Extremely Luminous Atmospheres

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Abstract. We present the effects that inhomogeneities have on radiating atmospheres. It is shown that nonuniformities in a medium induce a reduction of the effective opacity which subsequently increases the Eddington Luminosity. The most striking effect however that arises from the dependence of the opacity on the inhomogeneities, is the possibility of a phase transition, where the atmosphere energetically favors exciting horizontally propagating waves due to large fluxes.

Atmospheres with a large radiative flux are extremely interesting as they are important for the behavior of objects such as luminous stars, novae or accretion disks. Moreover, they exhibit many effects in radiative hydrodynamics. We summarize here two of them. We first show that the effective bulk opacity of an inhomogeneous medium is changed and that this can subsequently induce a phase transition in a very luminous atmosphere.

The important effect that arises when a system becomes inhomogeneous is the change of its effective opacity. Shaviv (1998a) has shown that the effective opacity relevant for the calculation of the average radiative force is not necessarily a simple mass or volume weighted average of the effective opacity. Instead, the opacity per unit mass is generally given by:

$$\kappa_m^{eff} = \langle H \kappa_m \rho \rangle \,/\, (\langle H \rangle \langle \rho \rangle). \tag{1}$$

It is found by comparing the total radiative force on the system with the total flux. For small amplitude perturbations of the form $\delta\rho/\rho = \delta \cos(kx - \omega t)$, the effective opacity becomes[1]: $\kappa_m^{eff} \approx \kappa_m^{(0)} / \left(1 + \nu \delta^2\right)$, where ν is a constant that depends on the wave type and on the form of the opacity. If for example $\kappa_m \propto \rho^\alpha T^\beta$ and $T \propto \rho^\mu$ then optically thin and thick waves respectively have (Shaviv 1998b):

$$\nu_{thin} = -\left[(\alpha(\alpha+1) + \beta\mu^2(\beta-1) + 2(\alpha+1)\beta\mu\right]/4. \tag{2}$$

$$\nu_{thick} = \left[(\alpha+2)(\alpha+1) + \beta\mu^2(\beta+1) + 2(\alpha+1)\beta\mu\right]/4. \tag{3}$$

Evidently, the bulk opacity can under various circumstances be reduced. We now show that this induces a phase transition. To see it, we calculate the atmosphere's energy when a perturbation is added to it. If we find that the homogeneous equilibrium is not a minimum of the energy when small adiabatic perturbations are added, then it will be unstable as the latter will grow. The system will then have a new equilibrium state.

[1] The expression is accurate only for small amplitudes; since we wish to qualitatively analyze also large ones and strong fluxes, we choose an expression that correctly gives the quantitative behavior, namely, that $\kappa_{eff} \to 0$ for $\delta \gg 1$.

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For simplicity, we assume that the relative perturbation is not a function of height and that the atmosphere resides on a rigid surface. Although generally not the case, these assumptions simplify the derivation as the specific energies become independent of height and of global energy changes in the system (e.g. a star) as a whole when the opacity is changed.

The basic equation is the hydrostatic equation with the radiation forces:

$$\frac{dp}{dz} = c_T^2 \frac{d\rho}{dz} = (-g + g_{rad})\rho \quad \text{with} \quad g_{rad} = g_{rad}^{(0)} \frac{\kappa_{eff}}{\kappa_0}. \tag{4}$$

Here c_T the isothermal speed of sound and $g_{rad}^{(0)}$ is the unperturbed acceleration due to radiation. The latter is changed from its unperturbed value when the opacity has corrections due to inhomogeneities. After integration, one finds that $\rho = \rho_0 \exp\left(-g_{eff}/c_T^2 z\right)$, with $g_{eff} = g - g_{rad}$. Even if the atmosphere is perturbed, the total mass of it per unit area – Σ should be constrained to remain the same. Using this condition we find that $\rho_0 = \Sigma g_{eff}/c_T^2$.

Two terms contribute to the total energy when a wave of amplitude δ is excited. The first is the acoustic energy in the wave per unit area. It is $A = (\epsilon/2)\Sigma\delta^2 c_T^2$, where ϵ is a constant that depends on the type of wave. The second is the potential energy. It is composed of the interaction with both the gravitational and radiation fields. For an optically thin atmosphere, one finds after proper integration that (Shaviv, 1998b):

$$U + A - U(\Gamma_0 = 0, \delta = 0) = \Sigma c_T^2 \left[-\ln\left(\frac{1 - \Gamma_0 + \nu\delta^2}{1 + \nu\delta^2}\right) + \frac{\epsilon}{2}\delta^2 \right] \tag{5}$$

where we have introduced Γ_0 – the ratio between the radiation pressure gradient and the unperturbed Eddington Limit. For small amplitudes, we have:

$$U + A - U(\Gamma_0 = 0, \delta = 0) \approx \Sigma c_{T^2} \left(\frac{\epsilon}{2} - \frac{\nu\Gamma_0}{(1 - \Gamma_0)}\right)\delta^2 \tag{6}$$

Evidently, the total energy of the system is quadratic in the amplitude of the perturbations. However, the coefficient of the second order term can be either positive or negative depending on the value of Γ_0, if ν is positive. For small values of Γ_0, the sign is clearly positive and a zero amplitude wave is clearly the least energetic, however, above the critical value of:

$$\Gamma_{crit} = 1/(1 + 2(\nu/\epsilon)), \tag{7}$$

it is apparent that by exciting a small amplitude wave, the total energy of the system will be lowered and thus more favorable! Moreover, the most favorable excitation or the one that is excited first when the flux is increased, is the one that has the lowest Γ_{crit}. In other words, the least stable mode is the one that has the lowest ϵ to ν ratio – a larger opacity change with a lower energy excitation cost.

We also see that one cannot find the equilibrium from the linear or small amplitude analysis. Although eq. 5 is quantitatively valid only for small am-

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plitudes ($\delta^2 \ll 1$), we can use its qualitative behavior to understand what happens at large amplitudes as well. This behavior can be seen in figure 1.

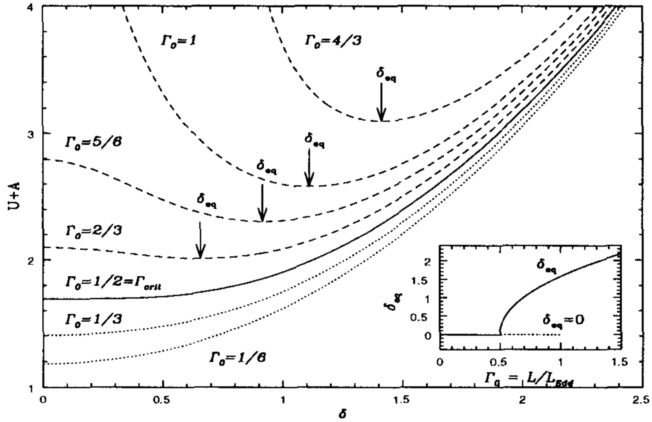


Fig. 1. The total energy of an atmosphere for different Γ_0 (fluxes) when g, c_T, ϵ and $2\nu = 1$ (according to eq. 5). For values of Γ_0 that are less than the critical value, the minimum of the total energy is at the origin. When Γ_0 is larger, the minimum energy is obtained for a finite amplitude wave. The inset depicts the equilibrium value as a function of Γ_0. It is of course *accurate* only for small amplitudes. The dotted line represents the unstable equilibrium of $\delta_{eq} = 0$ that exists for $\Gamma_{crit} < \Gamma_0 < 1$.

When Γ_0 is smaller than the critical value, the equilibrium amplitude is 0. When Γ_0 is larger, the total energy has a minimum for a finite amplitude of δ given by:

$$\delta_{eq}^2 \left(\Gamma_0 > \Gamma_{crit}\right) = \left(\Gamma_0 - 2 + \sqrt{\Gamma_0^2 + 8\Gamma_0(\nu/\epsilon)}\right) \Big/ \nu. \tag{8}$$

It approaches 0 for $\Gamma_0 \to \Gamma_{crit}^+$. Namely, it is similar to a second order phase transition: The order parameter δ is continuous but its derivatives are not.

We have seen two interesting characteristics of radiative hydrodynamic flows. First, inhomogeneities can decrease the effective opacity of a medium, and second, that this change can reduce the potential energy of the system and induce a phase transition. The effects have interesting implications to luminous objects. They affect phenomena like wind acceleration, change characteristics such as the Eddington luminosity and induce both spatial and temporal variability. A numerical simulation exhibits the qualitative results found here. A more general treatment can be found in Shaviv (1998b).

References

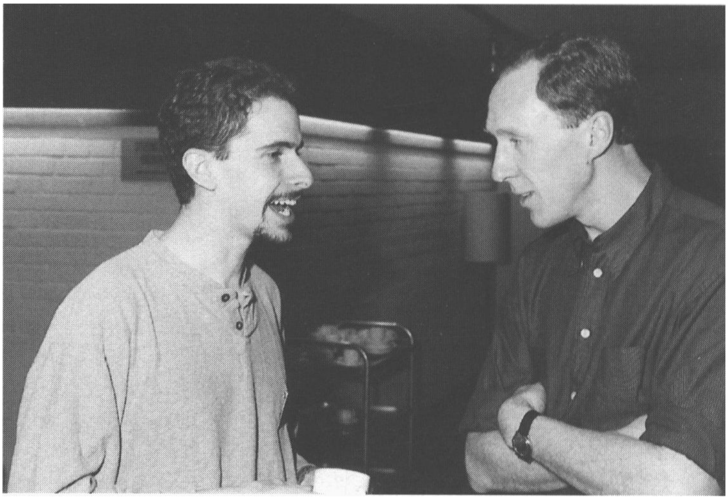
Shaviv, N. J., (1998a): *The Eddington luminosity limit for multi-phased media.* The Astrophys. J., **494**, L193.

Shaviv, N. J., (1998b): *A phase transition of luminous atmospheres.* Submitted to the Astrophys. J.

Discussion

S. Owocki: In your simulation, you eventually find only one horizontal structure in the periodic box. In a real star, what sets the limiting horizontal scale?
N. Shaviv: From the analytical treatment we know that different wavelengths are preferred for different opacity laws. Often, as was the case in the simulation, the preferred scale is $\sim 2\pi H_p$. Since the horizontal extent is roughly that, the periodic condition forces a wavelength that is exactly the width of the box. A larger horizontal extent gives a simulation that results in two wavelengths in the box.

S. Shore: What is the effective viscosity of your calculations? In other words, what is your effective Reynolds number?
N. Shaviv: The viscosity is of course limited by the finite resolution of the simulation (100x100). An increase in the resolution reduces the viscosity and could theoretically introduce more phenomena. This is why an analytical treatment is done as well.



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