26
The standard model

The development of a unified theory of the electroweak interactions surely must be regarded as one of the great intellectual achievements of our era. We assume the reader is familiar with the basic phenomenology of the weak interactions [Cu83, Wa95] and turn to the so-called *standard model* of the electroweak interactions [We67, We72, Sa64, Gl70]. The discussion follows [Wa95]

Most leptons (l, ν_l) are light, or massless, and they can be created and destroyed in weak interactions. This indicates that they *must* be described with relativistic quantum fields. In the interaction representation, fermion fields take the following form [Bj65, Fe71]

$$\psi(x) = \frac{1}{\sqrt{\Omega}} \sum_{\mathbf{k}\lambda} \left[a_{\mathbf{k}\lambda} u(\mathbf{k}\lambda) e^{ik\cdot x} + b^{\dagger}_{\mathbf{k}\lambda} v(-\mathbf{k}\lambda) e^{-ik\cdot x} \right] \tag{26.1}$$

In this expression a destroys a lepton, $b^{\dagger}$ creates an antilepton, and λ denotes the helicity with respect to the accompanying momentum variable.[1]

A spinor field can always be decomposed as follows

$$\begin{aligned} \psi &= \frac{1}{2}(1+\gamma_5)\psi + \frac{1}{2}(1-\gamma_5)\psi \equiv \psi_L + \psi_R \\ \bar{\psi}\gamma_\mu \frac{\partial}{\partial x_\mu}\psi &= \bar{\psi}_L \gamma_\mu \frac{\partial}{\partial x_\mu}\psi_L + \bar{\psi}_R \gamma_\mu \frac{\partial}{\partial x_\mu}\psi_R \\ \bar{\psi}\psi &= \bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L \end{aligned} \tag{26.2}$$

The lepton fields for the electron and electron neutrino[2] will be com-

[1] Hole theory implies that $v(-\mathbf{k}\lambda)$ is a negative-energy wave function with helicity λ with respect to $-\mathbf{k}$.

[2] And similarly for the other leptons.

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bined in the following fashion:

$$\psi_l = \begin{pmatrix} \psi_{\nu_e} \\ \psi_e \end{pmatrix} \equiv \begin{pmatrix} \nu \\ e \end{pmatrix} \tag{26.3}$$

The fields (L, R) are defined by

$$\begin{aligned} L &\equiv \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = \frac{1}{2}(1+\gamma_5)\psi_l \\ R &\equiv e_R = \frac{1}{2}(1-\gamma_5)\psi_e \end{aligned} \tag{26.4}$$

The kinetic energy of the leptons is then given by[3]

$$\begin{aligned} \mathscr{L}^0_{\text{lepton}} &= -\left[\bar{\psi}_e\gamma_\mu\frac{\partial}{\partial x_\mu}\psi_e + \bar{\nu}_L\gamma_\mu\frac{\partial}{\partial x_\mu}\nu_L\right] \\ &= -\left[\bar{L}\gamma_\mu\frac{\partial}{\partial x_\mu}L + \bar{R}\gamma_\mu\frac{\partial}{\partial x_\mu}R\right] \end{aligned} \tag{26.5}$$

This lagrangian is invariant under a global $SU(2)_W$ symmetry — a weak (left-handed) isospin — which treats the field L as a weak isodoublet and R as a weak isosinglet. The generators for this $SU(2)_W$ symmetry can be immediately written in terms of the above fields as

$$\begin{aligned} \hat{T}^i_W &= \int L^\dagger(\mathbf{x})\frac{1}{2}\tau_i L(\mathbf{x})d^3x \\ &= \int \psi_l^\dagger(\mathbf{x})\frac{1}{2}\tau_i\frac{1}{2}(1+\gamma_5)\psi_l(\mathbf{x})d^3x \end{aligned} \tag{26.6}$$

It follows immediately from the canonical (anti)commutation relations that these generators satisfy an $SU(2)$ algebra

$$[\hat{T}^i_W, \hat{T}^j_W] = i\varepsilon_{ijk}\hat{T}^k_W \tag{26.7}$$

The finite symmetry transformations are given by

$$\begin{aligned} \exp\{i\boldsymbol{\theta}\cdot\hat{\mathbf{T}}_W\}\,L\,\exp\{-i\boldsymbol{\theta}\cdot\hat{\mathbf{T}}_W\} &= [e^{-\frac{i}{2}\boldsymbol{\theta}\cdot\boldsymbol{\tau}}]\,L \quad ;\ \text{doublet} \\ \exp\{i\boldsymbol{\theta}\cdot\hat{\mathbf{T}}_W\}\,R\,\exp\{-i\boldsymbol{\theta}\cdot\hat{\mathbf{T}}_W\} &= [1]\,R \quad ;\ \text{singlet} \end{aligned} \tag{26.8}$$

These equations follow from the projection properties of $(1 \pm \gamma_5)/2$.

The mass term for the electron has the following form

$$-m_e\bar{\psi}_e\psi_e = -m_e[\bar{e}_L e_R + \bar{e}_R e_L] \tag{26.9}$$

[3] There is only one neutrino field in the standard model $\nu_L \equiv \frac{1}{2}(1+\gamma_5)\psi_\nu$; it describes left-handed neutrinos and right-handed antineutrinos. This is put in by hand, as is the fact that this neutrino is massless $m_\nu = 0$.

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This expression is *not* invariant under $SU(2)_W$. Hence if one wants to build on this symmetry, it is necessary to start with *massless fermions.*

The corresponding lagrangian for point Dirac nucleon fields is relatively simple and illustrates the general structure of the theory [We72]; we present this first. Matrix elements for physical nucleons then follow from general symmetry considerations. The somewhat more complex formulation in terms of quarks is then given [Gl70].

Proton and neutron fields are included in a manner analogous to the above

$$\begin{aligned} N_L &= \begin{pmatrix} p_L \\ n_L \end{pmatrix} = \frac{1}{2}(1+\gamma_5)\psi_N \qquad ; \text{ doublet} \\ & p_R, n_R \qquad\qquad\qquad\qquad ; \text{ singlets} \\ \mathscr{L}^0_{\text{nucleon}} &= -\left[\bar{N}_L \gamma_\mu \frac{\partial}{\partial x_\mu} N_L + \bar{p}_R \gamma_\mu \frac{\partial}{\partial x_\mu} p_R + \bar{n}_R \gamma_\mu \frac{\partial}{\partial x_\mu} n_R\right] \end{aligned} \tag{26.10}$$

This lagrangian is now also invariant under $SU(2)_W$; again this is true only if one starts with massless fermions.

The standard model introduces an additional global $U(1)_W$ symmetry *weak hypercharge* defined so that the fields transform according to

$$\exp\{i\alpha\hat{Y}_W\}\,\phi\,\exp\{-i\alpha\hat{Y}_W\} \;=\; e^{-i\alpha Y_W}\phi \tag{26.11}$$

Now assign quantum numbers to the fields (and corresponding particles) so that the lagrangian is invariant and the *electric charge* is still given by the Gell-Mann–Nishijima relation[4]

$$Q \;=\; (T_3 + \frac{1}{2}Y)_W \tag{26.12}$$

Conservation of electric charge will be imposed as an exact symmetry of the theory. Assignments of the weak quantum numbers for the fields introduced so far are shown in Table 26.1.

As with QCD in chapter 25, this is now made into a Yang–Mills local gauge theory based on the symmetry group $SU(2)_W \otimes U(1)_W$ [Ya54, Ab73]. The only slight new complexity is that now one has the direct product of two symmetry groups with commuting generators; however, an examination of the basic concept shows that this is an inessential complication. The steps of the Yang–Mills construction are as follows:

1. Add *gauge bosons*, one for each of the generators $(\hat{T}^i_W, \hat{Y}_W)$

$$\begin{aligned} &A^i_\mu(x) \qquad ; \; i = 1, 2, 3 \\ &B_\mu(x) \end{aligned} \tag{26.13}$$

[4] This follows for the fermions by constructing the appropriate operators in second quantization [Wa95].

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Table 26.1. Weak symmetry quantum numbers in the standard model.

Particle/field	T_W	T_{3W}	Y_W	Q
$(\nu_e)_L$	1/2	1/2	−1	0
e_L	1/2	−1/2	−1	−1
e_R	0	0	−2	−1
p_L	1/2	1/2	1	1
n_L	1/2	−1/2	1	0
p_R	0	0	2	1
n_R	0	0	0	0
ϕ^+	1/2	1/2	1	1
ϕ^0	1/2	−1/2	1	0

2. Use the *covariant derivative* in the lagrangian

$$\frac{\partial}{\partial x_\mu} \to \frac{D}{Dx_\mu} \equiv \left(\frac{\partial}{\partial x_\mu} - \frac{i}{2}g' Y_W B_\mu - \frac{i}{2} g \boldsymbol{\tau} \cdot \mathbf{A}_\mu\right) \quad ; \text{ on doublets}$$
$$\equiv \left(\frac{\partial}{\partial x_\mu} - \frac{i}{2}g' Y_W B_\mu\right) \quad ; \text{ on singlets} \tag{26.14}$$

3. Include a kinetic energy term for the gauge bosons

$$\mathscr{L}_{\text{gauge}} = -\frac{1}{4}\left(\frac{\partial B_\nu}{\partial x_\mu} - \frac{\partial B_\mu}{\partial x_\nu}\right)^2 - \frac{1}{4}\left(\frac{\partial \mathbf{A}_\nu}{\partial x_\mu} - \frac{\partial \mathbf{A}_\mu}{\partial x_\nu} + g\mathbf{A}_\mu \times \mathbf{A}_\nu\right)^2 \tag{26.15}$$

Mass terms of the form $m_B^2 B_\mu B_\mu$ or $m_A^2 \mathbf{A}_\mu \cdot \mathbf{A}_\mu$ break the local gauge invariance; hence the gauge bosons must be *massless*.

The Yang–Mills lagrangian thus takes the form

$$\begin{aligned}
\mathscr{L}_{\text{lepton}} &= -\left[\bar{L}\gamma_\mu\left(\frac{\partial}{\partial x_\mu} - \frac{i}{2}(-1)g' B_\mu - \frac{i}{2} g\boldsymbol{\tau}\cdot\mathbf{A}_\mu\right) L \right. \\
&\left. + \bar{R}\gamma_\mu\left(\frac{\partial}{\partial x_\mu} - \frac{i}{2}(-2)g' B_\mu\right) R\right] \\
\mathscr{L}_{\text{nucleon}} &= -\left[\bar{N}_L\gamma_\mu\left(\frac{\partial}{\partial x_\mu} - \frac{i}{2}(1)g' B_\mu - \frac{i}{2} g\boldsymbol{\tau}\cdot\mathbf{A}_\mu\right) N_L \right. \\
&\left. + \bar{p}_R\gamma_\mu\left(\frac{\partial}{\partial x_\mu} - \frac{i}{2}(2)g' B_\mu\right) p_R + \bar{n}_R\gamma_\mu\left(\frac{\partial}{\partial x_\mu} - \frac{i}{2}(0)g' B_\mu\right) n_R\right] \\
\mathscr{L}_{\text{gauge}} &= -\frac{1}{4}B_{\mu\nu}B_{\mu\nu} - \frac{1}{4}\mathscr{F}^i_{\mu\nu}\mathscr{F}^i_{\mu\nu}
\end{aligned} \tag{26.16}$$

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The masses for the gauge bosons are now generated by *spontaneous symmetry breaking*. One proceeds to:

1. Introduce a weak isodoublet of *complex scalar mesons*

$$\underline{\phi} \;\equiv\; \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \tag{26.17}$$

2. Assign weak quantum numbers as indicated in Table 26.1;
3. Use the covariant derivative of Eq. (26.14);
4. Add a term to the lagrangian for this scalar field that is invariant under local $SU(2)_W \otimes U(1)_W$

$$\mathscr{L}_{\text{scalar}} = -\left(\frac{D\underline{\phi}}{Dx_\mu}\right)^\star \left(\frac{D\underline{\phi}}{Dx_\mu}\right) - V(\underline{\phi}^\dagger\underline{\phi}) \tag{26.18}$$

   Thus[5]

$$\left(\frac{D\underline{\phi}}{Dx_\mu}\right)^\star \left(\frac{D\underline{\phi}}{Dx_\mu}\right) = \tag{26.19}$$

$$\underline{\phi}^\dagger \left(\frac{\overleftarrow{\partial}}{\partial x_\mu} + \frac{i}{2}g'B_\mu + \frac{i}{2}g\boldsymbol{\tau}\cdot\mathbf{A}_\mu\right)\left(\frac{\partial}{\partial x_\mu} - \frac{i}{2}g'B_\mu - \frac{i}{2}g\boldsymbol{\tau}\cdot\mathbf{A}_\mu\right)\underline{\phi}$$

5. Assume the most general form of the scalar self-interaction potential V for a renormalizable theory

$$V \;=\; \mu^2\underline{\phi}^\dagger\underline{\phi} + \lambda(\underline{\phi}^\dagger\underline{\phi})^2 \tag{26.20}$$

Assume that $\mu^2 < 0$ and $\lambda > 0$ so that the potential V has the shape shown in Fig. 26.1. The minimum of the potential no longer occurs at the origin with $\underline{\phi} = 0$, but now at a finite value of $\underline{\phi}$. Hence the scalar field acquires a *vacuum expectation value*. Only the neutral component of the field can be allowed to develop a vacuum expectation value in order to preserve electric charge conservation. Furthermore, the (constant) phase of the field can always be redefined so that this vacuum expectation value is real. Thus we write

$$\langle\phi^0\rangle \;=\; \langle\phi^{0*}\rangle \equiv \frac{v}{\sqrt{2}} \tag{26.21}$$

[5] The metric is not complex conjugated in $v_\mu^\star \equiv (\mathbf{v}^\dagger, +iv_0^\dagger)$.

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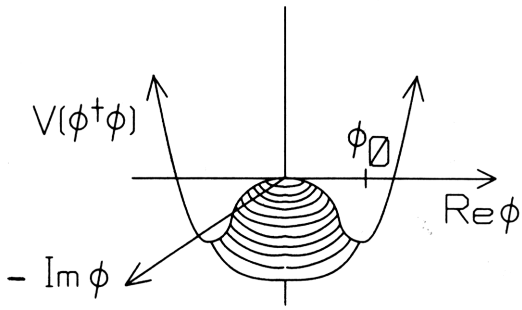

Fig. 26.1. Form of the scalar self-interaction potential to generate mass for the gauge bosons by spontaneous symmetry breaking. The illustration is for a single, neutral, complex ϕ.

At the minimum of the vacuum expectation value of the potential one finds

$$v^2 \;=\; -\frac{\mu^2}{\lambda} \tag{26.22}$$

Without loss of generality, one can now *parameterize* the complex scalar field $\underline{\phi}$ in terms of four real parameters $\{\xi(x), \eta(x)\}$ describing the fluctuations around the vacuum expectation value in the following fashion [Ab73]:

$$\underline{\phi} \;\equiv\; \exp\left\{\frac{-i}{2v}\boldsymbol{\xi}\cdot\boldsymbol{\tau}\right\}\begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v+\eta) \end{pmatrix} \tag{26.23}$$

The theory has been constructed to be locally gauge invariant. Make use of this fact to simplify matters. Make a gauge transformation to eliminate the first factor in this equation. Define

$$\underline{\phi}' \equiv \exp\left\{\frac{+i}{2v}\boldsymbol{\xi}\cdot\boldsymbol{\tau}\right\}\underline{\phi} = \underline{U}(\boldsymbol{\xi})\underline{\phi} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+\eta \end{pmatrix} \tag{26.24}$$

Written in terms of the new field $\underline{\phi}'$, the three scalar field variables $\{\boldsymbol{\xi}(x)\}$ now no longer appear in the lagrangian; and, as we proceed to demonstrate, the free lagrangian has instead a simple interpretation in terms of massive vector and scalar particles. The lagrangian in this form is said to be written in the *unitary gauge* where the particle content of the theory is manifest. The procedure for generating the mass of the gauge bosons in this fashion is known as the *Higgs mechanism* [Cu83, Ab73].

Substitution of the expression in Eq. (26.24) in the scalar lagrangian in

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Eqs. (26.18)–(26.20) leads to

$$\mathscr{L}_{\text{scalar}} = -V\left[\frac{1}{2}(v+\eta)^2\right] - \frac{1}{2}\chi_\downarrow^\dagger\left[\frac{\partial\eta}{\partial x_\mu} + \frac{ig'}{2}(v+\eta)B_\mu + \frac{ig}{2}(v+\eta)\boldsymbol{\tau}\cdot\mathbf{A}_\mu\right]$$
$$\times\left[\frac{\partial\eta}{\partial x_\mu} - \frac{ig'}{2}(v+\eta)B_\mu - \frac{ig}{2}(v+\eta)\boldsymbol{\tau}\cdot\mathbf{A}_\mu\right]\chi_\downarrow \qquad (26.25)$$

Here $\chi_\downarrow \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. An evaluation of the potential term, utilizing the minimization condition in Eq. (26.22), gives

$$\begin{aligned} V\left[\frac{1}{2}(v+\eta)^2\right] &= \frac{\mu^2}{2}(v+\eta)^2 + \frac{\lambda}{4}(v+\eta)^4 \qquad (26.26) \\ &= v^2\left(\frac{\mu^2}{4}\right) + \eta^2(-\mu^2) + \eta^3(\lambda v) + \eta^4\left(\frac{\lambda}{4}\right) \end{aligned}$$

Note that there is no term linear in η when one expands about the true minimum in V. The coefficient of the term linear in $\partial\eta/\partial x_\mu$ similarly vanishes in Eq. (26.25).

The remaining boson interactions in $\mathscr{L}_{\text{scalar}}$ are proportional to

$$\begin{aligned} &\chi_\downarrow^\dagger(g'B_\mu + g\boldsymbol{\tau}\cdot\mathbf{A}_\mu)(g'B_\mu + g\boldsymbol{\tau}\cdot\mathbf{A}_\mu)\chi_\downarrow \\ &\quad = \chi_\downarrow^\dagger(g'^2B_\mu^2 + g^2\mathbf{A}_\mu^2 + 2gg'B_\mu\boldsymbol{\tau}\cdot\mathbf{A}_\mu)\chi_\downarrow \\ &\quad = (g'^2B_\mu^2 + g^2\mathbf{A}_\mu^2 - 2gg'B_\mu A_\mu^{(3)}) \qquad (26.27) \end{aligned}$$

Hence the scalar lagrangian in the unitary gauge is given by

$$\begin{aligned} \mathscr{L}_{\text{scalar}} &= -\frac{1}{2}\left[\left(\frac{\partial\eta}{\partial x_\mu}\right)^2 + (-2\mu^2)\eta^2\right] - \frac{\lambda}{4}(4v\eta^3 + \eta^4) - \frac{1}{4}\mu^2v^2 \\ &\quad -\frac{1}{8}(v+\eta)^2(g'^2B_\mu^2 + g^2\mathbf{A}_\mu^2 - 2gg'B_\mu A_\mu^{(3)}) \qquad (26.28) \end{aligned}$$

The term in v^2 in the second line now provides the sought-after mass for the gauge bosons. The coefficient of this term is a quadratic form in the gauge fields, which can be put on principal axes with the introduction of the following linear combinations of fields:

$$\begin{aligned} W_\mu^{(+)} &\equiv W_\mu^\star \equiv \frac{1}{\sqrt{2}}(A_\mu^{(1)} + iA_\mu^{(2)}) \\ W_\mu^{(-)} &\equiv W_\mu \equiv \frac{1}{\sqrt{2}}(A_\mu^{(1)} - iA_\mu^{(2)}) \end{aligned}$$

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$$
\begin{aligned}
Z_\mu &\equiv \frac{-gA_\mu^{(3)} + g'B_\mu}{(g^2 + g'^2)^{1/2}} \\
A_\mu &\equiv \frac{g'A_\mu^{(3)} + gB_\mu}{(g^2 + g'^2)^{1/2}}
\end{aligned}
\tag{26.29}
$$

The fields $(W_\mu^\star, W_\mu)$ will create particles (W_μ^+, W_μ^-), respectively, the third field describes a neutral Z_μ^0 vector boson, and the fourth is the photon field. The relation between $(B_\mu, A_\mu^{(3)})$ and (Z_μ, A_μ) is an *orthogonal transformation*. Note in particular that the weak angle is defined by

$$
\sin\theta_W \equiv \frac{g'}{(g^2 + g'^2)^{1/2}} \tag{26.30}
$$

The scalar lagrangian can thus finally be written in the unitary gauge as

$$
\begin{aligned}
\mathscr{L}_{\text{scalar}} = & -\frac{1}{2}\left[\left(\frac{\partial\eta}{\partial x_\mu}\right)^2 + (-2\mu^2)\eta^2\right] - \frac{\lambda}{4}(4v\eta^3 + \eta^4) - \frac{1}{4}\mu^2 v^2 \\
& -\frac{1}{4}v^2(g^2 + g'^2)\frac{1}{2}Z_\mu^2 - \frac{1}{4}v^2 g^2 W_\mu W_\mu^\star \\
& -\frac{1}{8}\eta(2v + \eta)[(g^2 + g'^2)Z_\mu^2 + 2g^2 W_\mu W_\mu^\star]
\end{aligned}
\tag{26.31}
$$

The first term in the first line is the lagrangian for a free, neutral scalar field of mass $-2\mu^2$ — the *Higgs field*; this is the only remaining physical degree of freedom from the complex doublet of scalar fields introduced previously, in this unitary gauge. The second term describes cubic and quartic self-couplings of the Higgs field; the third term in the first line is simply an additive constant. The terms in the second line proportional to the constant v^2 represent the quadratic mass terms for the gauge bosons. Note, in particular, that no mass term has been generated for the photon field, which thus remains massless, as it must. Finally, the terms in the last line proportional to $(2v\eta + \eta^2)$ represent cubic and quartic couplings of the Higgs to the massive gauge bosons.

Since the transformation in Eqs. (26.29) is orthogonal, the quadratic part of the kinetic energy of the gauge bosons remains on principal axes and Eq. (26.15) can be rewritten as

$$
\begin{aligned}
\mathscr{L}_{\text{gauge}} = & -\frac{1}{2}W_{\mu\nu}^\star W_{\mu\nu} - \frac{1}{4}Z_{\mu\nu}Z_{\mu\nu} - \frac{1}{4}F_{\mu\nu}F_{\mu\nu} \\
& -\frac{g}{2}\mathbf{F}_{\mu\nu}\cdot(\mathbf{A}_\mu \times \mathbf{A}_\nu) - \frac{g^2}{4}(\mathbf{A}_\mu \times \mathbf{A}_\nu)^2
\end{aligned}
\tag{26.32}
$$

Here the field tensors are defined by the linear Maxwell form $V_{\mu\nu} \equiv \partial V_\nu/\partial x_\mu - \partial V_\mu/\partial x_\nu$ and the original gauge field $\mathbf{A}_\mu$ in the nonlinear terms

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must still be expressed in terms of the physical fields defined through Eqs. (26.29). The second line in the above result represents cubic and quartic couplings of the physical gauge fields.

The *particle content* of the theory is now made manifest in this unitary gauge, since the free lagrangian has the required quadratic form in the kinetic energy and masses. In addition to the original (still massless!) fermions, the theory evidently now contains

1. A massive neutral weak vector meson Z_μ^0 with mass given by

$$M_Z^2 = \frac{v^2(g^2 + g'^2)}{4} \tag{26.33}$$

2. Massive charged weak vector mesons $W_\mu^{(\pm)}$ with masses

$$M_W^2 = \frac{v^2 g^2}{4} = M_Z^2 \cos^2\theta_W \tag{26.34}$$

3. A massless photon

$$M_\gamma^2 = 0 \tag{26.35}$$

The lagrangian retains the exact local $U(1)$ gauge invariance generated by the electric charge $\hat{Q}$, corresponding to QED.

The total lagrangian for the standard model as presented so far is the sum of the individual contributions discussed above

$$\mathcal{L} = \mathcal{L}_{\text{lepton}} + \mathcal{L}_{\text{nucleon}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{scalar}} \tag{26.36}$$

Note that this lagrangian now contains all the electroweak interactions [Cu83, Wa95]. The coupling of the leptons to the gauge bosons follows immediately from Eqs. (26.16) and (26.29)[6]

$$\begin{aligned}
\mathcal{L}_{\text{lepton}}^{(\pm)} &= \frac{g}{2\sqrt{2}}[j_\mu^{(+)}W_\mu + j_\mu^{(-)}W_\mu^\star] \\
\mathcal{L}_{\text{lepton}}^{(0)} &= -\frac{g}{2\cos\theta_W} j_\mu^{(0)} Z_\mu \\
\mathcal{L}_{\text{lepton}}^{\gamma} &= e_p j_\mu^\gamma A_\mu
\end{aligned} \tag{26.37}$$

Here the electric charge e_p is defined by

$$e_p \equiv \frac{gg'}{(g^2 + g'^2)^{1/2}} \tag{26.38}$$

[6] The details of this algebra are provided in [Wa95].

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The lepton currents are given by the following expressions

$$
\begin{aligned}
j_\mu^{(\pm)} &= i\bar{\psi}_l\gamma_\mu(1+\gamma_5)\tau_\pm\psi_l \\
j_\mu^\gamma &= i\bar{\psi}_l\gamma_\mu\left[-\frac{1}{2}(1-\tau_3)\right]\psi_l \\
j_\mu^{(0)} &= i\bar{\psi}_l\gamma_\mu(1+\gamma_5)\frac{1}{2}\tau_3\psi_l - 2\sin^2\theta_W j_\mu^\gamma
\end{aligned}
\tag{26.39}
$$

The interaction of the point nucleons with the gauge fields takes exactly the same form as in Eqs. (26.37), with hadronic currents given by

$$
\begin{aligned}
\mathscr{J}_\mu^{(\pm)} &= i\bar{\psi}\gamma_\mu(1+\gamma_5)\tau_\pm\psi \\
J_\mu^\gamma &= i\bar{\psi}\gamma_\mu\left[\frac{1}{2}(1+\tau_3)\right]\psi \\
\mathscr{J}_\mu^{(0)} &= i\bar{\psi}\gamma_\mu(1+\gamma_5)\frac{1}{2}\tau_3\psi - 2\sin^2\theta_W J_\mu^\gamma
\end{aligned}
\tag{26.40}
$$

The lepton and nucleon doublets appearing in these currents are defined by

$$
\psi_l = \begin{pmatrix} \psi_{\nu_e} \\ \psi_e \end{pmatrix} \qquad \psi = \begin{pmatrix} \psi_p \\ \psi_n \end{pmatrix}
\tag{26.41}
$$

An analysis of the S-matrix for single, heavy weak boson exchange shows how interactions with the gauge bosons of the form in Eqs. (26.37) lead to an effective current–current lagrangian in the low-energy, nuclear domain where $\mathbf{q}^2 \ll M_W^2, M_Z^2$. In particular, comparison with that analysis immediately establishes the following relationships between the gauge couplings and masses of the standard model and the traditional weak Fermi coupling constant [Wa95]

$$
\frac{G}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{g^2}{8M_Z^2\cos^2\theta_W}
\tag{26.42}
$$

It is also evident that the total weak currents here receive additive contributions from the leptons and hadrons

$$
\begin{aligned}
\mathscr{J}_\lambda^{(\pm)} &= \mathscr{J}_\lambda^{(\pm)}(\text{hadrons}) + j_\lambda^{(\pm)}(\text{leptons}) \\
\mathscr{J}_\lambda^{(0)} &= \mathscr{J}_\lambda^{(0)}(\text{hadrons}) + j_\lambda^{(0)}(\text{leptons})
\end{aligned}
\tag{26.43}
$$

The semileptonic parts of this effective low-energy lagrangian form the basis of most of the nuclear applications. Formulation in terms of quarks, discussed below, simply changes the underlying structure of $\mathscr{J}_\lambda$(hadrons).

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The corresponding effective four-fermion lagrangians are [Wa95]

$$\begin{aligned}
\mathscr{L}_{\rm eff}^{(\pm)} &= \frac{iG}{\sqrt{2}}\Big\{[\bar{\psi}_e\gamma_\lambda(1+\gamma_5)\psi_{\nu_e}+(e\leftrightarrow\mu)]\mathscr{J}_\lambda^{(+)}(\text{hadrons}) \qquad (26.44)\\
&\quad+[\bar{\psi}_{\nu_e}\gamma_\lambda(1+\gamma_5)\psi_e+(e\leftrightarrow\mu)]\mathscr{J}_\lambda^{(-)}(\text{hadrons})\Big\}\\
\mathscr{L}_{\rm eff}^{(\nu)} &= \frac{iG}{\sqrt{2}}\left[\bar{\psi}_{\nu_e}\gamma_\lambda(1+\gamma_5)\psi_{\nu_e}+(e\leftrightarrow\mu)\right]\mathscr{J}_\lambda^{(0)}(\text{hadrons})\\
\mathscr{L}_{\rm eff}^{(l)} &= -\frac{iG}{\sqrt{2}}\left[\bar{\psi}_e\gamma_\lambda(1+\gamma_5)\psi_e-4\sin^2\theta_W\bar{\psi}_e\gamma_\lambda\psi_e+(e\leftrightarrow\mu)\right]\mathscr{J}_\lambda^{(0)}
\end{aligned}$$

The theory as formulated assumes massless fermions. The fermion mass is now *put in by hand*. One adds Yukawa couplings of the fermions to the previously introduced complex scalar field that preserve the local $SU(2)_W\otimes U(1)_W$ local gauge symmetry. One such coupling is introduced for each fermion field. The fermions then acquire mass when the scalar field develops its vacuum expectation value. As a consequence of this procedure, each fermion also has a prescribed Yukawa coupling to the *fluctuation* of the scalar field about its vacuum expectation value — the real scalar Higgs. We illustrate the procedure in the case of leptons.[7]

Start with the following lagrangian with Yukawa couplings of the fermions to the complex scalar field and invariant under local $SU(2)_W\otimes U(1)_W$

$$\mathscr{L}_{\rm int} = -G_e\bar{R}(\underline{\phi}^\dagger\underline{L})+\text{h.c.} \qquad (26.45)$$

Each term is a weak isoscalar, and each term is neutral in weak hypercharge (Table 26.1). Now with the previously discussed spontaneous symmetry breaking, and in the unitary gauge $\underline{\phi}$ is given by Eq. (26.24). Substitution into Eq. (26.45) and the use of Eq. (26.9) then gives

$$\begin{aligned}
\mathscr{L}_{\rm int} &= -G_e\bar{e}_R\left\{[0,\frac{1}{\sqrt{2}}(v+\eta)]\begin{pmatrix}\nu_L\\ e_L\end{pmatrix}\right\}+\text{h.c.}\\
&= -\frac{v}{\sqrt{2}}G_e\bar{e}e-\frac{\eta}{\sqrt{2}}G_e\bar{e}e \qquad (26.46)
\end{aligned}$$

The first term is the sought-after fermion mass (there is one adjustable coupling constant for each fermion mass in the theory). The second term is the remaining Yukawa interaction with the real scalar Higgs particle, with a prescribed coupling determined by the mass of the fermion.

The deeper formulation of the electroweak theory is in terms of quarks. At first glance, one might expect that the first quark weak isodoublet would just be that constructed from (u,d) quarks. The actual quark weak

[7] For point nucleons see [Wa95].

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isospin doublets that couple in the electroweak interaction have a more complicated form [Ca63, Gl70]. They are

$$q_L = \begin{pmatrix} u_L \\ d_L \cos\theta_C + s_L \sin\theta_C \end{pmatrix} \equiv \begin{pmatrix} u_L \\ d_{cL} \end{pmatrix} \tag{26.47}$$

$$Q_L = \begin{pmatrix} c_L \\ -d_L \sin\theta_C + s_L \cos\theta_C \end{pmatrix} \equiv \begin{pmatrix} c_L \\ D_{cL} \end{pmatrix} \quad \text{; weak doublets}$$

The fact that it is a rotated combination of fields in the charge-changing current, which includes a small strangeness-changing component, was first noted by Cabibbo [Ca63]. The discovery that one requires a second doublet with an additional c quark and the orthogonal rotated combination is due to Glashow, Iliopolous, and Maiani (GIM) [Gl70] who *predicted* the existence of the c quark on the basis of the arguments given below.[8]

As before, the right-handed quark fields form weak isosinglets

$$u_R, d_R, s_R, c_R \qquad \text{; weak singlets} \tag{26.48}$$

The quarks are assigned the weak quantum numbers in Table 26.2. The assignments are again made so that the electric charge operator is

$$\hat{Q} = (\hat{T}_3 + \frac{1}{2}\hat{Y})_W \tag{26.49}$$

Because one has two orthogonal linear combinations, the following (GIM) identity holds

$$\begin{aligned} \bar{d}_c d_c + \bar{D}_c D_c &= (\bar{d}\cos\theta_C + \bar{s}\sin\theta_C)(d\cos\theta_C + s\sin\theta_C) \\ &\quad + (-\bar{d}\sin\theta_C + \bar{s}\cos\theta_C)(-d\sin\theta_C + s\cos\theta_C) \\ &= \bar{d}d + \bar{s}s \end{aligned} \tag{26.50}$$

No off-diagonal, strangeness-changing terms appear in this expression; as a consequence, the neutral currents generated in the standard model have no lowest-order strangeness-changing components — an empirical observation that was the primary motivation for the introduction of the c quark in [Gl70].

The GIM identity can be used to rewrite the non-interacting quark kinetic energy as

$$\begin{aligned} \mathscr{L}^0_{\text{quark}} = -\Big[&\bar{q}_L \gamma_\mu \frac{\partial}{\partial x_\mu} q_L + \bar{Q}_L \gamma_\mu \frac{\partial}{\partial x_\mu} Q_L \\ &+ \bar{u}_R \gamma_\mu \frac{\partial}{\partial x_\mu} u_R + \bar{d}_R \gamma_\mu \frac{\partial}{\partial x_\mu} d_R + \bar{s}_R \gamma_\mu \frac{\partial}{\partial x_\mu} s_R + \bar{c}_R \gamma_\mu \frac{\partial}{\partial x_\mu} c_R \Big] \end{aligned} \tag{26.51}$$

[8] The extension to include still another (heavy) quark family is discussed in [Wa95].

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Table 26.2. Weak isospin and weak hypercharge assignments for the quarks.

Field /particle	q_L	Q_L	u_R	d_R	s_R	c_R
T_W	1/2	1/2	0	0	0	0
Y_W	1/3	1/3	4/3	−2/3	−2/3	4/3

The covariant derivatives acting on the quark fields are as before (see Table 26.2)

$$(\frac{\partial}{\partial x_\mu} - \frac{i}{2}g' Y_W B_\mu - \frac{i}{2} g \boldsymbol{\tau} \cdot \mathbf{A}_\mu) \qquad ; \text{on isodoublets}$$
$$(\frac{\partial}{\partial x_\mu} - \frac{i}{2}g' Y_W B_\mu) \qquad ; \text{on isosinglets} \tag{26.52}$$

The gauge boson and Higgs sectors of the theory are exactly the same as discussed above. The electroweak currents representing the interaction with the physical gauge bosons can also be identified exactly as before [Wa95]. The charge-changing weak current is given by

$$\begin{aligned} \mathscr{J}_\mu^{(\pm)} &= i\bar{q}\gamma_\mu(1+\gamma_5)\tau_\pm q + i\bar{Q}\gamma_\mu(1+\gamma_5)\tau_\pm Q \\ \mathscr{J}_\mu^{(+)} &= i\bar{u}\gamma_\mu(1+\gamma_5)(d\cos\theta_C + s\sin\theta_C) \\ &\quad + i\bar{c}\gamma_\mu(1+\gamma_5)(-d\sin\theta_C + s\cos\theta_C) \end{aligned} \tag{26.53}$$

Note it is the Cabibbo-rotated combination that enters into these charge-changing currents. The electromagnetic current of QED is just the point Dirac current multiplied by the correct charge

$$J_\mu^\gamma = i\left[\frac{2}{3}(\bar{u}\gamma_\mu u + \bar{c}\gamma_\mu c) - \frac{1}{3}(\bar{d}\gamma_\mu d + \bar{s}\gamma_\mu s)\right] \tag{26.54}$$

The weak neutral current is

$$\begin{aligned} \mathscr{J}_\mu^{(0)} &= i\bar{q}\gamma_\mu(1+\gamma_5)\frac{1}{2}\tau_3 q + i\bar{Q}\gamma_\mu(1+\gamma_5)\frac{1}{2}\tau_3 Q - 2\sin^2\theta_W J_\mu^\gamma \\ \mathscr{J}_\mu^{(0)} &= \frac{i}{2}[\bar{u}\gamma_\mu(1+\gamma_5)u + \bar{c}\gamma_\mu(1+\gamma_5)c - \bar{d}\gamma_\mu(1+\gamma_5)d - \bar{s}\gamma_\mu(1+\gamma_5)s] \\ &\quad -2\sin^2\theta_W J_\mu^\gamma \end{aligned} \tag{26.55}$$

The second equality follows with the aid of the GIM identity. Terms of the form $(\bar{s}d)$ or $(\bar{d}s)$ have been eliminated; hence there are no strangeness-changing weak neutral currents in this quark-based standard model, as advertised.

The quarks can be given mass in the same fashion as were the leptons above, although the argument is somewhat more complicated in the case of quarks [Ab73, Cu83, Wa95].

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How does the standard model of electroweak interactions get combined with QCD, the theory of the *strong* forces binding quarks into hadrons? Consider for simplicity the nuclear domain of (u, d) quarks. Quarks now carry an additional color index that takes three values (R, G, B), and the quark field gets extended to

$$\psi = \begin{pmatrix} u \\ d \end{pmatrix} \rightarrow \begin{pmatrix} u_R & u_G & u_B \\ d_R & d_G & d_B \end{pmatrix} \equiv (\psi_R, \psi_G, \psi_B) \tag{26.56}$$

These get combined into a three-component (actually multicomponent) field $\underline{\psi}$

$$\underline{\psi} \equiv \begin{pmatrix} \psi_R \\ \psi_G \\ \psi_B \end{pmatrix} \tag{26.57}$$

Let $\underline{\mathcal{O}}$ be a matrix that is the *identity* with respect to color, but an *arbitrary* matrix O with respect to flavor

$$\underline{\mathcal{O}} \equiv \begin{pmatrix} O & & \\ & O & \\ & & O \end{pmatrix} \tag{26.58}$$

Then under the extension of the quark fields to include color, all *electroweak currents* are defined to be correspondingly extended to

$$\begin{aligned} \bar{\psi}\gamma_\mu O \psi &\rightarrow \bar{\psi}_R \gamma_\mu O \psi_R + \bar{\psi}_G \gamma_\mu O \psi_G + \bar{\psi}_B \gamma_\mu O \psi_B \\ &\equiv \underline{\bar{\psi}} \gamma_\mu \underline{\mathcal{O}} \underline{\psi} \end{aligned} \tag{26.59}$$

Such currents are evidently invariant under strong $SU(3)_C$.

The full lagrangian of the strong and electroweak interactions thus takes the form (see [Do93] for an extended discussion)

$$\mathscr{L} = \mathscr{L}^0 + \mathscr{L}^{\text{int}}_{\text{QCD}} + \mathscr{L}^{\text{int}}_{\text{EW}} \tag{26.60}$$

This lagrangian is locally gauge invariant under the full symmetry group

$$SU(3)_C \bigotimes SU(2)_W \bigotimes U(1)_W \tag{26.61}$$

This full theory is renormalizable. It has the following characteristic properties:

- The electroweak interactions are colorblind — they are the same, independent of the color of the quarks;
- The gluons are absolutely *neutral* to the electroweak interactions — the electroweak interactions couple to the quarks.

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Let us examine the implications of this development for nuclear physics. To summarize the weak and electromagnetic quark currents in the standard model, we have

$$\begin{aligned}
\mathscr{J}_\mu^{(+)} &= i\bar{u}\gamma_\mu(1+\gamma_5)[d\cos\theta_C + s\sin\theta_C] \\
&\quad + i\bar{c}\gamma_\mu(1+\gamma_5)[-d\sin\theta_C + s\cos\theta_C] \\
\mathscr{J}_\mu^{(0)} &= \frac{i}{2}[\bar{u}\gamma_\mu(1+\gamma_5)u + \bar{c}\gamma_\mu(1+\gamma_5)c \\
&\quad - \bar{d}\gamma_\mu(1+\gamma_5)d - \bar{s}\gamma_\mu(1+\gamma_5)s] - 2\sin^2\theta_W J_\mu^\gamma \\
J_\mu^\gamma &= i\left[\frac{2}{3}(\bar{u}\gamma_\mu u + \bar{c}\gamma_\mu c) - \frac{1}{3}(\bar{d}\gamma_\mu d + \bar{s}\gamma_\mu s)\right] \qquad (26.62)
\end{aligned}$$

Each current is actually a sum over three colors $\sum_{\text{colors}}(\cdots)$ leading to an operator which is an $SU(3)_C$ - singlet as discussed above.

To a good approximation, the hadrons that make up the nucleus are composed of (u, d) quarks. As a starting point for nuclear physics, consider that subspace of the full Hilbert space consisting of any number of (u, d) quarks and their antiquarks $(\bar{u}, \bar{d})$. We refer to this as the *nuclear domain.* The quark field in this sector takes the form

$$\psi \doteq \begin{pmatrix} u \\ d \end{pmatrix} \qquad ;\ \text{nuclear domain} \qquad (26.63)$$

Assume that the (u, d) quarks have the *same mass* in the lagrangian; they are in fact both nearly massless. In this case, the lagrangian of the strong interactions, with the full complexity of QCD, has an *exact symmetry* — the $SU(2)$ of strong isospin. This is the familiar isotopic spin symmetry of nuclear physics. It is important to note that one still has the full complexity of strong-coupling QCD with colored quarks and gluons in this truncated flavor sector of the nuclear domain; nevertheless, one can draw conclusions that are exact to all orders in the strong interactions using this strong isospin symmetry.

The quark field ψ in Eq. (26.63) forms an isodoublet under this strong isospin. The quark currents in Eqs. (26.62) can then be written in terms of this isospinor in the nuclear domain as follows

$$\begin{aligned}
J_\mu^\gamma &= i\bar{\psi}\gamma_\mu\left(\frac{1}{6} + \frac{1}{2}\tau_3\right)\psi \\
\mathscr{J}_\mu^{(\pm)} &= i\bar{\psi}\gamma_\mu(1+\gamma_5)\tau_\pm\psi \\
\mathscr{J}_\mu^{(0)} &= i\bar{\psi}\gamma_\mu(1+\gamma_5)\frac{1}{2}\tau_3\psi - 2\sin^2\theta_W J_\mu^\gamma \qquad (26.64)
\end{aligned}$$

The properties of these currents under general symmetry properties of the

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theory now follow by inspection [Wa95]

$$
\begin{aligned}
\mathscr{J}_\mu &= J_\mu + J_{\mu 5} && ;\ V-A \\
\mathscr{J}_\mu^{(\pm)} &= \mathscr{J}_\mu^{V_1} \pm i\mathscr{J}_\mu^{V_2} && ;\ \text{isovector} \\
J_\mu^\gamma &= J_\mu^S + J_\mu^{V_3} && ;\ \text{EM current} \\
J_\mu^{(\pm)} &= J_\mu^{V_1} \pm iJ_\mu^{V_2} && ;\ \text{CVC} \\
\mathscr{J}_\mu^{(0)} &= \mathscr{J}_\mu^{V_3} - 2\sin^2\theta_W J_\mu^\gamma && ;\ \text{standard model}
\end{aligned}
\tag{26.65}
$$

Here the Cabibbo angle has been absorbed into the definition of the hadronic weak charge-changing Fermi coupling constant

$$
G^{(\pm)} \equiv G\cos\theta_C \qquad\qquad \cos\theta_C = 0.974 \tag{26.66}
$$

Note that the numerical value of $\cos\theta_C$ is, in fact, very close to 1 [Cu83].

The first of Eqs. (26.65) indicates that the weak current is the sum of a Lorentz vector and axial vector, the second that the charge-changing weak current is an isovector, and the third that the electromagnetic current is the sum of an isoscalar and third component of an isovector. The fourth equation is the statement of CVC. The conserved vector current (CVC) relation states that the Lorentz vector part of the weak charge-changing current is simply obtained from the other spherical isospin components of the same isovector operator that appears in the electromagnetic current. As a consequence, one can relate matrix elements of the Lorentz vector part of the charge-changing weak currents to those of the isovector part of the electromagnetic current by use of the Wigner–Eckart theorem applied to isospin. The resulting relations are then independent of the details of hadronic structure; they depend only on the existence of the isospin symmetry of the strong interactions. CVC is a powerful, deep, and far-reaching result, for it established the first direct relation between the electromagnetic and weak interactions which *a priori have nothing to do with each other!* All known applications of CVC are consistent with experiment. The last of Eqs. (26.65) exhibits the structure of the weak neutral current of the standard model in the nuclear domain.

If the discussion is extended to that sector of the full theory with no *net* strangeness or charm, and the electroweak interactions are treated in lowest order, then the first four of Eqs. (26.65) still hold; however, the weak neutral current is modified by the addition of an isoscalar contribution

$$
\delta\mathscr{J}_\mu^{(0)} = \frac{i}{2}[\bar{c}\gamma_\mu(1+\gamma_5)c - \bar{s}\gamma_\mu(1+\gamma_5)s] \tag{26.67}
$$

In this sector of the theory, (s, c) quarks and their antiparticles $(\bar{s}, \bar{c})$ enter through loop processes.