LINEAR AND NONLINEAR CONVECTION WITH AN ALIGNED MAGNETIC FIELD

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Abstract: The effect of horizontal magnetic field on the onset of three-dimensional convection in a horizontal fluid layer is studied. It is found that the two-dimensional solutions are unstable to three-dimensional disturbances. A detailed bifurcation study is reported.

Introduction: Convection in the presence of a vertical magnetic field has been extensively investigated because of its relevance in astrophysics and geophysics [1]. It is known that the penumbra of the sunspot is a well developed region for temperature about 5000°K where the magnetic field lines expand rapidly to become more horizontal of strength about 2 KG. Though small, they are important in providing a mechanism to cool the photosphere and upper convection zone [1-3]. Therefore, the effect of horizontal magnetic field on convection has been studied in [2-3] by considering only two-dimensional disturbances. Its effect on three-dimensional convection has not been given attention. This is considered in this paper with the object of showing that the two-dimensional solutions may become unstable to three-dimensional disturbances which possess added degree of freedom in the study of heat transfer. We have investigated both linear and nonlinear convection in the presence of horizontal magnetic field. Due to want of space we present here only the results pertaining to linear theory.

Mathematical Formulation: We consider a Boussinesq electrically conducting fluid layer bounded by stress-free surfaces. The basic equations of motion are conservation of mass, momentum, energy and magnetic field as given in [4]. The usual process of linearization and employing normal mode analysis, we get

$$R = k^2[k^4 - p^2/\sigma + (p^2 + Bk^4)G/(p^2 + B^2k^4)]/\pi^2\alpha^2 + ipk^2N, \qquad (1)$$

with $N = k^2(\sigma+1)/\pi^2\alpha^2\sigma + (B-1)G/(p^2+B^2k^4)$. Here $R = \beta g\Delta Td^3/\nu\kappa$, $Q = \mu H_0^2 d^2/\rho_0\nu\nu_m$,

$B = \nu_m/\kappa$, $\sigma = \nu/\kappa$, $k^2 = \pi^2(\alpha^2+1)$, $\alpha^2 = l^2 + m^2$, $G = QB\pi^2\ell^2$. Where ℓ and m are wavenumbers in x and y directions and p is the frequency. R is a physical quantity, therefore (1) demands that either p = 0 or N = 0. These conditions help us to study the types of bifurcations.

Direct bifurcation (p = 0): Now from (2), we get

$$R^{(s)} = k^2[k^4 + Q\pi^2\ell^2]/\pi^2\alpha^2. \qquad (2)$$

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This becomes minimum, for a fixed value of ℓ, when $\alpha = \alpha_c$ is a root of $2\alpha_c^6 + 3\alpha_c^4 = 1+Q\ell^2/\pi^2$. As $Q \to \infty$, $\alpha_c \sim \sqrt[6]{Q\ell^2/2\pi^2}$ and $R^{(s)} \sim Q\pi^2\ell^2$. As $Q \to 0$, $\alpha_c \sim 1/\sqrt{2}$, $R^{(s)} \sim 27\pi^4/4$.

Hopf bifurcation ($p \neq 0$, $N = 0$): There is a Hopf bifurcation at

$$R^{(0)} = k^6(\sigma+B)(1+B)/\sigma\pi^2\alpha^2 + k^2(\sigma+B)G/\pi^2\alpha^2(1+\sigma), \tag{3}$$

provided $p^2 = \alpha^2\sigma B(R^{(s)} - R^{(0)})/(\alpha^2+1)(1+\sigma+B) = -B^2k^4 + (1-B)\sigma G/(1+\sigma) > 0$. So the necessary condition for the existence of a Hopf bifurcation is $B < 1$, $Q > B(1+\sigma)k^4/\pi^2\ell^2\sigma(1-B)$. We note that when $\ell = \alpha$ (i.e. $m = 0$), (2) and (3) tend to those given in [2] for two-dimensional disturbances.

Conclusions: The critical Rayleigh numbers for direct and oscillatory motions are computed for different values of B and Q and the results are compared with those of two-dimensional disturbances in Fig. 1. This shows that the two-dimensional solutions become unstable to three-dimensional disturbances. The nature of bifurcation, for different values of σ, is depicted in Fig. 2. The bifurcation from steady to oscillatory solutions occurs earlier for small values of σ compared to the large values of σ. These conclusions are used in the study of nonlinear theory.

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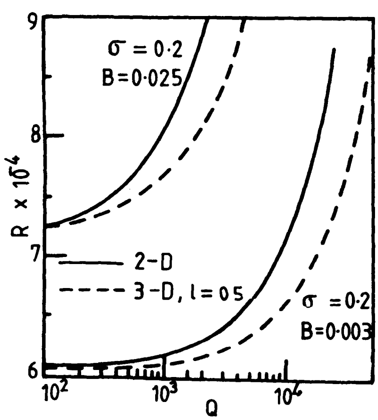

Fig.1 Critical Rayleigh number Vs.Q

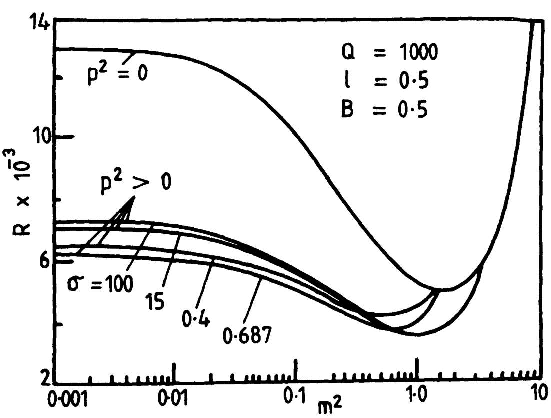

Fig.2 Curves of neutral stability

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