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ARTICLE

Response to Shieh

Charles Travis

Institute of Philosophy, University of Porto, Porto, Portugal, and Department of Philosoopphy, Tronghua University, Beijimg
Email: c.s.travis@gmail.com

Abstract
This article is a response to Shieh from Travis.

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Sanford has helped me to understand my own book better, both as to its matter and as to its point. He has brought out its relation to a number of other issues, some of which I had not been thinking about much, or at all, when writing it.

1. He has done me the great compliment of reading my last book (but one), (Travis 2000), before the present one and trying to relate the two. As I view myself, my thoughts about present things underwent a big change just around the time that the previous book appeared, that is, just at the turn of the millennium. For me the change is captured by Frege turning for me from target into friend; one with a major contribution to philosophy, extending well beyond "formal science." In that respect I rather thought of *Unshadowed Thought* as philosophy in a key I had abandoned. But Sanford has identified something in that former book which my new framework allows me to capture exactly right. The tip comes in the first sentence of this remark:

> Since the truth and falsity of *Gedanken* are absolute, since there is no such thing as a *Gedanke* which is true or false only relative to person, place, time, or (possible) circumstances, there are no modes of truth founded on such relativization of truth. In particular, no *Gedanke* is *necessarily true* because true no matter how the world might be, or only *possibly true* because false in some alternative circumstances, as opposed to *true*, *Punkt.* (pp.? ¿?)

Consider that first sentence alone. A *Gedanke* (here a thinkable (truly or falsely)) cannot be true or false *relative* to person, time, place, or whatever else. How could one ever think otherwise? Sanford identifies something which might inspire an idea of "otherwise." It lies in what is/was called a *Carnapian intension.* As I knew it, the work first assigned such an intension was to span the distance between *words* (centrally, a declarative *sentence*) and a thinkable (something to be thought truly or falsely, the sort of thing to be truth-evaluable). We can see this Carnapian idea as inspired by Frege's treatment of suggestions that a thinkable might be true relative to something, e.g., true at one time but not at another. For example, consider the sentence, "The population of Germany is 60 million and three." While a perfectly well-formed sentence, as such it seems to speak of what may be true of Germany at one time but not at another (for a start, one being born every minute). So, it seems, it is true that Germany has a population $\prod$, if at all, only at, or relative to, a time. From which Frege concludes that the candidate here is *not* a thinkable (truly or falsely).

The idea of a Carnapian intension is a way of rendering this point. Treat the sentence "The population of Germany (etc.)" as carrying with it an implicit n-tuple of parameters, one of which is time. Think of those parameters as taking on values on an occasion of (or for) using that sentence for expressing thought, or at least what it is *for* expressing in English. By default, in the central case, these parameters would be assigned the values to be found in that occasion for thought-expression. For example, the value of the time parameter would be (by default) the time of the relevant speaking. Time need not, and presumably would not, be the only such parameter. At the least, enough would be needed for covering the phenomena of deixis and demonstration. But the idea is that there is *some* set of parameters such that, on any possible assignment of values to all of them, one would have a particular thought, which was the one thus expressed and which, pleonastically, was true or false outright.

In this way, the idea is that we can view language as such, or anyway one such as English or German, as connecting by virtue of the meanings of its expressions with truth and falsehood as such. An n-tuple of relevant parameters would thus close the distance between the meanings of well-formed expressions and thinkables (truth-or-falsehoods). Thus the relation between language and truth. (Or so goes the thought.)

One might then aim for still stronger theses. Perhaps the meanings of expressions of, e.g., English, plus the values of sets of the parameters of some Carnapian intension, generate all the thinkables expressible in English, at least as it is at a time. Perhaps in that way language, conceived as containing such intensions, generates all the thinkables *punkt.*

In these terms, the main thesis of that 2000 book is that there is no such thing as a Carnapian intension, or at least what does the work thus assigned it. That is, there is no such thing as a determinate set of parameters by which, "Carnap-like," the distance can be spanned between what words contribute to thought-expression (by virtue of meaning what they do) and the thinkables there are thus to be expressed. Specify the words and the parameters attaching thereto, assign a value for each parameter, and it remains so that what we have is a vehicle of thought-expression capable in principle of expressing any of many truth-or-falsehoods (as well as failures to be either) depending on further factors in the circumstances of a given episode of thought-expression. Language provides means for a thinker to employ to make recognizable what thought s/he is to be understood to have expressed. Where thus employed, it contributes to determining how the relevant performance is to be understood. But, even given a fixed, determinate, Carnapian intension, it does not exhaust the determining factors. What matters to bearing an understanding, to be understood thus and so, reaches beyond anything a Carnapian intension could capture. The phenomenon of understanding how things were/are represented as being not thus reducing to something else.

2. So, of those two sentences of Sanford's quoted in **1** above, I think the first is not just correct but important. And it does, no doubt, bear on how one should think of modality. However, I also think that the second does not follow from the first. I'll begin here by expanding on what is correct in the first, then turn to the second.

A good place to begin is with a quote from Frege:

> I do not believe that concept formation can precede judging because this presupposes an independent existence of concepts, whereas I think that concepts arise through the decomposition of a judgeable content. (1980/1882: 118).

For Frege, whole thinkables, whole truth-or-falsehoods, come first in ontological order. A thinkable is not to be thought of as a construct made of independently existing parts. Rather, proper thought-parts—ones, that is, not themselves truth-or-falsehoods—are arrived at by decomposing a whole thought.

To see the point of this, consider an example. Suppose that there is this predicate of someone: that s/he eats snails. (For simplicity's sake I ignore temporal complications.) Here, then, is a proper predicative thought-part: something not itself a thinkable, but with a role to play therein, namely

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that of predicating being a snail-eater. The "someone" here indicates only indefinitely. And there is nothing in the thought-part itself (the predicable) here to convert the so far indefinite to something determinate. So to play that role, our thought-part needs something of which to predicate—say, Sid. But for it to play the role is thus *eo ipso* for there to be a whole thinkable, truly or falsely, e.g., that Sid eats snails. So a proper thought-part can function as such only in the context of a decomposition of a whole thinkable: only there can a proper predicative part predicate or a denotation denote. The "can" here need mean no more than this.

The role of a proper predicative thought-part is to represent *a thing* (whichever one) as being a certain way there is for something to be (or not). Both "something's" indicate only indefinitely. In the general case, each is an n-tuple of relevant n-ity.

A whole truth or falsehood represents things as being some way for them to be. So it, too, may be seen as predicating. But in this case the predicating *must* be of n-ity null. For the whole point of a whole thought (truth-or-falsehood) is to be true or false *outright*. Just so (as per above), that predicating within a decomposition of such may qualify as predicating at all.

By definition, a whole thinkable—a (putative) truth or falsehood—is *not* in need of completion. Which means: unlike a bit of language (mere means for *achieving* that sort of non-factive representing which is thought-*expression*), a whole thinkable cannot be susceptible to having a Carnapian intension, or anything of the sort, attaching to it. (Though, e.g., a specific spatiotemporal coordinate may be an ingredient *in* the way for things to be which it represents things as being). More may be needed to know whether a thought that Benno is eating snails is true, e.g., at what time this is meant to be happening. But no more can be needed to know of *what* the thought would be true (or false), e.g., of what it is true that Sid is now eating snails. Such is not something liable to vary across different zero-place predications. If it were, it would not support, as it must, the more particular projects of each of its proper parts: that of predicting something *of* a thing.

One-place and two-place predicates are true or false, *selon le cas*, of singletons or pairs, respectively—and of different such as occurring in different thoughts. A whole thought is true or false *outright, Punkt*. So, while it predicates, if we are to say that it predicates of something, what that something was would need to be independent of what thinkable was in question. It would always have to be the same something, no matter what the thinkable. Thus, that the thinkables are *per se* comparable: eligible for service as terms in the sorts of relations (e.g., truth-preserving ones) by which one thought may relate to another. Where, e.g., there is both a thought P and a thought Q, there is also the thought that P and Q—a prerequisite for there to be that phenomenon of truth preservation, which it is logic's business to underwrite.

Our topic here is non-factive representing, that peculiar sort that does not *per se* require for its occurrence that things *be* as represented. Thus the room for falsehood, thus truth. Where thinkables are thus truth-or-falsehoods, comparability comes as part of the package. The comparability provided at zero-place thus belongs to truth, intrinsically. So a truth-or-falsehood *cannot* call for values of further parameters to decide which, *true* or *false*, it is. Such cannot be relative to anything. Within the realm of the *thinkable*, as opposed to its expressing, there is no analogue to the task Carnapian intensions were meant to perform, no distance to be covered between the thinkable and what its true turns on, as there is distance to be covered between an episode of thought-expression and the thinkable thus expressed. Language is *not* logic's topic.

Sanford's first sentence is thus correct. A thinkable (a protagonist in non-factive representing) cannot be true in one "possible world," and false in another. It can only *be* thinkable of the way things are. Such underlines Frege's insistence (1918: 73) on the importance of winning an environment, thus accepting susceptibility to error.

The original intention of Carnapian intensions, at least around UCLA last century, was to bridge a distance between what the words of a language are to be understood to do as such and the particular thinkables they express (on an occasion, in a mouth). If I am right, they are not fit for *that* purpose. But the intensions they now inspire are meant to span a different supposed distance, that between a thinkable (already a truth-or-falsehood) and "that of which (in the case at hand) it is to be

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taken to be true or false." Modality may be *claimed* to make for such distance to be spanned. But in fact, there just is no such distance *punkt.* No matter what the thinkable, the truth is, pleonastically, the truth of the way things are.

A final word. Suppose we try to conceive of a possible world as a determinate countable among which *"our"* world is just one of kindred. An obstacle immediately arises. There is *nothing* at any n-ity of which for us to predicate except something's being as it *is.* For that on which truth turned in another world (as now conceived) would be, trivially, something of which we could not be aware and which we could not interact with. By hypothesis, any predication of this world would predicate what would be something entirely inaccessible to us. We would thus be ineligible to predicate it, just as I am ineligible to predicate anything of, e.g., a resident of Harbin of whose existence I am entirely unaware.

Pia is now wearing a tasteful light blue pantsuit. Might it have been red had Sid been more careless in making powder into Chinese ink? Such might be an intelligible question. But suppose we try to construe it as a question as to how things *are* in another possible world (conceiving worlds as above). Then, I suggest, what might be intelligible has been made unintelligible. "Possible world," I suggest, just belongs to that class of loose but imagistic terms mathematicians use for describing intricacies in broad strokes. This, however, is not quite to agree with Sanford through and through. The next section outlines why.

3. Truth outright is *truth*, *outright.* If it were not, what ought to be whole thinkables would fail to perform their assigned task of allowing predicables of higher n-ity actually to do what they are for doing. With which I turn to Sanford's second sentence:

> In particular, no *Gedanke* is *necessarily true* because true no matter how the world might be, or only *possibly true* because false in some alternative circumstances, as opposed to *true*, *Punkt.* (?¿?)

Sanford reads this idea as contained in this remark of *Begriffsschrift:*

> If I call a *Satz* necessary, I thus give a hint as to my grounds for so judging. *But thereby the conceptual content of the judgement remains untouched. Thus the form of an 'apodictic' judgement has no significance for us.* (Frege 1879, pp. 4–5).

And he takes Frege as so committed from then on. Three questions thus arise: First, does that second sentence follow from Frege's remark? Second, does it follow from the first sentence's good point? Third, is it correct?

As to the first, I think the answer depends on how one reads the "us" in *BS*§4. On one reading, the "us" is all those engaged in the project of *BS*, viz., to arrive at the laws of being true, thus at something a logic might be. The relevant point here: modalities need not, and should not, figure as variables over which any such law ranges. On the other reading, the "us" is rational beings, or at least all rational philosophers, full stop. The idea would then be: for a rational being, there is no such phenomenon as "modality" to be investigated, nothing there needing philosophical unfolding. Thus the conclusion Sanford draws.

I have always read *BS*§4 in the first way. Here I will just gesture at one reason for thinking that so doing is doing Frege a favor. First, an obvious textual remark. If one reads the passage in Sanford's way, work is needed to make logic's laws an exception or to explain what it means to call these "immovable boundary stones set in an eternal landscape" (1893: xvi), or again at (1918: 59), describing each law as a partial unfolding of (hence intrinsic to) the notion *true*, thus holding so long as there is such a thing as countable truths and falsehoods (thus as non-factive representing). All of which is not to say that Sanford's reading of that "us" is wrong, even less that it is uninteresting.

With which I turn to the third question. Is that second sentence correct? It certainly would be if the idea that a thinkable is necessarily true because true "no matter what" were incompatible with Sanford's first sentence, already contained in the relevant notion *no matter what.* One might think of this notion as containing an implicit quantification over some supposed determinate domain of

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all the ways things might otherwise be. Which might drive us back to some idea of "hyper-Carnapian" intensions attaching to what are already thinkable truth-or-falsehoods. Which, as said already, we cannot allow.

But perhaps that sort of thinking is starting down a false track. Perhaps there is another way of looking at modalities. Perhaps Frege himself hints at such, e.g., in 1884 in connecting analyticity (in his sense) with an idea of "ultimate grounds." (1884: 3). (I trod now on someone else's manor. I will try to be circumspect.) I start from an example of modality, (nearly) stolen from Frege, put by him to a quite different end. Imagine it being said in 1918 of someone accused of arson (in Jena) that if he was in Rome on the day of the fire, he could not have started it. *Could not*: He would have had no means for doing so. There *were* then no such means.

There's modality for you. Not that it is "part of the concept *fire-starting*" that such cannot be done at a distance. As things *now* are (2024), we know that such *can* be done at a distance; witness the burner phone. But in 1918, such could not be done. That it could not, or not by the defendant, is just a recognizable fact. Not that all modalities are treatable in the same way as this one, all resting on the same sort of fact that this one does. Suppose, though, that we ask whether, in this case, the defendant could not have started the fire "no matter how things were otherwise." First, it is not obvious how a question so worded might be understood. Second, *prima facie*, a question of this form is not to the point. There *were* no means then for the defendant to have done what he is accused of. This need not mean that their being is simply unimaginable, or even that things *could* not have been otherwise in the just-mentioned respect. Another point to harp on when it comes to what modality might be. Such modality would not fit the description of Sanford's second sentence. Correlatively, we have, at least as yet, no reason to suppose that "hyper-Carnapian intensions" are going to have any role to play in the facts of it. Again, I am not insisting that in such matters one size fits all. So far, though, explaining what the necessity is in *could not have* does not suggest *per se* an appeal to "possible worlds." So far modality lies in *the* world (the way things are) and the room *it* leaves for maneuver.

I now sketch an account of the importance of this sort of modality to non-factive representing *überhaupt*. For this I need a bit of non-standard terminology. Having disposed of "'hyper-Carnapian' intensions," we can now see a predicable (of whatever n-ity) as something extracted from the way things are. That a bowl of snails now sits on Sid's table is one way for things to be. For it to be so is for it to prescind from indefinitely much of how things are, which does not matter to whether things are *this* way. For example, whether the tabletop is metal or glass. Which, conversely, leaves indefinitely many ways for things to be the way in question. Still a bowl on a table, whether metal or glass.

With which the terminology. Suppose that a predicate is predicated of an item of suitable n-ity. And suppose that the item *is* as thus predicated. Then I will say that the item *falls under* the predicable. Suppose that for *an* item to be as some given one is is for it to be as a given predicate predicates. Then I will say that an item's so being *instances* the predicate (or is a case of being as so predicated). Where there is a case of failing to instance, I will speak of *counter-instancing*. To anticipate, it is in *instancing* rather than in *falling under* that the true nature of a predicable's generality stands out. In brief, it is part of the idea of a predicable that it can be instanced and/or counter-instanced in an indefinitely extendible variety of ways.

But if it is intrinsic to a predicable (more generally non-factive representing) that it prescind from the way things are as just described, it is equally crucial that *truth* restore factivity; at n-ity null, for a predicable to be *true* is for it to have a determinate presence in factive meaning (details depending on how things otherwise are). For example, that the accused was in Rome at the time means (factively) that he did not start the fire. Frege stresses the importance of such restitution of factivity in *Grundgesetze* 2 in criticizing the formal arithmetic of Eduard Heine and Johannes Thomae. In brief, suppose that whether someone was in Rome or not at a given time made, or could make, no difference to when one would be treating things as they are. So Sid might as well be in Rome as not for all the difference that makes to *anything* else. No reason, e.g., either to make or not to make an appearance at *Stammtisch* this evening. *Mutatis mutandis* for pursuing *any* course. Then one might as well treat, so count, the thought as true as count it as false. In which case it can be neither.

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If the accused was in Rome at the time, he *could not* have started the fire. Such is factive meaning. There is nothing epistemic in this *could not*. It is just a fact. At the time (or at Frege's time), there was no way of starting fires long-distance, or anyway none available to the accused. Such belongs to the way being in Rome on that occasion happens to matter (factively) to how things otherwise are. If it mattered in no such way, being in Rome would not be a genuine way for an item to be at all. (Not that so mattering need be part of what is thus predicated.) Finally, without factive meaning, there would be no non-factive representation.

In sum, it is part of the package in non-factive representing that truth has consequences, that things being as represented has a substantial role to play in factive meaning, the details of which depend in general on how things otherwise are, as does the fact that in 1918 one, or at least Frege's defendant, *could* not start a fire in Jena while in Rome. As this illustrates, modalities are just part of the package in factive meaning, thus part of what one signs on for in non-factive representing. Such is one source of at least one sort of modality.

Sanford (and Frege) help us see why there cannot be hyper-Carnapian intensions (possible worlds). Suppose one thought that there *must* be if there is to be modality at all. Such would be a mistake. We can *see* that it must be in observing, as now we see that there must be modality if there is to be thought in countables (non-factive representing). Put this all together and we are in a bind. I am suggesting here that the way out of the bind is to deny that modality requires hyper-Carnapian intensions: it is to be found, where at all, in the way things *are*. Details, naturally, are for elsewhere.

4. Agnosticism and Generality: In *Kernsatz* 4 (undatable/1983, pp. 189–190) Frege writes,

> The *Gedanke* always contains something reaching beyond the particular case, whereby this is presented as falling under a generality.

This section expands the idea of "reaching beyond." In present vocabulary, what does the reaching is a predicable (at n = 0, a thinkable). The idea can then be rendered thus: If a thinkable admits of instancing—if there is something that would count as things being as predicated—then there are indefinitely many different things that would so count. (For any given ways, another.) So if it *is* instanced, it is so in indefinitely many different ways. *Mutatis mutandis* for counter-instancing.

What a predicable predicates is being some given way there is for something to be. A predicable of n-ity null is a thinkable (truly or falsely). What it predicates of (no matter which predicable it is) is always the same: the way things are (or their so being), a mass with no plural, though here "things," for short. A predicable of higher n-ity predicates of *a* thing (at n-ity higher than 1, an n-tuple). Such indicates indefinitely. What it *is predicated of* may be any of indefinitely many different things, depending on its occurrence in a thinkable.

Extraction: Our interest here is in relations between the way things are and particular (countable) ways for something to be (or not). For things to be as they are may be for there to be those who smoke and those who do not. It may also be for *Sid* to smoke. But so much more remains so far unsaid as to how things are in being as they are, so much not yet decided by those just-cited participants in things being as they are. For example, it is so far left open whether *ham daan* (鹹蛋) costs more than 10 yuan in Guangzhou, or whether Pia takes tea at three, or whether Sid reeks (though in point of factive meaning, the likelihood is increased by what has so far been said as to how things are: *that he smokes*).

In general, there being such a thing as a way for something to be or not presupposes further particulars as to how things are. For smoking to be something Sid does or not, for it to be something there is to do at all, other things must be so. Tobacco must not have succumbed to the ice age (or to anything else). There must be fire. There must be lungs, or at least breath. There must be such a thing as inhaling. Given all that, we are at best only just begun to exhaust the way things are by means of countables. Indefinitely, much is still left open.

A Border Crossing: A transition from the way things are to a way for things to be is a move from mass to countable. In this particular such move, the countable shows up on the other side of the border between the two partners in non-factive representing: the represented (here the mass) and its

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representer (here a thinkable). The way things are lies on the first side of that border. It plays the role of universal represented at zero place (representations which call for no specifying of what). That extract of how things are, a given way for things to be, e.g., such that Sid smokes, moves across the border from represented to (non-factive) representer.

On this side of the border, cast in the role of representer, it is up to the countable carved from the way things are (that presumptive way for things to be) to make determinate just *how* truth is thus made hostage to the "mother"-mass things being as they are. And it will have performed the assigned task only if it is sufficiently determinate just when the represented would be as represented, what would so count, and when not.

For simplicity I am now restricting attention to predicables of n-ity 1. But *mutatis mutandis* for higher n-ity. For the zero-place case there is only one represented, and the question is only how *it* needs to be, the remaining question then being simply whether it is *thus* or not. Still, the reaching-beyond mentioned in *Kernsatz* 4 cashes out (and must) in terms of instancing. If the (here zero-place) way-to-be in question is instanced at all, such would remain so across indefinitely extensible variation in how things were. *Mutatis mutandis* for counter-instancing. If the way things are is true, things need not have been *just* as they are for this to be so. Truth (or falsehood) cannot depend on *everything.* And the point stands for arbitrary n-ity.

The issue arising: At zero place there *is* only one item, either to be or not as represented, to wit, the way things are. So *someone* might think (for a moment) that if a candidate bit of zeroplace representing did enough to decide whether the way things *are* is things being as represented—this, anyway, would instance (*casu quo* counter-instance) things being the way in question—then it has done what is demanded of it (at least demanded by the business of non-factive representing itself).

But to be told that, whatever else, a given candidate bit of non-factive representation represents things as, in any case, being as they are (aka truly) is to be told, if not quite nothing, close enough. There remains near enough *everything* left to be said before there is enough to determine *what* thinkable, what way for things to be, it is that is in question. To earn the right to the title *thinkable*, there must be something in its way of making truth hostage to how the represented (i.e., 'things') is, which, given things being as they are, either mandates counting "things" as a case of being as represented (as instancing the candidate way for things to be) or as not.

Suppose now that we are told what the way-to-be in question is. As may be, it is that Sid is a *Stammkunde* at *der alte Schnorrer.* If now all goes well (as it well might)—if what we are told bears a rich enough understanding and if we grasp it well enough—we may thus be equipped to recognize what would count as things so being, as so counting, and *mutatis mutandis* what not.

It may be that in such a happy case there is an identifiable constant feature in all instances of the relevant way-to-be and absent in all counter-instances. But without supposing this, we can at least say this: Suppose the question whether the way things *are* (the represented) is as represented to be settled. One thing for sure is that there are indefinitely many further questions which (for all it matters) remain to be settled, even given this—for example, whether Pia takes tea at three, whether you can find a 鹹蛋 in Guangzhou for under 10 yuan, whether *koevoet* is better than *boulie*, and whether salt water freezes at lower temperatures than sweet. It is at least conceivable that any of these that the question *is* settled by some fact of factive meaning. Conceivably, Sid's *Stammkundigkeit*, should it exist, means (factively) that salt water does not freeze at 0°C, or that *koevoet* is better than *boelie.* But the sure thing here remains sure modulo all that.

With which we have won, in one way, that on which *Kernsatz* 4 insists. Might it still count as Sid being a *Stammkunde* at *der alte Schnorrer* if in Guangzhou the price of *ham daan* sank (*casu quo* skyrocketed)? Such is conceivable. Or if Pia took tea at 6? Hard to see why not. And so on *ad inf.* From which we get an indefinitely extendible stock of ways the world might be, for all of which Sid's *Stammkundigkeit* remains fixed.

Of course, at n = 0, such *could* have no bearing on how many objects fall under the predicator in question since this number stands fast no matter what. It is 1. At positive n-ity, this point lapses. But the rest stands fast. For present purposes it is irrelevant how many objects fall under a predicable, or even conceivably could. Assign whatever positive number to that you like. For any object, if it

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satisfies the concept *ham daan*, there are indefinitely many ways for it to instance this. Which, on reflection, is simply another face of the fact that a predicable of positive n-ity, while it can be something there is to be predicated, *full stop*, can be predicated of something only in the context of a thinkable (a predicable of n-ity null).

Otherwise put: The point, now made once, can also be made more quickly. Predicables (along with other components in non-factive representations) are recurrables. Often the ways for something to be, which they predicate of things, recur in the way things are. Cats sometimes do hide with their tails sticking out. But they certainly recur in thought. Where a predicable recurs, there is one thing twice. Correspondingly, what identifies a predicable also fixes a sense for the *same again*. Which in turn makes for common elements in different decompositions, each, perhaps, of a different thinkable. Which, in turn, allows for assigning structures to thinkables, or sets of them, on which valid inference to turn.

Recurrence thus plays a crucial role in thought in countables. The phenomenon can also be portrayed in a different light. Consider an example, e.g., at n = 1, say, the predicable *mammal*. Suppose there is an ensemble of thinkables in which this predicable, a recurrent, occurs, say, five times, perhaps in the form of a lecture to be given on zoology. We thus have the same thing five times over. There is thus an element in the predicable, something in the way it represents a thing as being, which determines when, on the relevant understanding of "same" a case would be one of "same thing twice." I defer discussion of what this element may be; better to focus on what it would need to prescind from.

What the predicable *mammal* must prescind from here is, in short, all open to variation, while what would count as the *same twice* in a relevant sense remains untouched—in the present case, all in which one particular exemplar of mammality might differ from another. For example, some mammals have warts, and some do not. Of those who do, some have an odd number of them, and some do not. Warts may grow in various places on the body. And so on and so on. Just to scratch the surface at one small place. Moreover, agnosticism is called for. The predicable itself has no "inside" prior information as to just what candidate exemplars the world (things being as they are) in fact supplies. It may at least jog intuition here to point out, to understate shamelessly, that *in re* mammals there's one born every minute. In which the predicable *mammal* must be prepared to deal with all eventualities, both in assigning content to the *same again* and in the variation it allows for in validating the candidates it does.

And thus it is for a predicable (of no matter what n-ity) to bring a particular case under a generality (*thus* to reach beyond it). The predicable must abstract from all variation between candidates to reach some determinate content to serve for its purpose as identifying a case of the *same thing twice*. Just here is the generality: for any given case of instancing (here aka "same thing twice," there is always room for another. Once again (again in present vocabulary, the point holds of instancing, but need not hold of falling under). Some predicables allow for indefinitely many objects of such which are as predicated. Some do not. The point pointed to by *Kernsatz* 4 is indifferent to this distance. And to stress again (given the philosophical history of the term), here abstraction is not a process.

5. The Point in Frege: At various stages in his career, Frege goes to lengths to distinguish between predication and denotation (or in his 1882 terms (Frege, 1980: 118–119), 'Begriff' from 'Einzelne'). Two points stand out.

1. Suppose that in a decomposition of a given thinkable there is a predication (one-place for simplicity) and a denoting proper part meant to function in that decomposition of the thought to supply that predication with something of which to predicate. Suppose that the denotative part failed to identify the object, which was thus to contribute to the whole. Then it would have been a mere would-be denotation, not properly a thought-part at all. Which would make the *whole* a mere would-be and not thinkable at all. Failure of *denotation* to have an object destroys thought-hood.

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By contrast, it is fine for a proper predicative thought-part to fail to be fallen under—for nothing to be as *it* represents a thing. As above, such a thought-part is not charged with the mission of identifying any *particular* objects as with a role in the whole (even if what it predicates *could* be true of at most one thing). Hence, what predicates a predicable of positive n-ity, e.g., *being large but out of shape*, can do what it is for doing only given, as further input, something of which to predicate—perhaps Sid, perhaps Michael Caine. Thus, too, that the call for completion here is cancellable while still yielding a thinkable: Someone is large but out of shape. It does not matter who.

2. Were a predicate to work by enumeration (of the items in fact as predicated) rather than by its particular form of generality, some invalid arguments involving it would turn out to be valid. Frege (1914: 230–1) provides this example: All humans are mortal; therefore, Cato is mortal. A premise is conspicuously missing. But suppose 'human' were a name that named each human, or a concept identified as what it is by what fell under it. Then an argument that should be invalid would be valid. For the premise, all humans are mortal; it would be equivalent to this:

   H1, … Cato, …Hn … are all mortal.

   Which entails *eo ipso* that Cato is mortal.

   So predicables must not collapse into names, or sets of objects, e.g., their extensions, however multifarious or not these are. They are not identifiable by enumeration. The point, perhaps, stands out more clearly if we focus on instancing rather than enumeration. A predicate may admit of being fallen under by no more than one thing, even by no other than such-and-such thing. There is, e.g., being the one who burned the hole in Benno's carpet. The question remains how one must be to be the one; what would count as so being. For someone to have burned a hole in the carpet, it is neither necessary nor sufficient for that someone to be Sid. Nor either Sid or Pia, nor (keep naming others as long as you like). Sid did it with an artful flick at the end of his Murad. And *voilà*, bullseye. But *that* performance is not in any sense a *sine qua non*. Any set is the extension of some predicable and a subset of the extensions of various ones. *By which* no predicable is a unique owner of its extension.

6. Sanford on Reaching Beyond: Sanford raises at least these three questions as to what generality (if any) attaches *per se* to a predicable, or, at n-ity null, a thinkable:

1. Why need a predicable be identified by its possible instancings (and counter-instancings), i.e., by what *would* count (and what not) as being as predicated? Why not just by its actual instancings (or fallings under)?
2. Why does a predicable need to be general (or reach beyond) at all?
3. In particular, why cannot a predicable's reaching beyond be captured by its (actual) extension?

I. Sid goes on his long, intricate world tour. At each stop he lights up often and enthusiastically. No doubt *this* counts as being a smoker if anything does. Well, perhaps best to insert a *ceteris paribus*. Suppose, e.g., he does all this at gunpoint (or perhaps with gun pointed at Pia), swallowing his distaste for the weed. A smoker *malgré lui*? Or not really a smoker at all? Well, what *would* really count as being a smoker? The question arises, though it need not have an answer. But it is enough for the question genuinely to arise. For this alone is enough for us to see that the predicable is *not* identified by its actual instancings. For these neither answer the just-mentioned question nor show it not to arise.

Sid, let us suppose, is a smoker. But we are not to suppose that to be a smoker one needs to be Sid. On the one hand, by hypothesis, for him to have behaved just as he did throughout his lengthy world tour *is* for him to be a smoker (or at least to have been at that time). But if being just as Sid was is being what it would be for one to be a smoker, then being a smoker is something that could have been done

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once, and then only by Sid. Suppose we want to allow that the question also arises for Pia, that she, too, might have been a smoker throughout her lengthy world tour, or, again, perhaps not. Whether touring as she did counts as being a smoker is not decided by the fact that touring as Sid did so counts. So far, it is not decided at all. But suppose *we* decided that being a smoker is something both Pia and Sid do—by stipulation if necessary. Then, so far, there are two ways of being a smoker: being Sid (with exactly his history) or being Pia (with exactly hers). Now when we come to Benno, we want to entertain the possibility that he is a smoker; we need to stipulate all over again. And so on ad iinf. What remains undecided is what it is about Sid, and now about Pia, that matters to whether they count as a smoker or not.

'And so on *ad inf.*' But "*ad inf*" here fails to allow *a smoker* to fill the bill it is meant to, i.e., to be a way for *one* to be, that "one" indicating indefinitely. This bill cannot be filled if for each new replacement for that "one" new stipulation is called for. Granted, if we assume the finitude of existence (in our actual environment), substitutions for the "one" here are in principle exhaustible. But such does not yet help in answering such questions as those arising in the first paragraph above. Which brings us to question II.

II. Why does a predicate need to be general at all? Sanford's example: Sid lights up once (behind the barn, perhaps, when he is ten) and then never again, eschewing for the rest of his life "demon weed." Why could this not count as being a smoker?

First, whether a smoke behind the barn at age 10 makes one a smoker for life is not what is at issue here. The question is not about how spry Sid needs to be in producing the material that is to count or not as *someone* being a smoker. Whether that episode behind the barn makes Sid count as a present smoker is for the concept *smoker* to decide. But our present question is not about smoking in particular. It is rather about the notion of *predicable*, aka a *way for something to be* (*or not*). For all it matters to *this* question, *smoking* might just as well be something one could not do without being Sid, or at least lighting up on all the occasions he did. But the general point concerns the variety in and extensiveness of the questions to which a (any) concept is obliged to speak, specifically whether those questions must range beyond the scope of actual history.

A simple route to an answer was already set out and exploited in section 4. It is to start with what Frege marks as the ontologically "*ur*" case, n-ity null (where predicables are *thinkable*). At n = 0, what a (i.e., any) predicable represents as some way for it to be is the way things are (or things so being). If it predicates truly, then it is *this* which is as represented. If not, then nothing is. So here all true predicables share the same extension; all false ones have none. (Or, following Frege strictly, in the one case the extension is 'The True', in the other 'The. False'.)

The question now arises what distinguishes one such predicable (aka thinkable) from another. The answer to that is: the way each is carved from things being as they are, an answer elaborated above in section 4. But, as argued there, such carvings always reaches beyond the particular case so as to admit of indefinitely many ways of being instanced, *casu quo* counterinstanced. Thus, as Frege insists, they always present the way things are as falling under a generality. (Such is just the point of their extraction from the way things are.)

It remains only to add to this the following: A predicable, which is not a thinkable (one of positive n-ity) as engaged in predicating (*per se* within the context of a decomposition of a thinkable), is always a partial doing of what the whole thinkable does and identified accordingly as the predicable it is. *Quod erat demonstrandum.*

Predicables thus always reach indefinitely far beyond the particular case, hence beyond what is in fact so. But such need not be commitment to "hyper-Carnapian intensions," attaching to *thinkables* with parameters on values of which their truth value depends. Nor is it intrinsic to a predicable to *have* instancings. Counter-instance will do. There is nothing wrong with a predicable that predicates round-square-hood.

III. Could a predicable be identified by its actual extension as predicating what it does? Frege and I agree in saying no. I have already sketched our reasons. For a start, different ways-to-be may coincide in their extensions within a given domain but, for all that, diverge within a larger one. All the losing numbers on Sid's 2024 lottery tickets may be primes up to the end of January, while the extension of "2024 losers" over the whole year contains non-primes.

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I *think* that Sanford is attracted to rejecting an idea Frege and I accept (that a predicable is the predicable it is by virtue of its proprietary way of reaching beyond). And I think this is because he thinks that otherwise one flirts (or worse) with a *verboden* idea about modality. I detect this worry in the following, in which he scouts a particular argument he sees in favor of identifying a predicable by its proprietary 'reach beyond'):

> Suppose a concept is "identified" by what *does*, rather than what *would*, fall under it. Here, one might think, possibility enters the stage: the supposition might be understood as taking the identity of a concept to lie in what *is* the case rather than what *might be* the case. It follows then that there is no representing an object as falling under that concept unless it does fall under it, and so there's no false representation. (15?¿?)

Sanford does not find the argument convincing. But convincing of what? And why does he need it not to be? I take the idea to be something like this: There *must* be false representation (or there is no such thing as non-factive representing). So if identifying a concept by what *does* fall under it would entail that there is no false representation, then we cannot suppose that a concept is identified (merely) by what *does* fall under it. We *must* suppose it to be identified by what *would*. And, I take it, Sanford does not want to be backed into that particular corner.

Why not? I take this to be because being so backed would threaten the truth of that second sentence:

> No *Gedanke* is *necessarily true* because true no matter how the world might be, or only *possibly true* because false in some alternative circumstances, as opposed to *true*, *Punkt*. (5?¿?)

As to the truth of *that* thought we agree on at least this much:

> Since the truth and falsity of *Gedanken* are absolute, since there is no such thing as a *Gedanke* which is true or false only relative to person, place, time, or (possible) circumstances, there are no modes of truth founded on such relativization of truth. (Ibid)

That is, we both reject any view that commits us to "hyper-Carnapian" intensions: ones designed not to span the distance between *language* and the thinkables expressed in its use but rather a supposed distance between thinkables and what they express. That is, intensions containing a parameter whose values can re-map into truth values what is already a truth or falsehood. What *is* true, *casu quo* false, is not so only relative to something. (And the vocabulary of Sanford's second sentence *might* suggest such a thing in the offing, that is, insofar as it invites us to imagine some totality of ways the world might be (or not), or of "alternative circumstances"—a totality over which such a parameter might range.)

But, with Frege, I insist that a predicable, and in particular a thinkable, cannot be identified as the one it is by its extension. So hyper-Carnapian extensions will have to come to grief in some other way. Sanford asks us at least to imagine otherwise:

> Suppose concepts are (actual) extensions, i.e., something like sets of objects that we think of as falling under them. So the concept of being a prime number is the set of all primes. What stands in the way of representing, e.g. 26, as a member of this set? (Shieh (?¿?))

As to the specific question, *nothing*. But I do not think we can suppose what Sanford asks us to here. For a start, I consider this initial segment of the primes (and the set thereof): {2,3,5,7,11,13,17,19, 23, 29}. To begin with, a set is not a concept (if a concept is a predicable, a way for something to be), but rather, a set is an object, something of which to predicate. So far we have just two objects: the number 26 and the just-mentioned set. How are these meant to relate? Sanford wants the set to be the predictable. Absurdity is no hindrance to predication. But here grammar is. Try to predicate the above set of the number 26. The set being an object (the denotatum of its above introduction), so far,

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the joint work of 26 and the mentioned set is just "sign salad." To predicate the set of 26, we need something like *26 is the set {2,3,5,7,11,13,17,19, 23, 29}*.

Again, absurdity is no bar to predication. But I take it that this is not what Sanford had in mind. (And if it were, then the relevant predicable is not the set but rather *being the very thing, that set*.) I take it that if it is a concept's extension that identifies it as the concept it is, then what we want to predicate of 26 is being *in* that extension (i.e., that set); yet another way for an object to relate to a concept. Set theory kindly offers us a predicable-forming operator for forming just such predicables. As usually written, it looks like this: "ε." The thinkable (i.e., zero-place representing) we thus arrive at is: *26 is a member of the set* {2,3,5,7,11,13,17,19, 23, 29}—no worse than falsehood, unless obvious falsehood is worse.

So far the predictable, which yields this falsehood, predicates of 26 belonging to a certain set—which it patently does not. To identify that predicable one must, *inter alia*(!), specify the set. And *thus* we have the predicable. But the trouble now is that {2,3,5,7,11,13,17,19, 23, 29} (that set) is the extension of indefinitely many different predicables. For example, being one of the first ten primes, being one of the primes less than 30, being one of the primes less than 31, also, e.g., being a losing number on Sid's first lottery ticket of 2024, being 16 less than the mystical number 42, being contained in the house numbers of the houses due to be painted in Nightshade Lane, or (qua ordinal) each a day of some month on which Sid failed to show at *der alte Schnorrer* in 2023, and so on *ad inf.* And these all need to be distinguished, each from the others, by something else that differs while extension remains constant. How *do* they all differ? For one thing, by their consequences, the role of a given thing's so being in the business of *factive* meaning, correlatively by when they would be true. To conclude, it belongs to the very idea of predication that a predicable cannot be identified by its extension.

Back, then, to Sanford's second sentence. "No *Gedanke* is *necessarily true* because true no matter how the world might be …" On the view I set out here, every *Gedanke* (aka thinkable) represents the same thing, each in its proprietary way there is for that thing to be. I have dubbed this universal represented *the way things are* (or their so being). If the *Gedanke* is true, this is what it is true of. (Correlatively, there is no need to say so.) Not every *Gedanke* is necessarily true. What could necessity come to if not truth, no matter *what* the universal represented (i.e., the way things are) might have been? I do not suppose that necessity comes, so to speak, only in one flavor. But if we look at examples, it seems that in one sort of case, necessity consists in there being something in the way things are that blocks things being otherwise. For example, in 1918 there was simply no way for someone in Rome to start a fire in a house in Jena. That was before the burner phone and remote-controllable domestic appliances, among other things. For another example, there cannot be a largest prime. A simple proof shows what it is that excludes that possibility. In such ways, no doubt among many others, the way things *are* may exclude their being many ways they can be represented as being.

As Sanford reads Frege, Frege's hostility to modality does not reach as far as epistemology. "Possible for all we *know*'" remains in the cards. I remark only that the kind of modality with which the defendant could not have started the fire if he was in Rome at the time is not epistemic. There was just no way for the defendant to do it from Rome.

As Sanford points out, there is a connection between what I now say about Frege and what I did say in 2000 about occasion sensitivity. Albericht (in 1918, head of the Thuringian mafia) did not start the fire. Mob chieftains do not do such things. They delegate. It is underlings who spread the accelerant, etc.—while Albericht is free to feast in Rome. (But in the circumstances, might giving the order not count as starting the fire? What does a general, or emperor, have to do to start a war? *Shoot*?) There are various things fire-starting might be understood to be, different ones on different occasions for assigning blame. What predicables there are (for a thinker to predicate) may thus vary across occasions. But *this* sort of variation is not variation in the represented; that of which all representing truly or falsely is true or false.

7. Functions: I had, for yonks, thought of a function as a set of ordered pairs—a subset of the Cartesian product of a set (its 'domain') with a set (its 'range'), the which with a special property: for any a, b, c, if <a, b> is in the set and <a, c> is in the set, then b = c. ('The function-property') And as any such set.

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Sanford reminded me of another conception of a function, on which what identifies it as the function it is is not just its domain and range but also a law, that is, a proprietary condition on membership. For example, there is a function over the integers mapping each into its square. For it to be *that* function is for the ordered pairs that belong to it (on my original conception) to be just those such that the first member is an integer and the second its square. Similarly for a function mapping pairs of integers into their sum, the first member of its ordered pairs being itself an ordered pair. Similarly for a function mapping the degree of an angle into its sine. (For a relatively clear presentation of this idea *vide* his 1904: 683.)

So conceived, we may see a function as *relating*, lawfully, one thing to another, e.g., integers to their squares. And we may then, simultaneously, try to conceive a relation as one of the things it may sometimes be understood to be, to wit, a predicable of n-ity 2. I think that, at least in the first half of the 1890s, Frege does try to do both simultaneously. What this yields is the main bone I have to pick with him *in* his conception of logic. The main point has already been scouted. A *predicable* would be, for example, of an ordered pair that it belongs to the function such-and-such, or that its second member is the value of the function for its first member as an argument. But (to repeat), a *set* of ordered pairs, or of whatever, is not a predicable (a way for something to be). Thus one of the few bones I have to pick with Frege. It concerns this step in his story:

> We see from this how closely what in logic is called *concept* corresponds to what we are calling a function. One could just say directly: a concept is a function whose value is always a truth value. (1891: 15)

And so Frege does speak, at least in the first half of the '90s. I think there is a not-so-deeply hidden agenda here. But I save this for last. The prior question is, how could there be anything wrong with that simultaneous two-step just described? Why cannot we use "relation" to speak of a function and of a predicable all at once? The core of an answer: A set (whatever its members) has, so to speak, no representational intent. It does not represent anything as anything. Nor is it even the way something would be in being so represented. It does not "reach beyond," as per above, as a concept does.

A comparison at n-ity 1. Blue is a color. It is visibly present in certain familiar articles of clothing, flags, walls, the sky (in daytime), and so forth. It has a place in the visible spectrum. It interacts with other colors. It does not predicate anything of anything. *Being* blue (a way for something so to be) is another thing entirely. It is something *predicable.* Things may be so represented either truly or falsely.

Predicating being blue of something is doing some of the work done in some zero-place thinkables. I focus here on what this work leaves undone. The task assigned to such a predicable is to bring something under a certain generality, to identify something as an *instance* of a way for something to be which reaches beyond it on an understanding of *reaching beyond* unfolded earlier in this story. One thing striking about this task is that, at positive n-ity, it is not something the predicable, or its way-to-be, can do all on its own. Something else must determine for it what it is that is, e.g., to be blue or not. (And when such a something else is in place, one *eo ipso* is at zero-place.)

This feature, intrinsic to a predicable of positive n-ity, is, I suggest, one face of something else I mentioned in trying to unfold the here-relevant idea of reaching beyond. It is, namely, what I call the agnosticism of a predicable, or again, of what predicates. Fix a candidate for instancing, e.g., the predicable (*being*) *blue/a smoker/legal tender*, and it is the task of the predicate to deliver verdicts on whether such is what would count as a case of being the relevant way or not. If so, there is a falling under; if not, not. But what such a verdict turns on is not what candidate (what object of predication) the one in question is—whether it is this one, or rather that—but rather on how the item in question is. The question for the predicable (or predication) to answer is whether *so* being is being what it (the predicable) is of being. (Instancing, not falling under.)

Just this is what requires a predicable to reach beyond. It does not belong to its being the predicable it is that there are just *these* items (i.e., objects of predication) that are candidates for being or not what it is of being, all the less that it is determined of any candidate just *how* it is, thus

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what the candidate instancings of the way in question are. To inject a bit of imagery, when a predicable (such countable) is separated out of the mass, *things being as they are*, questions what items there are to instance it or not—all the more, *how* such items are in being as they are—are left largely behind (still embedded in the mass). For all that there is *this* way for something to be, there need not be precisely *this* thing, or *this* instancing (*casu quo* counter-instancing) for there to be such a thing as this way-to-be at all.

And yet, confronted with a candidate, and its being as it is, we expect that, indefinitely often, what we *have* extracted from the mass in moving to the countable in question is enough for this countable to render a verdict. We expect what was extracted, whatever this may be, to be enough for drawing a distinction between an instancing and a counter-instancing of the predicable extracted; that the predicable bears an understanding that, indefinitely often, makes it recognizable, or at least determinate, what, when it came to cases, would count as an instancing of it, and what not. A would-be predicable need not give up the claim because it lacks an extension. But it would give up the claim if, on inspection, it did not bear any such understanding, or, as set out above, an item's instancing the predicable confers on it no determinate role in the business of factive meaning.

Here predicables differ from functions, even conceived as per Frege (1904). Or at least they do if a function is a set of ordered pairs or, equally, maps a determinate domain onto a determinate range. The function is law-governed. What it records is what is so of each thing the relevant law in fact governs. If, e.g., the law states, map all and only lager-lovers into The True, then <Sid, The True> belongs to the function iff Sid is a lager-lover. Whereas the predicable *lager lover* does not, in being the predicable it is, *register* its successes and failures (the items which instance it, those which do not). If it did, then it would belong to its predicating what it did that it is true of Sid: Without Sid of which to predicate, there would be no such predicable at all. In which case there would be no call to reduce its n-ity, nothing needed to complete it into a *thinkable*. 'There is such a thing as loving lager' would already represent things as such that Sid loves lager. Were a predicable to register the outcomes of its applications, there would be no call for it to predicate at all. A function has, as such, its proprietary domain and range. A concept does not.

Frege's term is, granted, not "predicable" but rather "Begriff." A more accurate translation of *that* word is, admittedly, "concept." The two are more nearly equally waffly. In 1891, Frege draws his parallel between "was in der Logik Begriff genannt wird" and a function of a certain kind: "One could simply say: a concept is a function whose value is always a truth-value." (1891: 15). And he goes on to do so. Granted, for a determinate predicable of positive n-ity, there is an associated function. It takes on the value (*the*) *true* for any object of which the predicate is true, and *mutatis mutandis* for (*the*) false and that of which the predicate is false. But I do not think such justifies the identification of these two things that Frege makes. The next section sketches one case against.

8 A Diagram: In a letter to Husserl of 1891 Frege included this diagram.[1]

Satz	*Eigenname*	*Begriffswort*
↓	↓	↓
Sinn des Satzes	*Sinn des Eigennames*	*Sinn des Begriffswort*
↓	↓	↓
Bedeutung des Satzes	*Bedeutung des Eigennames*	(*Bedeutung des Begriffsworts*)
(*Wahrheitswerth*)	(*Gegenstand*)	— → *Gegenstand der unter den Begriff falt*

[1]The main idea here was inspired by discussion with David Hunter.

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His aim, he explained, was to make clear to Husserl his (Frege's) account of non-factive representing, the main notions it involved, and the contrast between this account and Husserl's picture of the phenomenon. Frege was keen to stress via the diagram the objectivity of all the elements in his view of the phenomenon, a point made by the horizontal arrow in the lower right corner, meant especially to stress this.

Explaining this arrow he writes in the letter,

> With *Begriffswörter* there is a step further to take to the object than with proper names, and this last can fail to happen … without making the concept word cease to be scientifically applicable. I have drawn this last step from concept to object sideways in order to indicate that it occurs on the same level, that objects and concepts have the same objectivity. (1891/1980: 33)

And he refers in particular to §47 of his *die Grundlagen der Arithmetik* of 1884 in which he writes,

> That a statement about a number expressed something factual, independent of how we view things is surprising only to one who supposes a concept to be something subjective, equivalent to a mental image or idea.

So it is clear that the objectivity at stake here is that of a concept. How might the diagram bear on that? In outline, it would be if a concept were the *Sinn* of a *Begriffswort* and that *Sinn* determined a *function, inter alia* a mapping (set of ordered pairs which had the function property, and which fixed for the concept a determinate (pleonastically 'objective) extension (set of objects).

But my first step here, with just one more preliminary, is just to fill the blanks in the diagram—to identify the relevant *Sinn* and *Bedeutungen.* I will do so, working columns left to right.

The preliminary. As Frege writes,

> If a *Gedanke* is not observable by the senses, it is not to be expected that its generality should be. I am not in a position to be able to present a *Gedanke* as a mineralogist is able to display a 'mineral'.

> Language may seem to open up a way out; for, on the one hand, its sentences are perceivable by the senses, and on the other they express *Gedanken.* … but appeal to language requires care. We must never neglect the great gulf which separates the domain of the linguistic from that of the thinkable, and there are definite limits to the mutual correspondence of the two domains. (1923: 279)

Though Frege is very clear that language is not his topic, often where there is occasion for him to introduce a notion, or notions, which will figure significantly in his account of non-factive representing, he starts from language, as he does in the present diagram. Spanning the gulf as we now must in moving down columns may leave rough edges here and there. I will gesture at some more important ones.

At the top left, then, stands the word "Satz," a particularly promiscuous one. Here we are starting from language, so the relevant notion would be roughly of the same ilk as in "Behauptungssatz"—a well-formed sentence of, e.g., German. Or it may be the use of such a "Satz" in some act or episode of thought-expression—a *Satz* in the sense of a (completed) performance. I will not stop to choose between these.

The next question, then, is "What is the *Sinn* of a *Satz*?" Again, confirmation of first choice. Since the phenomenon, non-factive representing, is that of truth and falsehood, the *Sinn* ought to be a truth-or-falsehood, i.e., a thinkable. This, too, is a *Satz*, but on another reading of the term as in, e.g., "Lehrsatz." The Pythagorean theorem is a "Lehrsatz": a *Satz*, but, e.g., with no way in which it is spelled.

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So in a given filling of the leftmost column, what fills the space, *Sinn* of a *Satz* should be a truth-or-falsehood, a *Satz* in the *Lehrsatz* sense if, that is, the *Satz* in the *Behauptingssatz* sense did in fact express a truth-or-falsehood. Otherwise, the game is over at this point.

(Note 1: A thinkable cannot *have* a *Sinn*, but only be one. *It* cannot be something which might yet be to be understood in any of a variety of ways. If it were, then it would not be that, let alone *just* that, which brings truth into question. What would do *that* would be the particular understanding in fact borne.

(Note 2: Sentences may afford themselves of means of expression which a thinkable cannot, e.g., indexicality or demonstration. To that extent, a sentence cannot bear the *same* understanding as the sentence which expressed it calls for. Making thought recognizable as representing things as it does and so representing things are different activities with different goals, thus, inevitably, different means of achieving them.)

Modulo these notes, we now arrive where the leftmost column bottoms out. What *should* appear here? The answer to this question will tell us a lot as to what *Sinn* and *Bedeutung* are meant to be (for Frege). It helps that it's the simplest case where such issues arise. For here the predication is of n-ity null. So there is one thing which is always represented by any representer of null n-ity. Either it or nothing would have to play the role of *Bedeutung—if* to be the *bedeutete* is to be what some relevant bit of representing is representing as being thus and so.

But there is another option. It is that the *Bedeutung* of a thinkable (or, if you like, the *Satz* of which it is the *Sinn*) is a truth-value (if there is any *Bedeutung* at all). And this, of course, is what Frege chooses for this role: The *Bedeutung* of a truth-or-falsehood is either (The) True or (The) False. How do these relate (relevantly) to the *Sinn* (a thinkable)? Well, a thinkable is in the business of being true or false. That is, it is one principal in a two-party interaction (between *it* (the non-factive representer) and what it represents as something, to wit, the way things are). A truth value is the outcome of such an interaction. So if we choose truth-values for *Sinne* of whole *Sätze* (no matter the reading of *Satz* so long as the business at hand is being true), we have here a guide for what *Bedeutungen* in other cases are going to have to be.

With which we reach the middle column. What we find here are objects, that is, objects of which to predicate. Not that there are items of which one cannot predicate. But what we are apt to focus on first are items which, in principle, one could bronze and put on the mantel (if one had a big enough mantel). And ones which cannot predicate. Granted, starting at the top, we have items engaged in non-factive representing. But, in general, non-factive representing is the sort of thing which can only be done where there is the relevant thing to represent. One cannot represent Tower Bridge either as London Bridge or as blocked with traffic as usual unless there is such a bridge of which to represent. So here, at the top, a "name," e.g., a piece of language (or utterance thereof, or a certain rude gesture). For its *Sinn*, an instance of a certain sort of proper thought-part, one whose job is to combine with a predicable to reduce its n-ity by one (in the paradigm case, thereby to zero). And at bottom, the denoted (as such there is if denoting has in fact transpired). As *Sinn* what demands of what wins the post of designatum, e.g., that it have eaten the whole thing. If Sid is the one who did this, then he is the *Bedeutung* of such *Sinn.* Sid thus the outcome of a certain interaction between a demand (itself non-factive representing) and that which supplies (and thus limits) what there *is* to denote. So far our generalization holds.

With which we move rightmost. At the top, a well-formed verb phrase, something an object short of a sentence. Alternatively, what Frege calls a "concept-word," i.e., what speaks of a way for something to be as "lager" speaks of lager (or being some), "lion" of lions, or being one, "legal tender" of (being) legal tender. The *Sinn* of such a bit of language would then reasonably be a predicable (of positive n-ity), e.g., what predicates being lager (of *a* thing). This presupposes, of course, that the relevant language really does express a predicable. If not, then, again, just move on to a more interesting endeavour.

For the moment, so much for *Sinn.* What *Bedeutung* might match up with such a *Sinn*? Well, a predicable's work is representing. What a *Bedeutung* is charged with is registering how the relevant

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representing fares when it is of what there is for it to be of. In the case of a predicable of positive n-ity, what there is for the representing it does to be of is a vast, and in principle always extendible, range of items. For any item of which it might be predicated, there is an outcome to be registered (one of the two just mentioned). Registering all that might seem like a Herculean task were it not for functions. Suppose that, for a given predicable, Π, there is a function which maps a candidate for the role of represented into The True just in case the candidate is as represented and false just in case it is not. Such would register all and only that which, by the above, there is to register. So a function of the just-mentioned sort seems well suited for the role a *Bedeutung* seems to need to play here. There are rough edges here. Whatever else, sets cannot have "indefinitely extendible" membership. Here we need to idealize a bit. Also, I allow a function here, as in Frege 1904, and suggested by Sanford, to be identified as the function it is by, *inter alia*, a governing law, such as Map it into The True if it is lager into the False if it is not. For all of which a mapping remains a mapping.

Summing up: Starting with the rightmost row, center column (as indicated by Frege's labels, the level of *Sinn*), we find "what in logic is called a concept" (Frege 1891: 15). i.e., a predicable of positive n-ity. At the column's base (the level of *Bedeutung*), we find a function into the truth values that, *inter alia*, identifies those items that are what the predicable above it) represents an item as being.

Such continuing themes which already run through the first two columns. At leftmost center (the level of *Sinn*) we find a thinkable—a predicable of n-ity null. At center-center we find a name: a bit of representing which combines with a predicable of positive n-ity to reduce its n-ity by one. Where such reduces n-ity to zero, the result is a thinkable. So at the center-center, what we have is something that operates on representers to form new representers. It is itself a bit of representing. For it also represents some object as available for n-ity reduction.

From this perspective, the suggestions above for the middle and bottom row rightmost *Sinn* and *Bedeutung* continue the theme. In the middle column, we have a predicate of positive n-ity and at the bottom a function from objects to truth-values—a means of identifying outcomes of predications—given sorts of non-factive representings—for given items in the role of represented. So, in the rightmost center we are dealing with the representer side of that two-party enterprise, non-factive representing; at the bottom we are dealing with a function that identifies things of which to predicate at higher n-ity, thus things belonging to the represented.

But if such a line of thought appeals, at this point Frege tosses a rather large sabot into the works. Because if concepts and functions (of a certain sort) are one, or if there are no significant differences between them, then so far there is only one item to fill two (still) open spaces in the schema. That is, just one item, both to *do* the non-factive representing, say, to represent an item—the ink in Pia's pen for one—as blue, and to register which items, in being as they are, *are* as thus represented and which are not. Representing non-factively and *registering* the *facts*, all wrapped into one.

Or so things would be if we let concepts (aka functions) occupy both so-far empty places (the rightmost *Sinn* and the rightmost *Bedeutung).* Not, of course, if we let this one item occupy but *one* of the still-open spaces. So then we would have non-factive representation but nothing to register when it was getting things right, or a way to register when things were being got right, but nothing *to* go right or wrong.

It *might* help at this point to pause for a moment over what it would be to identify what is spoken of *thus* with what is spoken of *so.* Not, of course, to identify one thing with another. One cannot make two one by fiat. Nor do I think Frege is trying to do such a nonsensical thing. In fact, as Frege insists, one cannot confer *any* property on a thing that it does not already have (or at least would not have anyway). The most stipulation could achieve is to make a given sign signify something there is anyway to signify.

But this still leaves things one might be able to do. Where we now think in terms of things being F and in terms of things being G, we can cease speaking, or thinking, in those terms and instead start to think in terms of things being a single thing, H, where H does the work we expected of F and also that we expected of G. For this to succeed, of course, there must be such an item, H, to think in terms of.

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Suppose we let F be a *concept* (i.e., predicable/predication thereof) and G a *function* (mappings, as per 1904 if you like) Now H obviously needs to be baptized. We need a new term. I suggest *Funcept*/*Funriff*. How, now, is this to work?

So far, so good. But now what about a function do we want also to find in a funcept? Whatever else, a function maps objects into objects. For it to map what it does into what it does is intrinsic to its being the function it is. It could not be that a function which maps Sid into The True, that very one, might have mapped him into the false. Thus it is representable as a set of ordered pairs.

Now how are we to fill in the top and spaces in our diagram? At the top sits a bit of language (a 'concept-word', as Frege puts it.), Midway, what Frege calls a *Sinn*. At bottom, what he calls a *Bedeutung*. We can think of the tasks assigned to these last two items thus: The task for what sits midway is to impose a requirement. It is then for what sits at the bottom to record the outcome of the imposition (*in re* truth and falsehood). Now, a funcept is fit for the first task by virtue of what it preserves of a concept. A concept (predicable) does indeed impose a requirement (on being true, or true of, *selon* the n-ity). But it is also so of a concept that, in general, it is agnostic as to what items it is true, what not. That is, in general, a concept does not fix an extension independent of how things are. If Sid falls under the concept of *smoker*, he need not have at least so far as the concept *smoker* is concerned. It was all up to him to choose to smoke or not. (Well, maybe.) So there is one feature of a concept as usually conceived that we do not want here.

So far, still so good. But that unwanted feature is exactly what is wanted of what fills the bottom space in the column. It is to be a record keeper, keeping track of what happens when a given requirement is imposed: what satisfies it and what does not. How does it keep track? Not by imposing a further requirement on pain of regress. Nor by closing off in any other way what it must leave open if to serve mid-right. So our attempt to find an all-in-one participant in non-factive representation *scheitert*. And so would any other attempt to satisfy these two demands at once, one calling for an engager in non-factive representing, the other calling for something unsuitable either to engage in representing or to be non-factive. One might, with Frege, remark, "how closely what in logic is called concept fits together with what we call function." But I advise against continuing, "One can just directly say: "a concept is a function." It is not."

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