subtler, at least for the analysed portfolio. When Jahr employs a FAA quarter table after quarterly-rounding ages, there is a redistribution of policyholders among companies, compared to the baseline markets, with Jahr losing significant portions of its policies, mainly related to men. The redistribution, however, has the opposite asymmetric impact by sex in terms of relative income statements. It has almost no effect in the case of the

https://doi.org/10.1017/asb.2023.16 Published online by Cambridge University Press

men subportfolio, but it significantly impacts on the women subportfolio (compare Table 2 with Table S18). The distances of profitability between Jahr and Ungerfahr reduce for the women subportfolio. Although the use of SAIs quarterly tables is still preferable, the distances between the companies can be reduced depending on the age-sex structure composition of the portfolio. To understand why this happens, it is necessary to decompose the relative income statements by age and sex groups (see Tables S23–S26) and relate this to the age-sex composition of the portfolio and the unequal impact that age has in both death risks and their intra-annual distributions by sex. On the one hand, in the portfolio, the percentage of men under 55 years of age is 75%, with that figure rising to 85% in the case of women. On the other hand, the hypothesis of uniform intra-annual distribution of death risks begins to manifest its inadequacy among women at later ages (Lledó *et al*., 2019), with women, moreover, having lower probabilities of dying than men at the same age. The combination of these three facts explains why the distances between profitability in the portion of the portfolio representing women are reduced when a FAA quarterly table is used as alternative to a shortcut table.

Finally, we should note that the results are also robust for the type of life-risk product marketed. In addition to considering a market of ART policy buyers, we also simulate markets where the insured buy three-year term renewable life-risk insurance policies. To make the analysis simpler, in these 3ART markets, we assume that interest rates are null and that companies just sell single premium policies. Tables S27 and S28 in Section S8 of the Supplementary Material offer the equivalents to Tables 1 and 2. As can be seen comparing the numbers in both pairs of tables, the market shares and the relative income statements remain practically the same.

5. The impact of loadings

Section 4 shows in the two and three competitors' markets the impact of using the shortcuts proposed in Section 2 to approximate quarterly life tables with no loading. Under these conditions, we have obtained that (i) it is preferable to use a quarterly table, even built using a shortcut, than not to use it and (ii), as expected, when a company uses quarterly tables derived from SAIs estimated using full information it reaches the best expected technical profits (income statements) in the market. In almost all the cases, however, no positive income statements have been obtained. In real insurance markets, nevertheless, companies use loaded tables to protect themselves from unfavourable deviations on the expected number of deaths in their portfolios and expected positive income statements tend to be the rule. In these cases, market shares play an important role in determining the total amount of expected technical profits.

In order to analyse the impact of loadings, in this section, we study how the expected values of technical profits (in percentages) evolve as a function of the security loadings, δ_x. In particular, we consider all the possible combinations of loadings from 0 to 30% for each of the two/three companies competing in a 1% step fashion and assume that the companies apply the same loading for all the ages and that the portfolio is distributed among companies employing the same rules as in the previous section. A total of 900 scenarios by shortcut are considered in the case of two competitors' markets and as many as 27,000 scenarios by shortcut in the three competitors' markets. Figure 3 summarises in terms of relative income statements the results attained for the two competitors' markets, while Figures 4–6 and Figures S3–S20 in Section 9 of the Supplementary Material show similar financial rates for the three competitors' markets. Additionally, Figure S21 in the Supplementary Material offers, from a loading of 0% to a loading of 100%, the relative technical profits under the constraint that loadings are constant by both company and age.

In our simulated markets, positive expected technical profits are obtained for all the companies with really small loadings as soon as their competitors also load their risks. For instance, with loadings assumed simultaneously the same for all the companies and ages, in the worst of the cases (Jahr strategy competing in a three players market with Ungefahr using shortcut 1), Jahr reaches positive expected technical profits even with a loading as small as 5% (see Figure S21 in the Supplementary Material). It should be noted, however, that Jahr always harvests the smallest relative income statements in these

https://doi.org/10.1017/asb.2023.16 Published online by Cambridge University Press

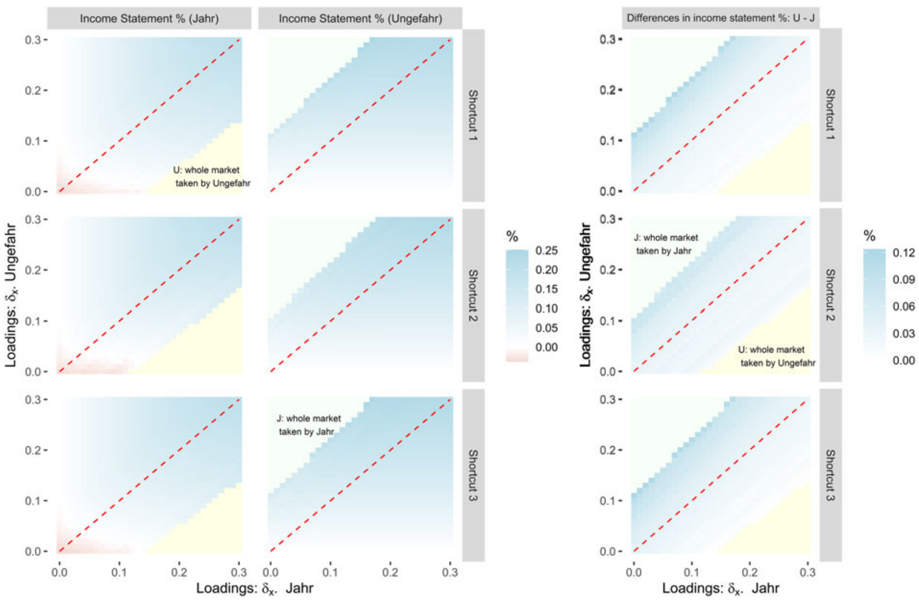

Figure 3. *Relative technical profits (income statements in %) by shortcut in the two competitors' markets in the actual portfolio as a function of the security loadings, δ_x. The far right panels show the differences between Ungefahr's and Jahr's relative income statements. Red lines signal the edges delimiting the different loading areas for the two displayed companies. The areas where one company gains the whole market are shaded using either the colour yellow or green.*

scenarios, with Viertel remaining the company with the largest value with the three shortcuts. Indeed, although relative income statements always grow and their gaps decrease as a function of the loadings, they are only equal when $\delta_x \to \infty$. In these restricted scenarios, however, as market shares remain constant among companies, Ungefahr and Jahr will eventually surpass Viertel in terms of total amount of technical profits.

In the richer scenarios in which each company uses a different loading, the possibilities expand significantly. The relationships among the companies' relative income statements are a function of all the loadings. Although in general all the companies increase their relative income statements as they increase their loadings, they must do so with some care by scrutinising the loadings/prices of their competitors. A company can get out of the market when the loadings it applies are relatively much higher than the ones applied by any of its competitors. This can be seen in Figure 3 for the two competitors' markets and in Figures 4–6 (and Figures S3–S20 of the Supplementary Material) for the three competitors' markets. In this regard, there are combinations of loadings for which the market is completely won by only one company (see, for instance, yellow and green shaded areas in Figure 3) or shared by two companies (see, for instance, wheat, grey and magenta shaded areas in Figures 4–6).

Indeed, in Figures 4–6 (and in Figures S3–S20 of the Supplementary Material), where the relationships between the relative income statements by shortcut for pairs of companies are presented at selected values for loadings of the third (non-displayed) company, one can observe that when a displayed company applies a loading which is quite high compared to the ones chosen by any of the other two companies, this company is rapidly out of the market. This is more evident when the level of loading chosen for the third company to plot the relationships is small, for example, as can be seen in Figures S3–S5 (when $\delta_x = 0$ for the third company) and in Figures S6–S8 (when $\delta_x = 0.05$ for the third company) of the Supplementary Material.

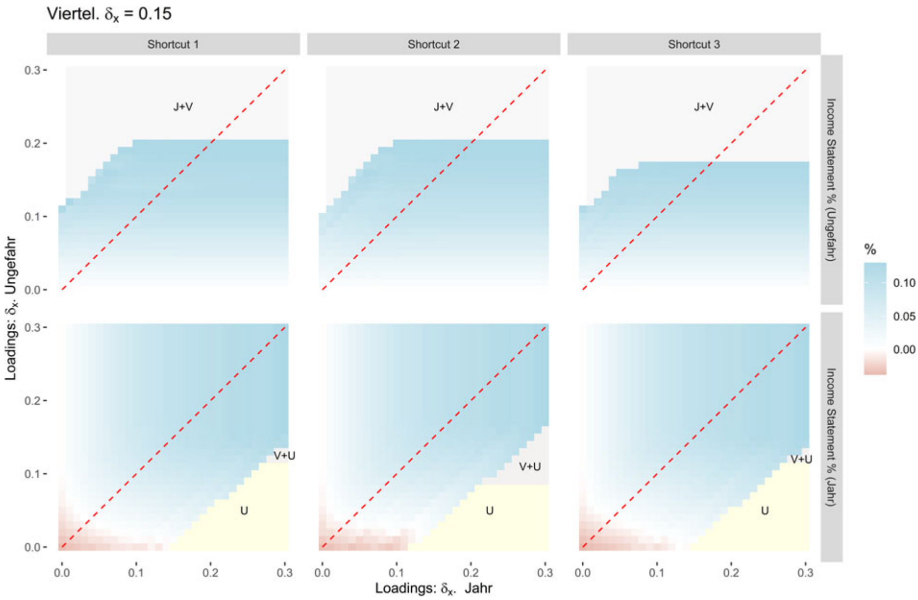

Figure 4. *Ungefahr's and Jahr's relative income statements by shortcut in the three competitors' markets in the actual portfolio as a function of the security loadings, δ_x, keeping Viertel's loading constant at 0.15. Red lines signal the edges delimiting the different loading areas for the two displayed companies. U: whole market taken by Ungerfahr. J+V: Ungerfahr gets out of the market. V+U: Jahr gets out of the market.*

6. Discussion and concluding remarks

Pavía and Lledó (2022a) have recently proposed a new methodology to build sub-annual (quarterly) life tables from annual tables using estimated SAIs. Their approach, however, implies the use of a large amount of data to estimate SAIs. In their application, they exploit a database composed of more than 186 million microdata records. In this paper, we have explored whether shortcuts could be used to approximate the estimators proposed by Pavía and Lledó (2022a) and have studied their impact in several simulated markets derived from a real-life insurance portfolio. To do this, in Section 2, we have proposed three different shortcuts whose data requirements are noticeably smaller. In particular, they only require 1616, 1818 and 5050 summary statistics by sex for each calendar year to compute the approximated SAIs, which contrasts with the million microdata records handled by Pavía and Lledó (2022a).

In our application, as we consider years from 2005 to 2008/9, in shortcut 1, we use 12,928 summary statistics: 16 quarterly death figures for each age times 101 ages (from 0 to 100 ages) times 2 sexes times 4 years. In shortcut 2, we deal with 13,938 statistics: the 12,928 summary statistics used in shortcut 1 plus 1010 population figures (corresponding to 101 ages, 2 sexes and 5 calendar years). And, in shortcut 3, we handle 39,794 summaries: the 13,938 numbers employed in shortcut 2 plus 12,928 quarterly summary statistics corresponding to immigrants and the same number for emigrants.

Our analyses clearly point to the shortcuts as good practical alternatives that can be used in real-life insurance markets. In the currently most plausible scenario in some countries where, due to personal data protection laws, lived big population microdata is not available (which in our application is exemplified by the two competitors' markets), building and using quarterly tables is, whatever the shortcut employed, economically preferable to using the associated annual table, ceteris paribus. It should be emphasized here that the use of general-population estimated SAIs in order to derive sub-annual life insurance tables from annual life insurance tables does not alter the annual mortality patterns of insured tables, but it preserves their well-known inter-age differences. It just assumes similar intra-annual fluctuations of deaths by age in insured and general populations, an issue whose suitability remains to be explored.

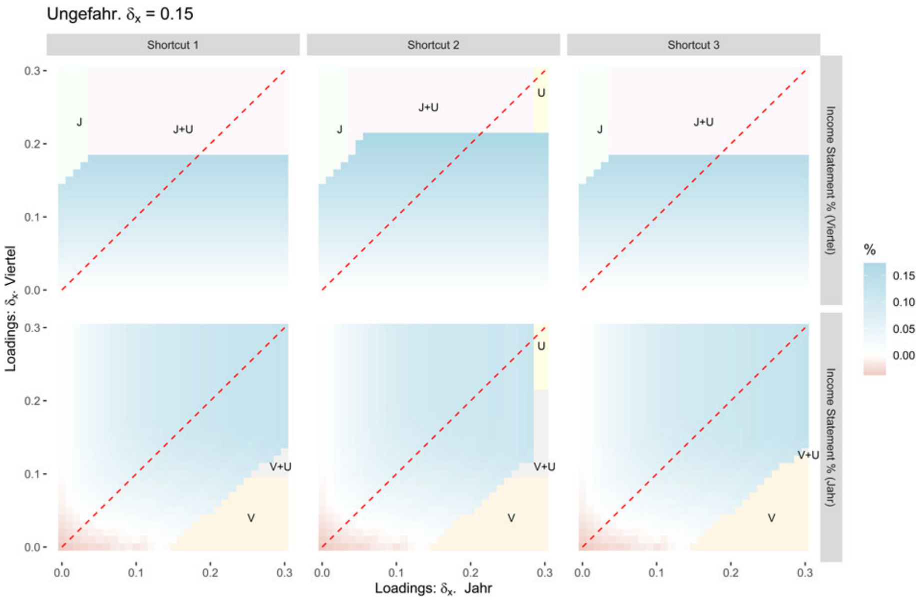

Figure 5. *Viertel's and Jahr's relative income statements by shortcut in the three competitors' markets in the actual portfolio as a function of the security loadings, δ_x, keeping Ungefahr's loading constant at 0.15. Red lines signal the edges delimiting the different loading areas for the two displayed companies. U: whole market taken by Ungerfahr. J: whole market taken by Jahr. V: whole market taken by Viertel. J+U: Viertel gets out of the market. V+U: Jahr gets out of the market.*

Whatever the shortcut, however, it should be noted that having quarterly death summaries is a requisite. All the approximations proposed here require them to be applied. Unfortunately, these summaries are not yet commonly provided by official statistical agencies. However, this does not represent an insurmountable barrier as death microdata records are usually available. Quarterly summary statistics can be very easily computed, by analysts or official statisticians, from the microdata with the help of the function count_events_quarter from the R-package qlifetable (Pavía and Lledó, 2022b).

In practical terms, therefore, the main problem arises when trying to use shortcut 3. When lived population microdata is not available (perhaps due to personal data protection laws), shortcut 3 is barely applicable as summary statistics of migration flows for each ageing-calendar quarter are not usually accessible. With no migration microdata, shortcut 3 could only be employed if quarterly flows were delivered by official statistical systems, an issue that currently, as far as we know, does not regularly happen. If, as usually happens with death records, migration microdata were available, quarterly migration flows could also be directly calculated by the analyst, although in this scenario this shortcut would lose some of its appeal as migration records tend to be more numerous than death microdata. In any case, the cost of dealing with microdata of (deaths and) migration flows is by far significantly smaller than the cost of dealing with the data of the whole population, which is needed to derive full-information quarterly tables. For instance, in the example presented in this paper, the full-information method means dealing with more than 186 million records, a figure that contrasts with the 1.5 million records handled in shortcuts 1 and 2, and with the 6 million records treated in shortcut 3 to compute the summary statistics. Shortcut 1 does not require population data, and in shortcuts 2 and 3, coping with individual records of population is unnecessary as the official statistical agencies ordinarily provide, at least once a year, the total number of people grouped by sex and age.

In our simulated markets with three competitors and constant loadings by company and age, we show that Viertel (the company that uses the full-information table for pricing) will be eventually outperformed by Ungerfahr and Jahr in the expected amount of technical profits due to their higher market

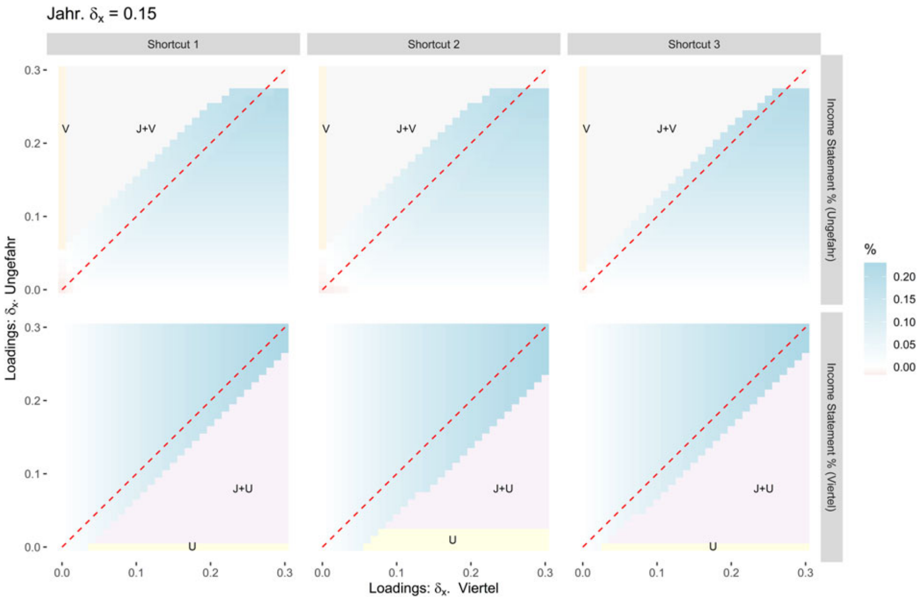

Figure 6. *Ungefahr's and Viertel's relative income statements by shortcut in the three competitors' markets in the actual portfolio as a function of the security loadings, δ_x, keeping Jahr's loading constant at 0.15. Red lines signal the edges delimiting the different loading areas for the two displayed companies. U: whole market taken by Ungerfahr. V: whole market taken by Viertel. J+U: Viertel gets out of the market. J+V: Ungerfahr gets out of the market.*

shares. This could mislead us into concluding that in real markets their approaches would work better than the full-information approach, a conclusion that we consider erroneous. In our view, the relatively better results that Ungerfahr and Jahr would attain as a consequence of them having more ability to gain market shares in markets where each company sets their prices without consideration of the competitors' strategy would not hold in the real world. In real markets, prices are set dynamically and by monitoring competitors' prices, so Viertel, managing more information, has more tools for properly setting its market strategies and adapting to the changes, maintaining profits even in the least profitable scenarios (see Figures S3–S8 in the Supplementary Material).

Finally, we would like to conclude by making a number of comments about our reasoning behind shortcuts 2 and 3 and on the possibilities of refining Equations (2.7) and (2.9). On the one hand, it should be noted that to arrive at shortcuts 2 and 3, we have assumed (i) a uniform intra-annual distribution of births (a hypothesis that can generally be accepted apart from in exceptional circumstances, such as after the end of a war) and (ii) the sizes of the $\frac{1}{8}$ triangles located in the upper-left and lower-right corners of the Lexis scheme of consecutive ages equal in terms of risk exposure times. The second assumption replicates the implicit hypothesis employed by the UK Office for National Statistics (ONS, 2012) to estimate its annual mortality tables. Compared to ONS, the difference is that we work with quarters instead of with blocks of 3 years. On the other hand, taking into account the inaccuracies in official statistics, we conjecture that in practical terms Equations (2.7) and (2.9) could be improved assigning different weights to the risk exposure times prompted by the stocks of population, specifically by allocating more weight the closer the stock of population is to the quarter we are dealing with. Instead of using a unique weight of $\frac{1}{8}$ for both stocks, we could use weights $(9-2s)/8$ and $(2s-1)/8$ for, respectively, a and $a+1$ summands. In this case, we should also add to/subtract from the stocks of population the entries and exits recorded in the quarter with weights equal to ¼. For example, the first term in Equation (2.7) would be $\frac{9-2s}{8}\left(P^a_{x-I(s>r)} - \frac{1}{4}{}^r_s D^a_x\right)$ instead of $\frac{1}{8}P^a_{x-I(s>r)}$. These additional terms do not appear in Equations (2.7) and (2.9) because there they cancel out.

https://doi.org/10.1017/asb.2023.16 Published online by Cambridge University Press

Declarations.

Availability of data and materials. The raw data used in this research include (i) an annual life table, (ii) birth statistics, (iii) stocks of population, (iv) microdata of deaths, (v) emigrant and immigrant microdata and (vi) a portfolio of life insurance policies. The annual life table is available in BOE (2020). Birth statistics were obtained from the section Vital Statistics of Spanish of the National Institute of Statistics (INE) in the link: <https://goo.su/JBrO1DY>. Stocks of population come from the Population Now Cast estimates and were obtained from INE by payment in advance. Death microdata was also obtained by payment in advance. These data were provided under a special agreement contract, according to which under no circumstances should the data be distributed to third parties. Emigrant and immigrant microdata come from the Statistics of Residential Variation and are available in the link: <https://goo.su/rjnoja>. The portfolio of life insurance policies is available in Lledó and Pavía (2022).

Competing interests. The authors declare that they have no competing interests.

Funding. The authors acknowledge the support of Generalitat Valenciana through the project AICO/2021/257 (Consellería d'Innovació, Universitats, Ciència i Societat Digital) and Ministerio de Economía e Innovación through the project PID2021-128228NB-I00. The funding bodies have played neither role in the design, collection, analysis and interpretation of data of the study nor in writing the manuscript.

Acknowledgements. The authors wish to thank three anonymous reviewers and the editors for their valuable comments and suggestions. The authors wish to thank Marie Hodkinson for revising the English of the paper.

Supplementary Material. To view supplementary material for this article, please visit https://doi.org/10.1017/asb.2023.16.

References

Berry-Stölzle, T.R., Hoyt, R.E. and Wende, S (2010) Successful business strategies for insurers entering and growing in emerging markets. *The Geneva Papers on Risk and Insurance - Issues and Practice*, **35**, 110–129.

BOE (2020) Resolución de 17 de diciembre de 2020, de la Dirección General de Seguros y Fondos de Pensiones, relativa a las tablas de mortalidad y supervivencia a utilizar por las entidades aseguradoras y reaseguradoras, y por la que se aprueba la guía técnica relativa a los criterios de supervisión en relación con las tablas biométricas, y sobre determinadas recomendaciones para fomentar la elaboración de estadísticas biométricas sectoriales. *Boletín Oficial del Estado*, **338**, 121566–121602.

Börger, M., Fleischer, D. and Kuksin, N. (2014) Modeling the mortality trend under modern solvency regimes. *ASTIN Bulletin*, **44**, 1–38.

Cairns, A., Blake, D., Dowd, K., Coughlan, G.D. and Khalaf-Allah, M. (2011) Bayesian stochastic mortality modelling for two populations. *ASTIN Bulletin,* **41**, 29–59.

Dong, Y., Huang, F., Yu, H. and Haberman, S. (2020) Multi-population mortality forecasting using tensor decomposition. *Scandinavian Actuarial Journal,* **2020**, 754–775.

Enchev, V., Klinow, T. and Cairns, A. (2016) Multi-population mortality models: Fitting, forecasting and comparisons. *Scandinavian Actuarial Journal,* **2016**, 319–342.

Haberman, S. and Renshaw, A (2012) Parametric mortality improvement rate modelling and projecting. *Insurance: Mathematics and Economics,* **50**, 309–333.

Hossain, S. (2011) Quadratic fractional age assumption revisited. *Lifetime Data Analysis,* **17**, 321–332.

INE (2016) Tablas de mortalidad. In *Metodología*. Madrid: Instituto Nacional de Estadistica. https://www.ine.es/metodologia/t20/t2020319a.pdf

Jones, B.L. and Mereu, J.A. (2000) A family of fractional age assumptions. *Insurance: Mathematics and Economics,* **27**, 261–276.

Jones, B.L. and Mereu, J.A. (2002) A critique of fractional age assumptions. *Insurance: Mathematics and Economics,* **30**, 363–370.

Lee, R.D. and Carter, L.R. (1992) Modelling and forecasting US mortality. *Journal of the American Statistical Association,* **87**, 659–671.

Ledberg A (2020) A large decrease in the magnitude of seasonal fluctuations in mortality among elderly explains part of the increase in longevity in Sweden during 20th century. *BMC Public Health,* **20**, 1674.

Lledó, J. and Pavía, J.M. (2022) Dataset of an actual risk-life insurance portfolio. *Data in Brief*, **45**, 108655.

Lledó, J., Pavía, J.M. and Morillas, F. (2017) Assessing implicit hypotheses in life table construction. *Scandinavian Actuarial Journal,* **2017**, 495–518.

Lledó, J., Pavía, J.M. and Morillas, F. (2019) Incorporating big microdata in life table construction: A hypothesis-free estimator. *Insurance: Mathematics and Economics,* **88**, 138–150.

ONS (2012) Guide to Calculating Interim Life Tables. Office for National Statistics (UK), Hampshire, [online]. Available at https://goo.su/VJA7Vs

Parks, R.M., Bennett, J.E., Foreman, K.J., Toumi, R. and Ezzati, M. (2018) National and regional seasonal dynamics of all-cause and cause-specific mortality in the USA from 1980 to 2016. *Elife,* **7**, e35500.

Pavía, J.M. and Lledó, J. (2022a) Estimation of the combined effects of ageing and seasonality on mortality risk. An application to Spain. *Journal of the Royal Statistical Society: Series A,* **185**, 471–497.

Pavía, J.M. and Lledó, J. (2022b) qlifetable: Managing and Building of Quarterly Life Tables. R package version 0.0.1-13. https://CRAN.R-project.org/package=qlifetable

Pavía, J.M., Morillas, F. and Bosch-Rodríguez, J.C. (2019) Using parametric bootstrap to introduce and manage uncertainty: Replicated loaded insurance life tables. *North American Actuarial Journal,* **23**, 434–446

Pavía, J.M., Morillas, F. and Lledó, J. (2012) Introducing migratory flows in life table construction. *SORT (Statistics and Operations Research Transactions)*, **36**, 103–114.

Porter, M.E. (1980) *Competitive Strategy: Techniques for Analyzing Industries and Competitors*. New York: Free Press

R Core Team (2021) *R: A Language and Environment for Statistical Computing*. Vienna: R Foundation for Statistical Computing. http://www.R-project.org/

Rau, R., Bohk-Ewald, C., Muszyńska, M.M. and Vaupel, J.W. (2018) *Visualizing Mortality Dynamics in the Lexis diagram*. Cham, Switzerland: Springer International Publishing

Richards, S.J., Ramonat, S.J., Vesper, G.T. and Kleinow, T. (2020) Modelling seasonal mortality with individual data. *Scandinavian Actuarial Journal*, **2020**, 864–878.

Rocco, L., Fumagalli, E., Mirelman, A.J. and Suhrcke, M. (2021) Mortality, morbidity and economic growth. *PLoS ONE,* **16**, e0251424.

Sandström, A. (2011) *Handbook of Solvency for Actuaries and Risk Managers: Theory and Practice*. Chapter 21, Chapman & Hall/CRC Finance Series.

Wilmoth, J.R., Andreev, K.F., Jdanov, D.A. and Glei, D.A. (2020) Methods Protocol for the Human Mortality Database. Human Mortality Database, University of California Berkeley and Max Planck Institute for Demographic Research [online]. Available at http://www.mortality.org/

https://doi.org/10.1017/asb.2023.16 Published online by Cambridge University Press