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MATHEMATICAL MODELLING AND ANALYSIS OF HARMFUL ALGAL BLOOM

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The phenomenon of phytoplankton blooms has garnered significant interest from the scientific community over several years, due to its profound impact on marine ecosystems and global environmental processes. It also has harmful effects on marine life (causes the death of fish and shellfish), human health (causes respiratory issues, liver damage and skin problems), economy (fisheries, tourism and water-dependent industries suffer losses) and environment. To study and analyse the bloom dynamics mathematically, various mathematical models have been employed. These models have explored several directions taking into account nutrients, toxic and nontoxic phytoplankton species, refuge taken by phytoplankton and additional food sources for zooplankton. This thesis primarily focuses on the study of the bloom dynamics through the construction of some phytoplankton–zooplankton interaction models, especially paying attention to some climatic or environmental factors namely solar radiation, sea surface temperature and flow-induced hydrodynamic disturbance.

At first, to study the bloom dynamics, especially focusing on the solar radiation (a climatic factor), we have analysed a Nutrient–Phytoplankton–Zooplankton (NPZ) model consisting of a periodic driving force in the growth rate of phytoplankton due to solar radiation, and investigated the dynamics of the corresponding autonomous and nonautonomous systems in different parametric regions. A detailed bifurcation analysis of the system with respect to the parameter N_T (the total nitrogen concentration in the system), including transcritical bifurcation analysis and the existence

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of Hopf bifurcation, has been presented. Then, we have introduced a novel aspect to extend the model by incorporating another periodic driving force into the growth term of the phytoplankton due to sea surface temperature (SST) (a climatic factor). Temperature dependency of the maximum growth rate ($\mu_{\max}$) of the phytoplankton is modelled by the well-known Q_{10} formulation: $\mu_{\max} = \mu_0 * (Q_{10})^{T/10}$, where μ_0 is the maximum growth at 0°C. System dynamics is explored through a detailed bifurcation analysis, both mathematically and numerically, with respect to the light and temperature dependent phytoplankton growth response (ρ_2). The bloom phenomenon is explained by the saddle point bloom mechanism. Solar radiation and SST are modelled using sinusoidal functions constructed from satellite data. The simulated result of the proposed model with two periodic driving forces due to solar radiation and sea surface temperature describes the initiation of the phytoplankton bloom better than the simulated result of the model with only one periodic driving force (due to solar radiation) for the region 25–35° W, 40–45° N of the North Atlantic Ocean. An improvement of 14 days (approximately) is observed in the bloom initiation time. The rate of change method (ROC) is applied to predict the bloom initiation.

Next, to study the bloom dynamics with a particular emphasis on the environmental factor of flow-induced hydrodynamic disturbance, we have analysed a mathematical model of phytoplankton–zooplankton interaction, considering the impact of flow-induced hydrodynamic conditions. In the model, we have introduced a mathematical function having hump-shaped characteristics to formulate the flow-dependent predation efficiency of zooplankton, a key point of novelty. Phytoplankton growth is assumed to solely rely on a constant nutrient level, N, within the aquatic ecosystem. Furthermore, we have incorporated the ability of phytoplankton to form patches and release toxins into the aquatic environment, and also taken account of environmental toxin-induced plankton mortality. Thus, to capture system dynamics more realistically, we have studied the corresponding seasonally forced model by introducing seasonality in some relevant model parameters. Initially, we have proved the positivity, boundedness and permanence properties of the autonomous model, and then analysed the model via equilibrium analysis, stability analysis and bifurcation analysis. In further analysis, for the seasonally forced system, we have explored the permanence property, extinction scenario and existence of a positive periodic solution and its global attractivity. To prove the existence of a positive periodic solution in the system, we have used the continuation theorem in coincidence degree theory, which is a powerful mathematical tool used to study the existence of solutions for systems of equations, particularly in the context of differential equations and dynamical systems. Furthermore, we have proved the global attractiveness of the positive periodic solution by defining a suitable Lyapunov functional. Through numerical simulation, in the absence of seasonality, the flow velocity has been found to act as a stabilising as well as destabilising factor in the system dynamics. Flow velocity and all other relevant model parameters have been found to play an important role behind the three most significant characteristics of planktonic dynamics, namely coexistence, recurrent blooms and monospecies blooms. Moreover, we have found that the system exhibits diverse dynamical characteristics ranging from a simple periodic solution to a 2-cycle, 3-cycle, 4-cycle and higher periodic solution in the presence of seasonal forcing. We have also investigated the potential strategies for controlling the complexity in the system induced by seasonality.

The importance of our research lies in the fact that our proposed model predicts the actual timing of the occurrence of the bloom with greater accuracy. We have also discovered that maintaining specific low and high flow conditions in aquatic environments can effectively mitigate the occurrence of blooms. By knowing the precise onset of a bloom and the conditions that can mitigate it, necessary proactive measures can be taken to prevent the occurrence of the bloom. Therefore, our findings on bloom timing prediction and identifying suitable flow conditions underscore the significance of our research.

Overall, the work in this thesis has enriched the study of bloom dynamics from both mathematical and biological standpoints. Therefore, we expect that the current study can be beneficial for biologists and mathematicians, facilitating a more comprehensive and realistic exploration of plankton dynamics.

Some of this research has been published in [1].

Reference

[1] M. Mondal and T. Zhang, 'Bloom dynamics under the effects of periodic driving forces', *Math. Biosci.* **372** (2024), Article no.109202.

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