A GENERALIZATION OF GLOBAL CLASS FIELD THEORY

TAE KUN SEO AND G. WHAPLES

Introduction. Let R be a field of rational functions of one variable over a field of constants R_0. Dock Sang Rim (**6**) has proved that the global reciprocity law in exactly the usual sense holds whenever R_0 is an absolutely algebraic quasi-finite field of characteristic not equal to 0: this was known before only when R_0 was a finite field. We shall give another proof of Rim's result by means of a noteworthy generalization of the usual global reciprocity law. Namely, let R_0 be a finite field and let F be the set of all fields k contained in some fixed $R^{\text{alg.clos.}}$ and of finite degree over R. The reciprocity law states that there exists a family $\{f_k\}$, $k \in F$, of functions f_k: $C_k \to G(k^{\text{abel.clos.}}/k)$ (where C_k is the idèle class group of k) enjoying certain properties such as the norm transfer law. Let F^* denote the set of all fields which are composite of a field in F and a quasi-finite algebraic extension of R_0, possibly of infinite degree. We shall show that if the idèle class groups C_k are replaced by their closures $\hat{C}_k$ under a certain topology, we can define a generalized norm $N_{L/k}$: $\hat{C}_L \to \hat{C}_k$ for all $k, L \in F^*$ with $k \subset L$, and a family $\{\hat{f}_k\}$, $k \in F^*$, of functions defined on the groups $\hat{C}_k$, such that the global reciprocity law holds for our much larger set of ground field F^* with the C_k replaced by the $\hat{C}_k$. Finally, let $k \in F^*$ and let $F(k)$ denote the set of finite extensions of k. The global reciprocity law in exactly the usual sense holds for the family $\{f_L\}$, $L \in F(k)$, of functions f_L obtained by restricting the $\hat{f}_L$ to the subgroup $C_L \subset \hat{C}_L$; this proves Rim's result.

Since our proof uses only routine topological constructions, our results are in a certain sense trivial.

Several papers (the most recent being **3**; **4**; **5**) have been written already on global reciprocity law over ground fields of infinite degree but they replace the usual idèle class group by something else.

1. Extended idèles. Let k be a local field (i.e., complete under a discrete rank 1 valuation) and call its residue class field $\bar{k}$ *good* if it is quasi-finite, absolutely algebraic, and of characteristic $p \neq 0$. We shall consider only such local fields. Topologize the multiplicative group $k^\cdot$ of k by taking the subgroups $\{k^{\cdot n} k_{(i)}^\cdot\}$, for all positive integers n and i, as a base for the open neighbourhoods of 1, where $k_{(i)}^\cdot$ denotes the group of elements congruent 1 mod π^i.

Received December 13, 1967.

PROPOSITION 1. *If $\bar{k}$ is good, then $k^\cdot$ is a Hausdorff space under this topology.*

Proof. We must show that $\bigcap_{n,i} k^{\cdot n} k_{(i)}^\cdot = \{1\}$. Let α be contained in this intersection. Its residue class $\bar{\alpha}$ is contained in $\bigcap_n \bar{k}^{\cdot n}$. Suppose that $\bar{\alpha} \neq 1$. Then since $\bar{k}$ is good, $\bar{\alpha}$ is a primitive mth root of unity for some $m > 1$ and prime to p. Let q be a prime different from p and dividing m. If $\bar{\alpha}$ were a q^jth power for every j, then $\bar{k}$ would contain primitive q^{j+1}th roots of unity for every j, the Steinitz degree of $\bar{k}$ over $\mathbf{Z}/p$ would be divisible by p^∞, and $\bar{k}$ would not be quasi-finite. Thus, $\bar{\alpha} = 1$, i.e., $\alpha \equiv 1 \bmod \pi$. Considering the neighbourhoods $k^{\cdot p^i} k_{(i)}^\cdot$ shows that $\alpha = 1$.

Let $\hat{k}$ be the completion of $k^\cdot$ under this topology. Proposition 1 shows that there is a monomorphism $k^\cdot \to \hat{k}$. We shall identify $k^\cdot$ with its image in $\hat{k}$ so that $k^\cdot \subset \hat{k}$. Artin (**1**) defined $\hat{k}$ in the case when $\bar{k}$ is finite: if that is so, then $\hat{k}$ is compact and the local norm residue symbol can be extended to an isomorphism of $\hat{k}$ onto the Galois group of $k^{\text{abel.clos.}}/k$; however, if $\bar{k}$ is not finite, then $\hat{k}$ is not compact and the local norm residue symbol can only be extended to a monomorphism into.

Let $\hat{\mathbf{Z}}$ denote the product over all primes p of the rings $\mathbf{Z}_p$ of p-adic integers with p-adic topology on the $\mathbf{Z}_p$ and product space topology on $\hat{\mathbf{Z}}$. There is an obvious injection $\mathbf{Z} \to \hat{\mathbf{Z}}$: namely, $n \in \mathbf{Z}$ goes into the element of $\hat{\mathbf{Z}}$ whose p-component is n at every prime p. We identify $\mathbf{Z}$ with its image under this injection and consider $\mathbf{Z} \subset \hat{\mathbf{Z}}$. Then (**1**) the topology on $\hat{\mathbf{Z}}$ induces on $\mathbf{Z}$ the topology defined by taking the subsets $\{m\mathbf{Z}\}$, $m \in \mathbf{Z}$, $m \neq 0$, as base for the open neighbourhoods of 0, and $\hat{\mathbf{Z}}$ is the completion of $\mathbf{Z}$ under this topology. The natural definition of $k^\cdot$ as $\mathbf{Z}$-module can be uniquely extended to a definition of $\hat{k}$ as $\hat{\mathbf{Z}}$-module.

Let k be a product formula field for a set of non-archimedean valuations $M(k)$ (i.e., a field of algebraic functions of one variable). Assume that the field of constants k_0 of k is good. Define the group $\hat{J}_k$ of *extended k-idèles* by replacing the $k_\mathfrak{p}^\cdot$ in the definition of idèles by the groups $\hat{k}_\mathfrak{p}$. That is, $\hat{J}_k$ is the subgroup of $\prod_{\mathfrak{p} \in M(k)} \hat{k}_\mathfrak{p}$ consisting of the elements which are units at all but finitely many primes. (Unit in $\hat{k}_\mathfrak{p}$ = limit of units of $k_\mathfrak{p}$.) We do not consider J_k or $\hat{J}_k$ as topological groups. Under natural identifications we consider $k^\cdot \subset J_k \subset \hat{J}_k$; since $\hat{J}_k$ is a $\hat{\mathbf{Z}}$-module, this defines an action of $\hat{\mathbf{Z}}$ on $k^\cdot$; let $\hat{k}$ denote the group of all α^n, $\alpha \in k^\cdot$, $n \in \hat{\mathbf{Z}}$; it is a $\hat{\mathbf{Z}}$-module contained in $\hat{J}_k$. Define $\hat{C}_k = \hat{J}_k/\hat{k}$: it is a $\hat{\mathbf{Z}}$-module and there is a natural homomorphism of C_k into $\hat{C}_k$.

PROPOSITION 2. *If k_0 is good, this homomorphism $C_k \to \hat{C}_k$ is a monomorphism; i.e., $J_k \cap \hat{k} = k^\cdot$.*

Proof. Suppose that $\mathfrak{a} = \alpha^m$ with $\mathfrak{a} \in J_k$, $\alpha \in k^\cdot$, $m \in \hat{\mathbf{Z}}$. Proposition 1 shows that $\mathfrak{a}$ is uniquely determined by α and m. If $m \in \mathbf{Z}$, then $\mathfrak{a} \in k$, and the proof is complete; thus, assume that $m \notin \mathbf{Z}$. If $|\alpha|_\mathfrak{p} \neq 1$ for any $\mathfrak{p} \in M_k$ we easily see that an equation $\alpha_p = \alpha^m$ with $\alpha_\mathfrak{p} \in k_\mathfrak{p}$ is impossible; therefore,

https://doi.org/10.4153/CJM-1969-069-8 Published online by Cambridge University Press

$|\alpha|_\mathfrak{p} = 1$ for all $\mathfrak{p}$, i.e., α is in k_0; hence, α is a root of unity, and hence α^m is also in k_0, and this completes the proof.

2. Fake degrees and fake norms. If k is any subfield of a field K, let $\deg K/k$ mean the Steinitz number $\prod p^{\nu(p)}$ (product taken over all positive prime integers p with $\nu(p)$ a non-negative integer or ∞) which is the l.c.m. of the degrees of all finite algebraic extensions K' of k with $K' \subset k$. We shall use this notion only when K/k is algebraic or when k is complete under a non-archimedean valuation and K is the completion of an algebraic extension of k. In these cases one can show that $k \subset L \subset K$ implies

$$\deg K/k = (\deg L/k)(\deg K/L). \tag{1}$$

Call a Steinitz number $\prod p^{\nu(p)}$ *quasi-finite* if $\nu(p) \neq \infty$ for every p. If k is any quasi-finite field and K/k algebraic, then K is quasi-finite if and only if $\deg K/k$ is quasi-finite.

From now on, let R be the field of rational functions of one variable over a finite field of constants R_0: one may take $R_0 = \mathbf{Z}/p$. Let F^* denote the set of all subfields k of some fixed $R^{\text{alg.clos.}}$ such that $R \subset k' \subset k$ with k'/R of finite degree and k/k' a constant field extension of quasi-finite degree. It is easy to see that F^* is closed under finite algebraic extension and quasi-finite constant field extension.

The field R is of course a product formula field for the set M_R of all primes (= equivalence classes of valuations) of R. If $k \in F^*$, then k is a product formula field for the set M_k of all its primes. Its field of constants k_0 is "good" in the sense of § 1. Every prime in M_R has only finitely many extensions to a $\mathfrak{p} \in M_k$: the ramification number of $\mathfrak{p}$ in k/R will be finite but the residue class degree will be a quasi-finite Steinitz number, not finite unless $\deg k/R$ is finite.

If $L, K \in F^*$, define $L \sim K$ to mean that $\deg LK/K$ and $\deg LK/L$ are finite. It is an equivalence relation.

PROPOSITION 3. *Let L, $K \in F^*$, let $\mathfrak{P} \in M_{LK}$, and let $K_\mathfrak{P}$, $L_\mathfrak{P}$, and $R_\mathfrak{P}$ be the completions of L, K, and R at $\mathfrak{P}$. Then*

$$\begin{aligned} L \sim K &\Leftrightarrow (\deg K/R)/(\deg (L/R)) \textit{ is rational} \\ &\Leftrightarrow (\deg K_\mathfrak{P}/R_\mathfrak{P})/(\deg L_\mathfrak{P}/R_\mathfrak{P})) \textit{ is rational} \\ &\Leftrightarrow L_\mathfrak{P} \sim K_\mathfrak{P}. \end{aligned}$$

Proof. These quotients of Steinitz numbers make sense since none of the exponents is ∞. Of course, such a quotient is rational if and only if the exponents in numerator and denominator are equal at all but finitely many p.

If $L \sim K$, then $(\deg K/R)/(\deg L/R) = (\deg LK/L)/(\deg LK/K)$ by (1); thus, it is rational. Let $R \subset L' \subset L$ and $R \subset K' \subset K$ with L'/R, K'/R finite and L/L', K/K' constant extensions. If $(\deg K/R)/\deg(L/R)$ is rational, then $\deg(KL'/K'L')/\deg(K'L/K'L')$ is rational; thus, $\deg KL/KL'$ and $\deg KL/K'L$ are finite since a constant extension is completely determined by its degree, therefore, $\deg KL/K$ and $\deg KL/L$ are finite. This proves the first "$\Leftrightarrow$" and the last "$\Leftrightarrow$" follows in the same way.

To prove the middle "$\Leftrightarrow$", notice that the residue class field of K' is of finite degree over the field of constants of K' and if d is the g.c.d. of this degree and $\deg K/K'$, then $\deg K/K' = d \cdot \deg K_{\mathfrak{P}}/K_{\mathfrak{P}'}$, where d is an ordinary integer; similarly for $\deg L/L'$. Therefore, the middle "$\Leftrightarrow$" holds.

If $a \in \hat{\mathbf{Z}}$ and p is any prime, define $v(a, p)$ by $|a|_p = |p|_p{}^{v(a,p)}$, where $|a|_p$ is the p-adic value of the p-component of a ($v(a, p) = \infty$ if this p-component is 0). The function h: $a \to \prod p^{v(a,p)}$ is a homomorphism, under multiplication, of $\hat{\mathbf{Z}}$ onto the Steinitz numbers: its kernel is the group of units of $\hat{\mathbf{Z}}$, it maps the set of non-0-divisors of $\hat{\mathbf{Z}}$ onto the quasi-finite Steinitz numbers, and it reduces to the identity on the positive integers.

PROPOSITION 4. *Let S^* denote the set of quasi-finite Steinitz numbers,* $\mathbf{P}$ *the positive integers. There exist functions ϕ: $S^* \to \hat{\mathbf{Z}}$ such that*

(4.1) *$h \circ \phi$ = identity on S^*;*

(4.2) *$\phi(a) = a$ for $a \in \mathbf{P}$;*

(4.3) *$a|b \Rightarrow \phi(a)|\phi(b)$ for $a, b \in S^*$;*

(4.4) *If a/b is rational, then $a/b = \phi(a)/\phi(b)$.*

Proof. Call two elements of S^* equivalent if their quotient is rational, choose representatives of the equivalence classes and define $\phi(r)$ so that $h\phi(r) = r$ for all representatives r; then define $\phi(r\rho) = \rho\phi(r)$ if ρ is rational and $r\rho \in S^*$.

Remark 1. It is an interesting problem, unsolved so far by us, whether ϕ can be chosen to be a homomorphism $S^* \to \hat{\mathbf{Z}}$.

Choose a particular fixed ϕ satisfying Proposition 4 and for $k \in F^*$ define the fake degree $\partial(k/R)$ to be $\phi(\deg k/R)$. If $k \subset K \in F^*$, define $\partial(K/k) = \partial(K/R)/\partial(k/R)$: this implies that $\partial(K/k) = \partial(L/k) \cdot \partial(K/L)$ whenever $k \subset L \subset K$, and $\partial(K/k)$ is the ordinary degree whenever this is finite. Similarly, for each prime spot $\mathfrak{P}$ of K define fake local degree $\partial(K_{\mathfrak{P}}/k_{\mathfrak{P}})$ to be

$$\phi(\deg(K_{\mathfrak{P}}/R_{\mathfrak{P}}))/\phi(\deg(k_{\mathfrak{P}}/R_{\mathfrak{P}})).$$

PROPOSITION 5. *Let $k, K \in F^*$ with $k \subset K$; then for all $\mathfrak{p} \in M_k$,*

$$\partial(K/k) = \sum_{\mathfrak{P}|\mathfrak{p}} \partial(K_{\mathfrak{P}}/k_{\mathfrak{P}}), \tag{2}$$

where the sum is taken over all $\mathfrak{P} \in M_K$ which divide $\mathfrak{p}$.

Proof. Since $\mathfrak{p}$ has only finitely many extensions to K we can find K' with K'/k finite and K/K' a constant extension such that each $\mathfrak{P}' \in M_{K'}$ which divides $\mathfrak{p}$ has only one extension $\mathfrak{P}$ to M_K; then $\partial(K_{\mathfrak{P}}/K_{\mathfrak{P}'}) = \partial(K/K')$ for each $\mathfrak{P}|\mathfrak{p}$ and (2) holds for K/k since it holds for the finite extension K'/k.

Now we define fake norms. Let $k, K \in F^*$ with $k \subset K$ and let $\mathfrak{P} \in M_K$. To simplify printing we shall sometimes denote completions at $\mathfrak{P}$ by k^* and K^* instead of $k_{\mathfrak{P}}$ and $K_{\mathfrak{P}}$. If A is any element of $K_{\mathfrak{P}}$ which is algebraic over $k_{\mathfrak{P}}$, then its fake local norm $N_{K/k,\mathfrak{P}}A$ is defined to be $(N_{K'/k^*}A)^{\partial(K^*/K')}$, where K'

is any finite algebraic extension of k^* containing A and contained in K^* and $N_{K'/k^*}A$ is the ordinary norm. This is independent of the choice of K' and defines $N_{K/k,\mathfrak{P}}$ on an everywhere dense subset of $\hat{K}_{\mathfrak{P}}$: extend it by continuity to a homomorphism $N_{K/k,\mathfrak{P}}$: $\hat{K}_{\mathfrak{P}} \to \hat{k}_{\mathfrak{P}}$.

PROPOSITION 6. *If* $k \subset L \subset K \in F^*$ *and* $\mathfrak{P} \in M_K$, *then*

$$N_{K/k,\mathfrak{P}} = N_{L/k,\mathfrak{P}} \circ N_{K/L,\mathfrak{P}}.$$

Proof. Let $L^* = L_{\mathfrak{P}}$. Suppose that A is in K^* and is algebraic over k^*. Let $f(x)$ be the monic irreducible polynomial with coefficients in L^* satisfied by A and let L' be the finite extension of k^* generated by the coefficients of $f(x)$. Then

$$\begin{aligned} N_{K/k,\mathfrak{P}}A &= (N_{L'(A)/k^*}A)^{\partial(K^*/L'(A))} \\ &= N_{L'/k^*}((N_{L^*(A)/L^*}A)^{\partial(K^*/L^*(A))})^{\partial(L^*/L')} \end{aligned}$$

since $N_{L^*(A)/L^*}A = N_{L'(A)/L'}A = \pm f(0)$ and

$$\begin{aligned} \partial(K^*/L'(A)) &= \partial(K^*/L^*)\partial(L'(A)/L')^{-1}\partial(L^*/L') \\ &= \partial(K^*/L^*)\partial(L^*(A)/L^*)^{-1}\partial(L^*/L') = \partial(K^*/L^*(A))\partial(L^*/L'). \end{aligned}$$

Now define $N_{K/k}$: $\hat{J}_K \to \hat{J}_k$ as follows: the $\mathfrak{p}$-component of $N_{K/k}\mathfrak{A}$ shall be $\prod_{\mathfrak{P}|\mathfrak{p}} I_{\mathfrak{P}/\mathfrak{p}} N_{K/k,\mathfrak{P}} A_{\mathfrak{P}}$, where $A_{\mathfrak{P}}$ denotes the $\mathfrak{P}$-component of $\mathfrak{A}$ and $I_{\mathfrak{P}/\mathfrak{p}}$ the canonical isomorphism $\hat{k}_{\mathfrak{P}} \to \hat{k}_{\mathfrak{p}}$. This is exactly like the usual definition of norm of an idèle and it defines a homomorphism, also called $N_{K/k}$, of $\hat{C}_K$ into $\hat{C}_k$. Our fake norm satisfies the transitivity law by Proposition 6.

Remark 2. Our fake degrees and fake norms have the following disadvantage: if K/k is normal separable and $K \cap L = k$, then deg LK/L = deg K/k and $N_{LK/L} = N_{K/k}$ need not hold for fake degree and norm, unless deg K/k is finite. If we could construct a ϕ which was a homomorphism (see Remark 1) we could remove this disadvantage.

3. Generalized reciprocity law. For $k \in F$ the usual norm residue function f_k can be uniquely extended by continuity to a $\hat{\mathbf{Z}}$-homomorphism $\hat{f}_k$: $\hat{J}_k \to G(k^{\text{abel.clos.}}/k)$. Let $k \in F^*$ and let K/k be abelian of finite degree n. We can find $k', K' \in F$ with $k' \subset k$, $\partial(k/k')$ prime to n, $K = K'k$, K'/k' finite abelian and $G(K/k) \cong G(K'/k')$ under restriction.

Let $\mathfrak{a} \in \hat{J}_k$. We can find $\mathfrak{a}_1 \in J_k$ such that $\mathfrak{a} \in \hat{J}_k{}^n\mathfrak{a}_1$. There is a finite $S \subset M_k$ such that K/k is unramified and $\mathfrak{a}_1$ is a local unit at all primes outside S. At each $\mathfrak{p} \in S$ the $\mathfrak{p}$-component of $\mathfrak{a}_1$ is congruent modulo the local conductor of K/k to a finite linear combination of powers of the local prime element times elements of local field of constants. Therefore, we can further assume that there is an $\mathfrak{a}' \in J_{k'}$ with $\mathfrak{a} \equiv C_{k/k'}\mathfrak{a}'$ mod $N_{K/k}\hat{J}_K$.

There is a unique $\sigma_K \in G(K/k)$ with $\sigma_K = \hat{f}_{k'}(N_{k'/k}C_{k/k'}\mathfrak{a}')$ on K'. It is independent of the choice of k', K', S, and $\mathfrak{a}'$. Define $\hat{f}_k$: $\hat{J}_k \to G(k^{\text{abel.clos.}}/k)$ by requiring that $\hat{f}_k(\mathfrak{a}) = \sigma_K$ for all $\mathfrak{a} \in \hat{J}_k$, all finite abelian K/k: this is possible in a unique way because of the compactness of the Galois group.

The function $\hat{f}_k$ could have been defined in many other equivalent ways:

$$\hat{f}_k(\mathfrak{a}) = \hat{f}_{k'}(\mathfrak{a}'^{\partial(k'/k)}) = f_{k'}(\mathfrak{a}')^{\partial(k'/k)} \tag{3}$$

for $\mathfrak{a}'$, k', K' as above. Also, $\hat{f}_k$ equals the product of the extended local norm residue functions. Its definition involves one arbitrary choice, namely the function ϕ satisfying Proposition 4.

THEOREM 1. *The family* $\{\hat{f}_k\}$, $k \in F^*$, *has the following properties:*

(a) $k \subset L \in F^* \Rightarrow \mathrm{rst}_{k^{\mathrm{abel.clos.}}} \circ \hat{f}_L = \hat{f}_k \circ N_{L/k}$ ("rst" *denotes the restriction);*

(b) *For* $\sigma \in G(R^{\mathrm{alg.clos.}}/R)$, $\sigma_G \circ \hat{f}_k = \hat{f}_{k\sigma} \circ \sigma$, *where* $\sigma_G(\rho) = \sigma\rho\sigma^{-1}$ *for* $\rho \in G(R^{\mathrm{alg.clos.}}/R)$;

(c) *For K/k finite abelian, the kernel of* $\mathrm{rst}_K \circ \hat{f}_k$ *is exactly* $k^{\cdot} N_{K/k}\hat{J}_K$.

Proof. (a) and (b) follow by routine constructions like those used to prove Proposition 6. From (a), it follows that if K/k is finite abelian, then $k^{\cdot} N_{K/k}\hat{J}_K$ is contained in the kernel of $\mathrm{rst}_K \circ \hat{f}_k$. Let $\mathfrak{a}$ be contained in this kernel. Then for K', k', $\mathfrak{a}'$ as above, we see that $\mathfrak{a} \equiv C_{k'/k}\mathfrak{a}'$ mod $N_{K/k}\hat{J}_K$ and (3) yields $\hat{f}_k(\mathfrak{a}) = f_{k'}(\mathfrak{a}')^{\partial(k'/k)} = 1$ on K' with $\partial(k'/k)$ prime to degree K'/k'. Therefore, $f_{k'}(\mathfrak{a}') = 1$ on K' and by the ordinary reciprocity law, $\mathfrak{a}' = \alpha' N_{K'/k'}\mathfrak{B}'$ with $\mathfrak{B}' \in J_K$. We easily see that $\mathfrak{a} \equiv \alpha' N_{K/k} C_{K'/K}\mathfrak{B}'$ mod $N_{K/k}\hat{J}_K$, completing the proof of (c).

COROLLARY. *Let $k \in F^*$, let $F(k)$ be the family of finite extensions of k, and let $\{f_L\}$, $L \in F(k)$, be the restrictions to the ordinary idèle class groups C_L of the functions $\hat{f}_L$. Then the global reciprocity law over k holds for this family.*

REFERENCES

1. E. Artin, *Algebraic numbers and algebraic functions.* I, Mimeographed lecture notes, Princeton University, Princeton, N.J.; New York University, New York, 1951 (printed ed., Gordon and Breach, New York, 1967).
2. E. Artin and J. Tate, *Class field theory*, Lecture notes, Harvard University, Cambridge, Massachusetts, 1961 (W. A. Benjamin, New York, 1968).
3. Y. Kawada, *On the class field theory on algebraic number fields with infinite degree*, J. Math. Soc. Japan *3* (1951), 104–115.
4. ——— *Class formations.* IV (*infinite extension of ground field*), J. Math. Soc. Japan *9* (1957), 395–405.
5. M. Mori, *Ueber Klassenkörpertheorie für unendliche Erweiterungen von einem p-adischen Zahlkörper*, Proc. Japan Acad. *33* (1957), 376–379.
6. D. S. Rim and G. Whaples, *A remark on quasi-finite field*, Nagoya Math. J. *27* (1966), 323–329.
7. Tae Kun Seo, *A generalization of global class field theory*, Ph.D. Thesis, Indiana University, Bloomington, Indiana, 1965.
8. G. Whaples, *Lecture notes on class field theory*, Bowdoin College, Summer, 1966.

Indiana University,
Bloomington, Indiana;
University of Kentucky,
Lexington, Kentucky;
University of Massachusetts,
Boston, Massachusetts