CORRIGENDUM TO:
"A LATTICE-POINT PROBLEM IN HYPERBOLIC SPACE"

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In the above paper [1] there is an error in the statements of Proposition 1 and Theorem 1 which we shall now correct. The principal results, Theorems 2 and 3, are unaffected.

The incorrect statement is that of equation (12). This should read

$$|h_X(r)| \leqslant c_1 X^{|\mathrm{Re}(\frac{1}{2}+ir)|}(1+|r|^2)^{-\frac{5}{4}}$$

and is valid only for $|r| \geqslant \frac{1}{3}$, say. From (11) one obtains directly the following statement for $|r| \leqslant \frac{1}{3}$, r real

$$h_X(r) - \tfrac{4}{3}X^{\frac{1}{2}}(X^{ir} - X^{-ir})/ir = O(X^{\frac{1}{2}})$$

and, consequently,

$$h_X(0) = (\tfrac{8}{3})X^{\frac{1}{2}}\log X + O(X^{\frac{1}{2}}).$$

This gives now

$$\sum_{g \in G} k_X\big(L(z_1, gz_2) - 4\big) = \sum_{\lambda_\mu \leqslant \frac{1}{4}} h_X(r_\mu)\,\phi_\mu(z_1)\,\phi_\mu(z_2)$$

$$+ \int_{-\frac{1}{3}}^{+\frac{1}{3}} \psi(z_1, z_2, r)\,\frac{X^{ir} - X^{-ir}}{ir}\,dr\,X^{\frac{1}{2}} + O(X^{\frac{1}{2}})$$

where $$\psi(z_1, z_2, r) = (\pi/3) \sum_{p \in P} E_p(z_1, \tfrac{1}{2}+ir)\,E_p(z_2, \tfrac{1}{2}-ir).$$

As (z_1, z_2, r) is analytic on $\mathrm{Im}(r) = 0$

$$\int_{-\frac{1}{3}}^{\frac{1}{3}} \frac{X^{ir} - X^{-ir}}{ir}\big(\psi(z_1, z_2, r) - \psi(z_1, z_2, 0)\big)\,dr = O(1).$$

But $$\int_{-\frac{1}{3}}^{\frac{1}{3}} \frac{X^{ir} - X^{-ir}}{ir}\,dr = 4\int_0^{(\log X)/3} \sin x\,dx/x = O(1).$$

Thus the right-hand side of Theorem 1 should contain the additional term

$$2\tfrac{2}{3} \sum_{s_\mu = \frac{1}{2}} \phi_\mu(z_1)\,\phi_\mu(z_2)\,X^{\frac{1}{2}}\log X,$$

the error term remaining as before. This term makes no difference to the Tauberian argument and Theorems 2, 3 are unaffected.

Reference

1. S. J. Patterson. "A lattice-point problem in hyperbolic space", *Mathematika*, 22 (1975), 81–88.

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10C30: *NUMBER THEORY; Forms; Arithmetical properties of classical groups.*

20H10: *GROUP THEORY; Other groups of matrices, Fuchsian groups.*

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