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ICE FORCES

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ABSTRACT. When large masses of ice, for example in the form of glaciers or floating ice fields, are moving against an obstruction, either natural or man-made, huge forces will develop. It is possible to derive formulae to estimate these forces from elementary mechanics of the rupture of ice. The basic formula may be written: $F = \phi(C)\, r_c e d$, where $\phi(C)$ is a function of the geometrical and physical parameters of the system, r_c is the compression strength of the ice, e is the thickness of the ice, and d is the width of the structure. The paper deals with the different types of rupture patterns that may develop in front of either vertical or inclined faces, and formulae are given for some typical cases.

RÉSUMÉ. *Forces de glace*. Lorsque de grandes masses de glace, par exemple sous forme de glaciers ou de champs de glaces flottantes, se meuvent et se heurtent à un obstacle, naturel ou construit par l'homme, d'énormes forces sont développées. Il est possible de tirer des formules pour estimer des forces sur la base de la mécanique élémentaire des ruptures de la glace. La formule de base peut s'écrire: $F = \phi(C)\, r_c e d$ ou $\phi(C)$ est une fonction de paramètres géométriques et physiques du système, r_c est la résistance à la compression de la glace, e est l'épaisseur de la glace, et d est la largeur de la structure. Cet article examine les différents types de ruptures qui peuvent se développer devant des façades verticales ou inclinées et on donne des formules pour quelques cas typiques.

ZUSAMMENFASSUNG. *Eiskräfte*. Wenn grosse Massen von Eis, z.B. in Form von Gletschern oder schwimmenden Eisfeldern, sich gegen ein natürliches oder künstliches Hindernis bewegen, entwickeln sich starke Kräfte. Mit Hilfe der grundlegenden Bruchmechanik für Eis lassen sich Formeln ableiten, mit denen diese Kräfte abgeschätzt werden können. Die Grundgleichungen lassen sich folgendermassen schreiben: $F = \phi(C)\, r_c e d$, wobei $\phi(C)$ eine Funktion der geometrischen und physikalischen Parameter des Systems, r_c die Druckfestigkeit des Eises, e die Eisdicke und d die Breite des Hindernisses bedeutet. Die Arbeit behandelt die verschiedenen Typen von Bruchmustern, die sich vor einer senkrechten oder geneigten Fläche bilden können, und gibt die Formeln für einige typische Fälle.

1. INTRODUCTION

This paper deals with the behaviour of ice-floes, considered as floating plates, which rupture against obstructions of various shapes. The velocity of contact is large enough to exclude plastic deformation, and the ice is assumed to behave as an elastic material until failure. It deals essentially with ice of a constant thickness, formed in water.

Ice formed over land—such as glaciers—will often move slowly and permit plastic deformation, but there are instances, where the forces acting on a glacier change at a rate too rapid for plastic deformation, and in these conditions, where elastic behaviour may be reasonably assumed, an analysis comparable to the investigations in this paper would be applicable, as for example when a glacier is broken up by entering the sea and thus in a violent encounter calves icebergs.

2. RUPTURE PATTERNS IN INDENTATION PROBLEMS

In problems involving the fracture of materials it is natural to study the type of rupture which is visible in the form of rupture lines and/or zones. This principle has been applied within the field of soil mechanics with great success, and it is a fact that the theory developed by Coulomb in 1776 is still being used.

Rupture planes of a type which have been observed in model experiments in front of a vertical obstruction (a pile or a bridge pier) are illustrated in Figure 1. The floe is moving towards the vertical face of the structure and the rupture takes place in shear along inclined surfaces as shown in Figure 1. The effect of the shear on the triangular vertical end faces becomes significant when the width d of the structure is of the same order as the thickness of the ice sheet.

The shear strength is assumed to be one-half of the compressive strength.

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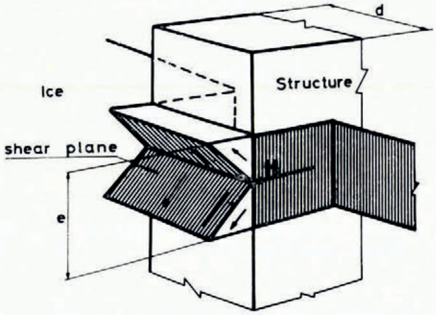

Fig. 1. Shear rupture.

By expressing the equilibrium of the forces in the rupture situation as illustrated in Figure 1, it is possible to show that the horizontal force may be written (Tryde, [c1976][a])

$$F_{\max} = r_i e d \quad \text{(kN)}, \tag{1}$$

where (for $0 \leqslant e/d \leqslant 2$)

$$r_i = k r_u = (1+1.5e/d)\, r_u \quad \text{(kN/m}^2\text{)}, \tag{2}$$

where $k = 1+1.5e/d$ is the indentation factor (dimensionless), r_i is the indentation strength (kN/m²), e the thickness of the ice (m), d the width of the structure (m) and r_u the reference indentation strength for $k = 1$ (kN/m²).

For r_u, the value $r_u = 0.8r_c$ has been proposed by the author, r_c being the uniaxial compressive strength of the ice.

A formula giving a better agreement with existing test results and theories derived by others may be written (Tryde, [c1976][a]; Assur, 1972; Afanas'yev, 1972; Schwarz and others, unpublished)

$$k = 1+2.1(0.4+d/e)^{-1} \quad (0.1 \leqslant d/e \leqslant \infty), \tag{3}$$

as shown in Figure 2.

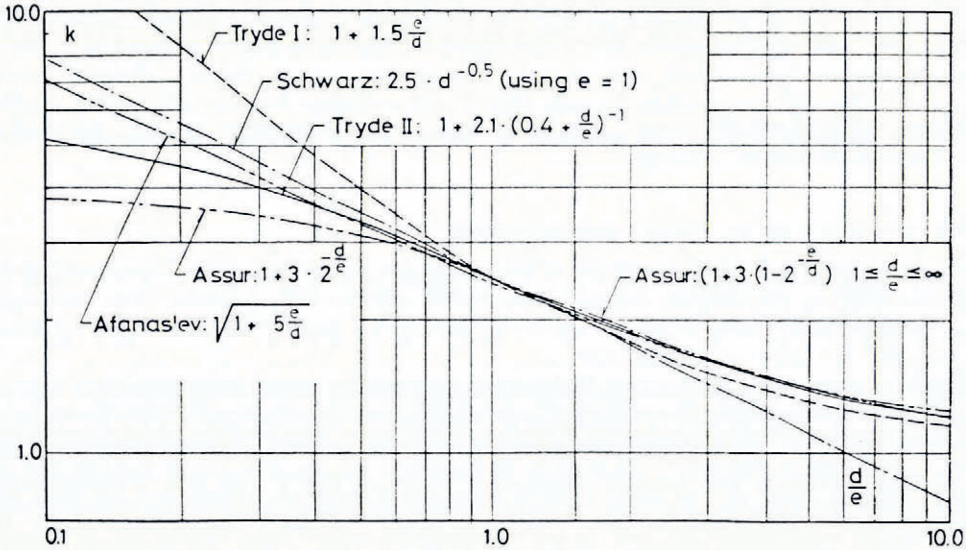

Fig. 2. Indentation strength as a function of d/e.

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For comparison with results of others see Figure 2. All curves have been normalized to give $k = 2.5$ for $e/d = 1$. For $d/e = 0.1$ the value of k in the above formula becomes 5.2.

This shows that for slender structures, such as piles, the indentation strength is considerably higher than the compressive strength. Neglect of this effect can therefore result in the failure of piles in ice such as has been observed in piles in Danish waters.

Finally it should be noted, that in any rupture theory some specific pattern of rupture is assumed to take place, and based on that it is possible to derive formulae giving the rupture forces. These formulae may not be valid if the rupture is of a different pattern, as for instance is the case at initial contact.

3. Rupture patterns at vertical wedges

Instead of a vertical face perpendicular to the motion of the floe (see Section 2) the obstruction may be formed as a vertical wedge.

What will the rupture pattern be in that case?

If the floe is rather small and hits the structure centrally, it may be split in two by a relatively small force.

If the floe is large, the wedge will cut into the ice sheet and the force will increase gradually, until it reaches its maximum value at full penetration. For a floe drifting against the wedge, the movement will be arrested when the kinetic energy is used up in the rupture process, which means that if the energy is not sufficient to produce full penetration, the floe will only move partly into the wedge and stop, and the force will be reduced correspondingly (Fig. 3).

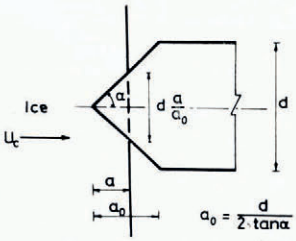

Fig. 3. Partial penetration.

For wide structures ($d/e > 3$) the force for penetration a may be written

$$F_a = r_c e d \frac{a}{a_0} = 2 r_c e a \tan \alpha. \tag{4}$$

The kinetic energy of the floe is equal to the work done by the rupture force

$$\tfrac{1}{2}\rho A e u_c^2 = \tfrac{1}{2}F_a a = r_c e a^2 \tan \alpha, \tag{5}$$

from which the penetration distance a may be found from the formula

$$a = u_c \left(\frac{A\rho}{2r_c \tan \alpha}\right)^{\frac{1}{2}} \quad \text{(m)}. \tag{6}$$

If $a < d/(2 \tan \alpha)$, the reduced force can be determined from

$$F_a = u_c e (2 A r_c \rho \tan \alpha)^{\frac{1}{2}} \quad \text{(kN)}, \tag{7}$$

where r_c is the compression strength (kN/m²), A the area of the floe (m²), a the partial penetration (m), 2α the included angle at point of wedge (°), u_c the velocity of the floe (m/s) and ρ the density of the ice (Mg/m³).

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The rupture patterns along the faces of the vertical wedge are somehow a little more complicated, as a three-dimensional stress condition prevails along the faces. If the coefficient of friction μ in a horizontal direction along the face is assumed to be nil, we can assume a shear failure as found for the vertical wall.

If the friction is included, the force will be increased as expressed by the formula

$$F = F_a(1+\mu \cot \alpha), \tag{8}$$

which means that for $\alpha = 45^\circ$ the force is increased 10% if the friction is 10%, to include the effect of friction. The actual coefficient of friction μ is probably very low, and is not likely to exceed 0.1.

4. Rupture patterns in front of inclined plane

In the previous cases the failure has been either in shear or in compression, the rupture zone being local at the vertical faces. When the floe strikes a sloping face, the rupture pattern is completely changed. The floe is now subject to a vertical component of force and this force will introduce bending moments in the ice sheet, and thus break the ice into large pieces. By observing ice pilings on shores it is possible to measure the actual size of the ice pieces. Sometimes the ice pieces are rather large, sometimes they are small. Why?

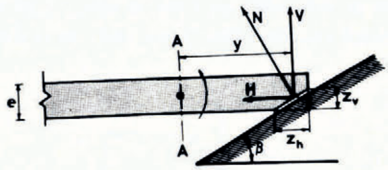

Fig. 4. Forces on inclined plane.

Consider a floe at rest against an inclined face (Fig. 4). The external force is assumed to be the wind shear on the surface of the ice, and this force may be found as a function of the wind velocity. The non-dimensional numerical coefficient is derived from the expected vertical profile of wind velocity

$$S = 4.8\times 10^{-3}\ \tfrac{1}{2}\rho_a U_v^2 lb \quad (\mathrm{kN}), \tag{9}$$

where U_v is the wind velocity (10 m above the ice) (m/s), l the length of the floe (m), b the width (m) and ρ_a the density of air equal to 1.25×10^{-3} Mg/m³.

By considering the combined axial and bending stresses in a cross-section a distance y from the line of contact with the inclined face, and applying a criterion of failure in bending tension, it is possible to derive the formula in which both the vertical and horizontal forces at the face, the eccentricity of the force, and the friction have been included. The fracture stress—compression or tension—is stated to be a function of r_c and the factor K stated below (Tryde, 1975).

$$\frac{y}{e} = \frac{\tan\beta}{12}\,\frac{1+\mu\cot\beta}{1-\mu\tan\beta}\left(K-\frac{7}{K}+6\right), \tag{10}$$

where $K = (er_c b)/S$ (dimensionless), β is the angle with the horizontal (°), r_c the compressive strength (kN/m²), y the breaking-off distance (m), e the thickness of the ice (m) and μ the friction coefficient.

This formula is illustrated by the curves drawn in Figure 5. By means of this relation, the dimensions of the fragments may be estimated.

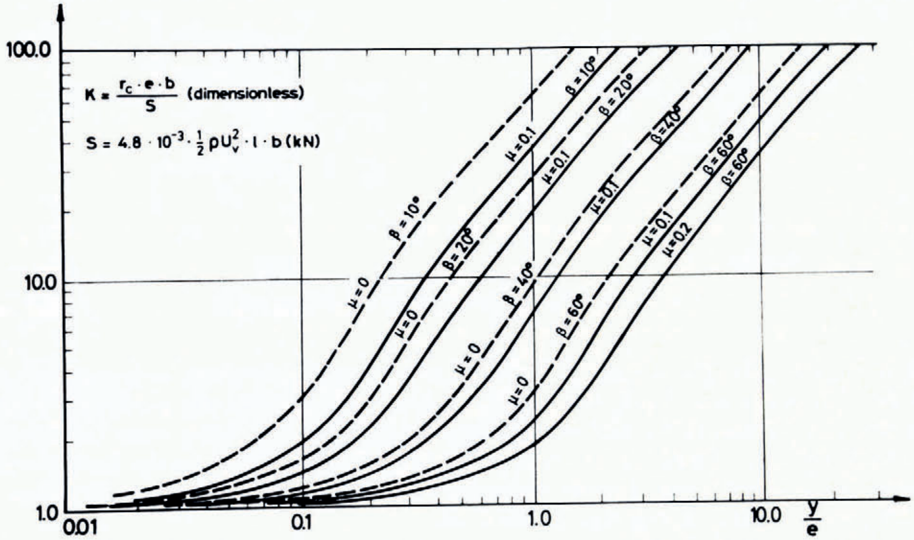

Fig. 5. y/e as a function of K, β and μ.

From observations of ice pilings it is known that the most likely size of the fragments is $y/e \approx 0.5$–3. If the pieces are too small the ice will tend to remain in the water, not causing any appreciable ice piling, and if the pieces are very large the ice may not pile up either, as the floe will be arrested by a few large ice pieces, upset into an almost vertical position, producing a counter force.

It is also interesting to note that the fragments are smaller for a gentle slope than for a steep slope for a given value of K.

For $\beta = 10°$ and $\mu = 0$, using $y/e = 1$, typical within the range of ice piling fragment sizes, we obtain from Figure 5 that $K = 60$. In order to determine the critical wind velocity, for which the ice piling is likely for a given situation on a shore, we shall use the following data: $r_c = 1\,000$ kN/m², $e = 0.3$ m, $\rho_a = 1.25 \times 10^{-3}$ Mg/m³, $l = 10\,000$ m. The wind velocity may be found from

$$U_v = \left(\frac{r_c\, e}{K\, 4.8 \times 10^{-3}\, \frac{1}{2}\, \rho_a\, l}\right)^{\frac{1}{2}} = \left(\frac{1\,000 \times 0.3}{60 \times 4.8 \times 10^{-3}\, \frac{1}{2}\, 1.25 \times 10^{-3} \times 10^4}\right)^{\frac{1}{2}} = 13 \text{ m/s.} \quad (11)$$

It is interesting to see that ice piling in this case will occur for a relatively small wind velocity. The ice force on the shore is in this case considered to be equal to the wind shear on the area.

Based on work done by Tsang (1973) it is possible to estimate the height H of ice piling from the formula

$$H = \frac{1}{30}\, U_v \left(\frac{2l \sin \beta}{g(1 + \mu \cot \beta)}\right)^{\frac{1}{2}} \quad (\text{m}), \qquad (12)$$

where g is the acceleration due to gravity, 9.8 m/s².

5. Rupture patterns in front of inclined wedge

If the wedge is inclined, the rupture pattern will also change from that for a vertical wedge, again instead of a crushing or shearing fracture the ice will be broken up in bending as illustrated in Figure 6 (Korzhavin, 1962; Tryde, [c1976][a], [b]).

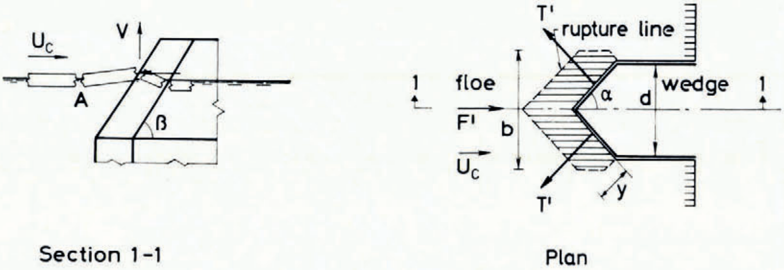

Fig. 6. (a) Vertical section of wedge. (b) Plan of wedge.

When the ice strikes the wedge the floe is lifted up and breaks along lines parallel with the axis of motion, one in the centre and one at each side, after which the ice breaks along lines parallel with the sloping faces, and located approximately two to three times the thickness of the ice from the faces. The vertical forces produce a bending rupture as the force reaches the maximum value, whereupon it drops suddenly to zero until a new contact has been established. An intermittent force is thus introduced.

By considering the forces and moments acting along the rupture lines it is possible to derive formulae giving the forces producing the failure. The forces are much smaller in this case, than those found at the vertical wedge. The derivation of the formula for the reduction coefficient C_F is rather lengthy; it may be found in Trydе (1975).

The maximum force acting on a vertical face may be written

$$F_{\max} = r_c e d \quad (\mathrm{kN}). \tag{13}$$

The actual force can be expressed as a fraction of $F_{\max}$

$$F = C_F F_{\max} \quad (\mathrm{kN}), \tag{14}$$

C_F is a reduction coefficient that can be determined from

$$C_F = \frac{5.2\epsilon^{\frac{1}{3}}}{C^{\frac{1}{2}}}, \tag{15}$$

where

$$C = 0.16\left[\frac{E}{\rho u_c^2 \sin\alpha}\right]^{\frac{1}{2}} \frac{C_1}{C_2} C_3^2. \tag{16}$$

This expression for C expresses the relationship between the physical and geometrical parameters of the system. $\epsilon = r_b/r_c$, where r_b is the bending strength ($\mathrm{kN/m^2}$), r_c the compression strength ($\mathrm{kN/m^2}$), E Young's modulus ($\mathrm{kN/m^2}$), ρ the density of the ice ($\mathrm{Mg/m^3}$) and u_c the velocity of the floe (m/s).

The parameters C_1, C_2 and C_3 are found from

$$C_1 = 1-\mu\frac{\tan\beta}{\sin\alpha}, \tag{17}$$

$$C_2 = \mu+\frac{\tan\beta}{\sin\alpha}, \tag{18}$$

$$C_3 = 6\left(\frac{e}{d}\cos\alpha+\frac{C_1}{C_2}\right), \tag{19}$$

where 2α is the included angle at the point of wedge in the horizontal plane, β the inclination of the face of the wedge to the horizontal and μ the friction coefficient.

For practical purposes the parameters may vary as follows

$$30^\circ \leqslant \alpha \leqslant 60^\circ, \quad 45^\circ \leqslant \beta \leqslant 70^\circ, \quad 0 \leqslant \mu \leqslant 0.2,$$
$$0.1 \text{ m/s} \leqslant u_c \leqslant 4.0 \text{ m/s}, \quad 0.2 \leqslant \epsilon \leqslant 0.5, \quad \rho \approx 0.9 \text{ Mg/m}^3,$$
$$0.1 \leqslant C_1/C_2 \leqslant 0.9, \quad 0.1 \leqslant C_3 \leqslant 4.0, \quad E \approx 2\times 10^5 \text{ kN/m}^2,$$

and the reduction coefficient $0.2 \leqslant C_F \leqslant 1.0$. The dots shown in Figure 7 are the values obtained in model tests with artificial and natural ice. These are plotted against the theoretical curves with various parameters. The test results show a scatter as is to be expected from variations which are bound to occur between the idealized theoretical conditions and the individual fracture circumstances.

As appears from Figure 7, the value of C_F is most likely to be from 0.1 to 0.3, which is a considerable reduction from that for vertical structures. From an engineering point of view it is important to reduce the forces, and the formulae stated here have already been applied for offshore lighthouses and for bridge piers.

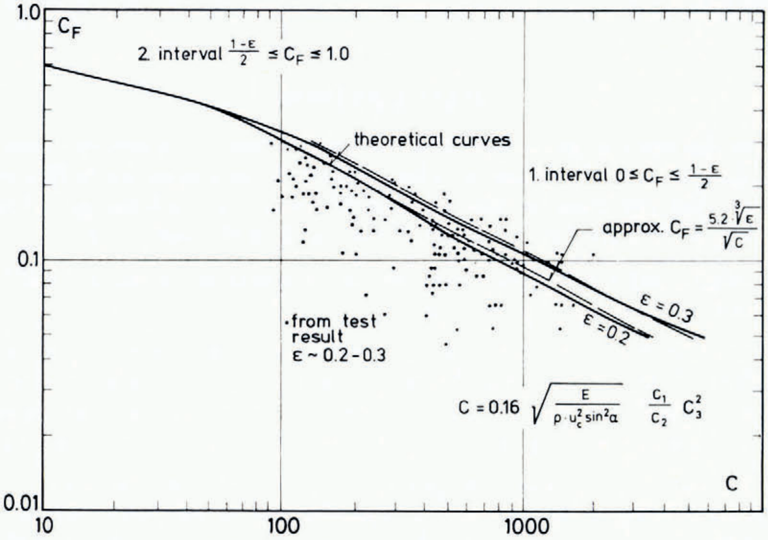

Fig. 7. Theoretical curves and test results.

6. Closing remarks

The cases which have been presented here are intended to illustrate the manner in which rupture theories can be applied in order to determine the forces involved.

The matter becomes more intricate when one moves away from some of the idealizations adopted, such as floe motion in precise alignment with the wedge, and refinements in the application of fracture initiation and propagation, elastic plate behaviour, and hydrodynamics of attached waters, to mention some of the aspects of ice dynamics, are yet to be made.

Although some of this may be of limited interest to glaciologists, it is my hope that some of the principles can be used within the field of glaciology.

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DISCUSSION

B. Michel: You did not discuss the problem of ice forces when ice is not behaving in an elastic manner but failing in a ductile fashion. This problem is even more important when you use model materials that are more ductile because of lower rates of loading at reduced model speed. How do you account for that in the theory and in the modelling?

P. Tryde: Strength and elastic properties are scaled according to the Froude law and with the material used this is correct up to scales of 50 : 1.

J. Schwarz: In modelling ice-force problems, we have to obey not only Froude's but also Cauchy's similarity law; i.e. the ratio E/σ has to be the same in model and full scale. The Hamburg Ship Model Basin (HSVA) has developed a method of creating a model ice (saline ice) which provides a E/σ-ratio up to 2 000 which is in the lower range of sea ice.

L. W. Gold: Various authors have suggested that $\sigma/\sigma_0 \propto bh^{-n}$, where b is the width of the structure, h the ice thickness and $0 < n < 1$. Considering the natural variability in σ and σ_0, the difficulty of this measurement, and evidence that n is, in fact, small, it would seem appropriate from the engineering point of view to develop, initially, the relationship between σ/σ_0 and b, emphasizing accurate measurement of σ and σ_0. In time, such information might allow an evaluation of the effect of ice thickness, if measurements are sufficiently accurate and the influence of h sufficiently significant. Would the author care for comment on this?

Tryde: All materials show increasing strength when smaller samples are used (limited by physical properties, crystal size, etc.). I agree that investigations should be performed to establish a functional relationship.

D. S. Sodhi: Does your analysis, which is perhaps based on plastic limit analysis, give you force history with respect to time or only the maximum forces?

Tryde: Based on certain assumptions the records provide a force–time record.

H. Ito: The stress in the model (ice) plate is to be measured. If the stress field in the model ice and that in the real ice are different, the force measured at the model wedge does not have much meaning.

Tryde: This is true, but it is very difficult correctly to measure the stress/field in an ice plate. However the stress field in the model conforms with the prototype.