Appendix A: Derivation of Cost Model

Let f_α be the estimated frequency of administration of antidote α, expressed in cases per person per year. The cost of the formulation stocked is denoted **$\$_\alpha$**, and the recommended shelf-life λ_α. Antidotes for which full credit is given by the supplier upon return of expired lots, therefore, have $\lambda = \infty$. For simplicity, antidotes with \$<\$20 had λ set to 1 year.

The initial cost required to assemble the antidotes is the sum $\sum \$_\alpha$ of all antidotes selected. This cost is independent of f_α (and hence the population base served and hospital size) and λ_α.

The annual maintenance cost is determined by considering the probability **$\Pr(\iota)$** of the antidote being administered $\iota = 0,1,2$ or more discrete times during the finite interval λ_α. Assuming independence of events, this probability is the Poisson distribution for rare events $\Pr(\iota) = \dfrac{e^{-p} p^{\iota}}{\iota\,!}$, where $\boldsymbol{p} = f_\alpha \lambda_\alpha N$ is the expected number of cases encountered during the interval λ_α in a population of $\boldsymbol{N}$.

For each antidote, the maintenance cost over the entire interval λ_α can be estimated by taking into consideration the conditional probability $\Pr(\iota)$ of ι uses of the antidote, multiplied by ι replacement costs, summed for each value of ι. This value must be corrected by the value of unexpired antidote remaining at the end of the original interval λ_α, since this fresh replacement antidote will expire some time later.

Thus, with a $\Pr(\iota = 0) = e^{-p}$, the antidote in question will go unused and expire at the end of its shelf-life, resulting in a maintenance cost during that λ_α equal to the annualized straight-line amortization of $\$_\alpha / \lambda_\alpha$. If the antidote is actually administered at least once during the interval λ_α, the maintenance cost then becomes the ongoing restocking plus replacement cost $(\iota+1)\$_\alpha$ minus the benefit of having unexpired product at the end of the original interval λ_α. For example, replacing the antidote after using it 18 months into its 24-month shelf-life effectively extends the expiry date by 18 months beyond the previous expiry date. It can be shown by counting that there is, on average, $\left(\iota/_{\iota+1}\right)\lambda_\alpha$ unused shelf-life in the last batch of antidote purchased, when ι doses of antidote are administered at random independent times during the interval λ_α.

Therefore, the model for annual maintenance cost to keep antidote α stocked, ignoring inflation and using straight-line amortization, is given by

$$\frac{\$_\alpha}{\lambda_\alpha} \sum_{\iota=0}^{\infty} \Pr(\iota)\left(\iota + 1 - \frac{\iota}{\iota+1}\right) = \frac{\$_\alpha}{\lambda_\alpha} \sum_{\iota} \Pr(\iota)\left(\iota + \frac{1}{\iota+1}\right) = \frac{\$_\alpha}{\lambda_\alpha} \cdot e^{-p} \sum_{\iota} \frac{\left(\iota + \frac{1}{\iota+1}\right)}{\iota!} \cdot p^{\iota} \quad .$$

The reader will note that the sum in this formula is now in the form of a power series, $\sum_{\iota=0}^{\infty} a_{\iota} x^{(k+\iota m)}$, which can be used to calculate the sum using standard techniques.

Of course, if the antidote has $\lambda_\alpha = \infty$ (is replaced free of charge if expires prior to use), the probability of expiry and the depreciation both are nil, and the annual cost is determined solely by the number of cases actually encountered, or $f_\alpha \$_\alpha N$.

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