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Truth without truths: Grim's Cantorian paradox and the ontology of the objects of omniscience

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Abstract

I argue that Grim's diagonalization argument against the possibility of omniscience is not sound by arguing that the properties of being a proposition or a truth are not legitimate sortal properties. Thus, the fact that there can be no set corresponding to the extension of these properties does not imply that there is no completed totality of the things possessing it. First, I demonstrate that a correspondence theory of truth implies that propositions are non-linguistic representations of a type that resist determinate and uniform individuation into units and allow for arbitrary division into parts that are also propositions. The property is, therefore, an abstract mass property with no determinate cardinality of individuals that possess it. I then sketch a new theory of omniscience with this as its basis.

Keywords: omniscience; coherence of; Cantorian paradoxes; propositions; mass terms

It's a dangerous thing to know too much. Or so Patrick Grim would have us believe. In a series of influential articles, he has argued that an omniscient being knows too much, so much in fact that it leads to contradiction. In order to show this, he adduces a Cantorian style diagonalization to prove that any collection of all truths must have a contradictory cardinality.[1] But, if an omniscient being were to exist, what it would know would constitute a collection of all truths. Hence, there can be no omniscient being as traditionally conceived. If there is no collection of all truths, there can be no collection of all propositions; for if there were such a collection it would have a determinate cardinality, and so, therefore, would every sub-collection, including the collection of all truths.

There is no doubt that Grim has brought to light an important paradox, one that challenges our conceptions of such metaphysical notions as propositions, truth, and omniscience. But a paradox threatens the whole system in which it is formulated; and, for that reason, threatens nothing in particular until a positive diagnosis is provided as to the source of the anomaly. Contrary to Grim, I shall argue that these cardinality problems are symptoms of an opposing ontological dysfunction. The contradiction arises in our traditional notion of truth, not from our supposition that there is a completed totality of Truth as such, but rather from our supposition that there are *truths*. Neither Truth nor the objects of our intentional attitudes come pre-wrapped in neat little propositional packages. Consequently, the terms 'proposition' and 'truth' do not correspond to legitimate sortal properties, despite the fact that they function as count nouns in our language.

Only sortals divide their extensions in such a way that it makes sense to say that they have definite cardinalities or numbers of instances.

I shall argue that the problems that prevent the things that fall under mass properties from being individuated into particulars with determinate cardinalities are the same problems that beset the things that fall under the property of being a proposition. Thus, it is more plausible to view the property as a non-sortal mass property. Next, I apply these considerations to the question of omniscience and its possibility. Here, I argue that, since an omniscient being must have knowledge of the totality of Truth as a completed whole, then attempted solutions to Grim's paradox that seek to describe the domain of truths as a mere plurality or indeterminate totality do not ultimately rescue omniscience from logical absurdity. I then sketch a plausible theory of propositions as quasi-pictorial representations that shows how it is possible for an omniscient being to know everything while only having knowledge of a single propositional mass.

Stuff about things

It seems to be a commonplace in the literature to make a distinction between things that can be counted, such as dogs, stars, and llamas, and stuffs that defy such unqualified quantification like water and air. As Burge said about the terms that refer to these property types, 'Mass terms are typically used to measure the masses – count nouns, to number the multitudes' (Burge (1975), 199). But what are the precise metaphysical underpinnings of this difference? Why can't the stuff qualified by mass properties be determinately numbered?

Our starting point for answering this question will be Frege's criterion for determining when a term picks out a mass rather than a sortal concept. Let's go into the details of his thinking in this matter. In his *The Foundations of Arithmetic*, Frege argues that only certain types of concepts individuate their extensions in such a manner that a definite cardinality can be assigned to the class of things falling under them. He gives the following criteria for determining when we have such a concept:

> Only a concept which isolates what falls under it in a definite manner, and which does not permit any arbitrary division of it into parts, can be a unit relative to a finite number. (Frege (1953), 66)

Here we are given two conditions that together suffice to delineate a concept as a sortal concept. Let's call them the Definite Isolation (DI) condition and the Ultimate Division (UD) condition, respectively. Consider a concept C. If C allows us to determine for any particular x whether or not x falls under C, then C isolates what falls under it from its environment in a definite manner. In other words, C is not a vague concept; the law of excluded middle applies to it. This is obviously a necessary condition for statements of number, precisely because such statements are assertions of measure, and any measurement requires a way of delimiting the exact amount of the item to be measured, a way of determining when we have come to the end of the things to be counted.[2] But one more thing is needed to assign a definite number to the extension: a way of individuating the things that fall under C into distinct and ultimate units. C must provide a principle that distinguishes one member of the extension from any other member of the extension of C. There are two ways that a concept could fail to do this. First, it could make it impossible to divide one member from another *tout court*, or it could allow for an arbitrary division into parts. It is this second sense that picks out a mass concept. A mass concept does isolate its extension from its environment. The concept of water does allow us to determine definitely when something is indeed water. But the concept, in and of itself,

allows for the arbitrary division of its extension into parts. This is ambiguous. What does Frege mean by an arbitrary division into parts? He clarifies with an example of the concept of red: 'We can . . . divide up something falling under the concept "red" into parts in a variety of ways, without the parts thereby ceasing to fall under the same concept' (Frege (1953), 66). So a mass concept does not preclude a number of distinct ways of dividing its extension into individual members. Moreover, each of these principles of individuation still allow for the parts of members to be members of the same extension. For example, a cup of water is water, but so are the two half cups that make up that cup of water, and so are the four quarter cups that make up those half cups, and so on. Hence, mass concepts never provide us with a single and ultimate unit of measure. And without an ultimate unit of measure, there can be no objective cardinality.

Notice that mass concepts do allow for measurement. They just don't suffice for it. They themselves do not *autonomously* provide a unit of measure that would objectively ground any statement of number concerning the stuff that falls under them.[3] Hence, they are compatible with a variety of incommensurate ways of dividing their extensions into measurable quantities, that is, collections with definite cardinalities. I want to argue that the concept 'proposition' fails to meet UD in a manner that is similar to the way that mass concepts fail to fulfil it.

One question remains to be answered before we leave this discussion of mass concepts. All of this applies quite well with regard to finite numbers,[4] but wouldn't the infinite divisibility of a stuff simply imply that there are an infinite number of things to which the term applies? It certainly does, if the division of the extension into things is determined solely by the application of the concept itself. But this is precisely the problem with a mass concept; it is powerless to divide its extension without the help of other *sortal* concepts. This highlights the fact that an *ultimate* unit of measure doesn't necessarily mean an atom. Rather it means a standard of measure that is both uniform, not allowing for conflicting judgements of cardinality, and doesn't rely on any more basic criteria of individuation for the efficacy of its measure.

The attempt to apply determinate cardinalities to concepts with infinite extensions actually exacerbates the need for a condition like UD. Now the cardinality must be determined via a one-to-one mapping of one collection onto another, and this cannot take place without uniform identity conditions that individuate each member of the extension so precisely that one can deductively prove that if $x \neq y$ for each x and y that are members of the preimage of $f(x)$, then $f(x) \neq f(y)$ for each member of the image. But, if the concept that defines the image of the injection fails to give us such precise and non-arbitrary identity conditions, then, by that same token, it fails to give us individuals that can be counted as distinct, let alone determinately counted.

Alternative solutions: plural quantification, plena and non-propositional accounts of omniscience

We might ask ourselves at the onset of such a task: why is such a departure from our intuitive understanding of propositions necessary, if there is another solution to Grim's paradox that does not require such cognitive contortions? The solution I have in mind here is one that Plantinga first suggests in his dialogue with Grim (Plantinga and Grim (1993)). Remember, Grim argues that, in order for the statement 'God knows all truths' to make sense, there must be an underlying domain over which the universal quantifier ranges and this domain must be a set, a complete and determinate totality. But, argues Plantinga and others following in his footsteps, this simply is not the case. Take ZFC set theory as a case in point. V, the universe of sets, is not a set, and yet one can

prove and provide models for universal statements about all sets, for example, 'Every set can be well-ordered.' Thus, it does not follow that every universe of discourse over which a universal quantifier ranges has to be collectable into a set. This is called plural quantification in the literature (see Menzel (1993), (2018); Boolos (1984)). To say that God knows every truth is only to say of each and every particular proposition that has the quality of being true that it has the further quality of being known by God. It does not commit us to the existence of a completed totality consisting of all truths as such.

I agree with the general logical assessment of the invalidity of this kind of inference from there being a plurality to there being a whole. The drawback in this strand of thinking is it leaves the problematic ontology of absolute totalities or wholes completely untouched. And yet theories concerning the God of traditional theism seem to require the existence of just such absolute entities. This is because God in such theories is the ultimate source of all contingent entities. Propositionally speaking, this implies that the actual truth of each contingent proposition traces back to a decision on the part of God to at least acquiesce to its truth. The basis for this choice rests on a deliberation between alternative ways the world would be if such and such a proposition were true. Thus, God isn't just choosing between the truth of a particular proposition and its contradictory; God is deciding among distinct maximal pluralities of propositions, ones that represent the universe as a whole; God is choosing to make a certain totality, as a totality, true. To see this, consider the principles by which God chooses. A great deal could be, and has been, said about this, but I won't say it. Suffice it to say that pretty much every theology of creation agrees that God chose to bring the universe into existence that best fulfils God's goals. Moreover, this judgement cannot be made at the level of individual truths of existence; it must be a judgement about the whole. For example, if one believes that there is a best of all possible worlds and that God creates according to the Principle of the Best, then God will have brought a world into being that instantiates the greatest possible amount of perfection or harmony.[5] This will be a truth about the whole that is emphatically not reducible to truths about its parts, as those as diverse as Boethius and Leibniz are at great pains to emphasize. Now translate all of this to talk about the deliberative process in God's mind 'before' any such world was brought into being, and you will have to invoke some type of synoptic representation of each possible cosmos, namely pluralities of *totalities* of possible truth from which God chooses. This process would be akin to the way scientists choose between cosmological theories in inferences to the best explanation. Theories that exhibit the greatest amount of simplicity, elegance, etc. are more likely to be true. Again, these are judgements about each theory (set of propositions) as a whole. Given this intuitively powerful way of talking about creation, it is, therefore, very difficult to see how there wouldn't be a completed totality of truth that God knows. Of course, you could eschew this way of talking about creation, but my alternative allows you to have the whole cosmological cake and eat it too.

Now, strictly speaking, Plantinga does not deny the existence of a completed totality of truth. Nevertheless, throughout his debate with Grim, he fervently avers that we do not need to posit the existence of such a totality in order to ground the truth of propositions about all propositions. Indeed, his agnosticism concerning the existence of a totality of all truths/propositions seems to slip into scepticism when he states, '[p]erhaps there is no totality (a set, a class) of propositions; sets and classes are a real problem anyway' (Plantinga and Grim (1993), 292). At the very least this implies that, as long as we conceive of the totality of truth as a collection of some sort, its existence can be called into question. This then begs the question: if the totality is not a collection, then what is it?

Rescher and Grim (2008, 2011), on the other hand, admit a solution to the diagonalization paradox that seems to let such totalities back into our ontology and hence our theorizing. Namely, deny that the collection of all propositions/truths is a Cantorian set.

They would call such a collection a *plenum.* In order to understand the nature of a plenum, we must first investigate collections *I* where their conditions for membership do not fully determine for every *x* whether $x \in I$. These are collections where bivalence fails in all of its forms.[6] Rescher and Grim call these collections *indeterminate collectivities.* Plena are collectivities *P* where every sub-collectivity of *P* can be mapped onto a distinct member of *P*. Hence *T*, the collectivity of all truths, is a plenum. Plena like *T* avoid being inconsistent totalities precisely because the existence of such a mapping of *T*'s sub-collectivities into *T* implies that there is at least one sub-collectivity of *T* that is an indeterminate collectivity, namely the diagonalization sub-collectivity C^*. Since C^* must be mapped onto a member of *T* $m(C^*)$, it will follow that neither $m(C^*) \in C^*$ nor $m(C^*) \notin C^*$. Thus, on pain of contradiction, C^* must be an indeterminate collectivity.

Rescher and Grim go on to demonstrate that not every plenum is an indeterminate collectivity, but, in the case of *T*, the very existence of C^* implies that *T* itself is indeterminate. Consider the fact that $m(C^*) \in C^*$ (DP) expresses a proposition. So, we may rightfully inquire: is DP a member of *T*? Alternatively, is DP a member of the plenum of all truths? But, given the fact that DP is neither determinately true nor determinately not true, it follows that the proposition $DP \in T$ is neither determinately true nor determinately not true. Therefore, *T* is an indeterminate collectivity.

T's indeterminacy, to my mind, raises at least two difficulties. First, it does not seem intuitively obvious that an indeterminate collectivity (IC) should be conceived of as an individual whole. A whole is an individual entity. Individuals are obviously fully individuated from other entities of the same or different types. The notion of an unindividuated individual is a contradiction in terms. Why might we think that IC's are not fully individuated? Suppose that I have the following collectivities $S = [a, b]$ and $S^* = [a, b, <c>]$, where $<c>$ stands for it being completely indeterminate in the ontological sense whether *c* is a member of S^*. On an extensional view of collectivity individuation, the proposition $S = S^*$ would inherit this indeterminacy, which would mean that neither S nor S^* is fully individuated from entities of the same kind. So, in order to preserve IC's as discrete individuals, we have to resort to an intensional criterion for collectivity individuation.[7] This is to state that collectivities with distinct qualifying conditions K and K^* are distinct regardless of the indeterminacy of their membership. But now consider two collectivity defining conditions D and D^*. Since they are two distinct conditions, they define two distinct collectivities C and C^*. And yet it certainly seems possible that D and D^* apply to all and only the same individuals. Maybe C is the collectivity of featherless bipeds, and C^* is the collectivity of humans. This seems to force us into the counterintuitive claim that it is possible for distinct collections to be completely coextensive; but surely each collectivity is nothing more nor less than the extension of its defining condition, and surely coextensive conditions share the same extension. Hence, on either conception of collectivity individuation, there are problems with conceiving an IC as a fully individuated whole.[8]

Rescher and Grim also note that a collectivity's indeterminacy infects the determinacy of the truth of propositions involving universal quantification over the members of that collectivity. More specifically, when the property attributed to every member of the IC applies only contingently, if at all, then the generalization will have no determinate truth value (Rescher and Grim (2008), 429; (2011), 67). This poses a special problem for the classical way of formulating God's omniscience, given *T*'s status as an indeterminate collectivity. The claim asserting God's omniscience may be expressed as follows:

CO: $\forall p \ (p \in T \supset$ God knows *p*).

Now, as long as God's existence is not a matter of conceptual/logical necessity, CO will be neither determinately true nor determinately false, even if God does exist, a decidedly

counterintuitive result.[9] So, although Plenum Theory as it is articulated by Rescher and Grim might hold promise for resolving many of the paradoxes that attend other mega-totalities, I do not think it will lend us much help in unravelling the knotty issues that arise with regard to the totality of Truth and its relation to omniscience.

One might think that, since God only has a choice about which *contingent* propositions to make true, contracting the collection of truths that God knows to all and only the totality of contingent truths might rescue such a collection from contradiction. But even so modest a collection as this will turn out to be too large to be a completed totality. Assume that the class of contingent truths C is a set. Then the power set P(C) is a set and is strictly larger than the set itself. But to each $c \in$ P(C) there will correspond a distinct contingent truth, namely $\forall p \in c(p$ is true). Each of these propositions will be contingently true, because, by hypothesis, every member of c is contingent and true, making it the case that it is not a necessary truth that every member is true. In addition, since each quantifies over a distinct set of truths, then each of these propositions is a distinct proposition. This demonstrates that there is a one-to-one mapping of each member of P(C) into C, contradicting the fact that P(C) is strictly larger. So, we need a way of talking about the contingent whole of reality and God's knowledge of it that is immune from paradoxes of cardinality like Grim's.

Before I leave this foray into alternative solutions to Grim's paradox, it behoves me to explain why I do not consider another seemingly straightforward way of eliminating the paradox: eschew propositions altogether and claim that God's knowledge of reality does not involve either beliefs or other mental states that require propositional mediation. William Alston (1986; 1989) put this view of God's knowledge on the map. He claimed that God knows facts via direct acquaintance with the facts themselves without the need to represent these facts in beliefs that could be true or false. Regardless of whether such an account does justice to the nature of knowledge,[10] it does not resolve the cardinality problems attending propositional accounts of omniscience; it merely kicks the problem down the road a bit. To see this, consider the set of all facts F. Let P(F) be F's power set. We may define the function $f(s)$ for $s \in$ P(F) such that $f(s) = $ *the fact that s exists.* In view of the fact that every member of s exists, it follows that s exists and hence that $f(s)$ is a member of F. Now let r and t be distinct members of P(F), then *the fact that r exists* and *the fact that t exists* are by that token distinct facts. Hence, $f(r) \neq f(t)$, and f is an injection of P(F) into F. But this implies that P(F) $\leqslant$ F, contradicting Cantor's Power Set Theorem. Thus, replacing truths with facts as the objects of divine knowledge doesn't help here. There will be too many facts as well.[11]

Propositions are quasi-pictorial representations

So as not to beg too many questions, I shall not provide an eliminative definition of the notion of a proposition, neither shall I provide a set of necessary and sufficient conditions for being a proposition. These, presumably, would entail principles of individuation that would be as controversial as the definition or criteria themselves. Instead, I shall merely rely on a sufficient condition for being a proposition that no one who is a propositional realist would care to deny:

(TB)[12] For all P, If P is a primary bearer of truth values, then P is a proposition.

To say that P is a primary bearer of truth values is to say that, necessarily, P can be either true or false, and P's possession of this truth value is not derivative. For example, when we state that the sentence 'Llamas are mammals' is true, as good propositional realists, we do not mean that the sentence itself has this property. We mean that the proposition

expressed by this sentence is true, and its possession of this property is intrinsic or original to it, whereas the sentence's possession of truth is at most relative to an interpretation that assigns a true proposition as the semantic value or meaning of it. I will now proceed to sketch an account of a kind of entity that fulfils TB.

My account is a natural consequence of two disputable but quite traditional assumptions about propositions and their relation to the rest of reality. The first assumption is that the truth of a proposition consists in the proposition's correspondence with reality. The second is that a proposition is a non-linguistic intentional entity, something that is *about* something. Once we view the truth of a proposition as a matter of correspondence, it follows that a proposition is a representation of some kind, for it makes no sense to say that something corresponds to the reality it is about unless that something presents something as a substitute for that reality, as something that thereby repeats the essential elements of that reality in the content of the presentation itself. But that is precisely the nature of a representation, something that intends, or refers to, an object by means of isomorphic similarity. This is why representations are primary truth bearers, because by their very nature they present something as bearing a likeness to some aspect of reality. They, thus, are either accurate in their presentation or not. Truth as correspondence is nothing more nor less than a proposition's accurate representation of some aspect of reality.

But now a question arises: how can non-linguistic entities represent reality? An obvious answer is by picturing it in some way. To picture something is to depict its nature by means of resembling it in relevant respects.[13] This resemblance could be sensory, such as visual resemblance or auditory resemblance, or it could be an abstract resemblance, such as a structural, functional, or logical resemblance. In the human case, this might take place via some mental image. And yet it is equally obvious that propositions are not literally pictures, since they are not physical objects and hence have no phenomenal characteristics. Indeed, no one among those who claim that mental images are pictorial is claiming that such images are actual pictures anyway. These images just represent in a way that is analogous to the way a picture represents. Kosslyn (1980; 1982; 1983) calls this mode 'quasi-pictorial'. A representation is quasi-pictorial just in case every part of the representation is also a representation of a part of the object represented. Paintings and photos paradigmatically exhibit this characteristic. For example, examine any arbitrary portion of a portrait of van Gogh and you will be presented with something that represents a part of van Gogh's face or some feature of his person. I want to claim that propositions are quasi-pictorial representations. My argument for this claim is simple:

(1) Any representation with non-representational parts is either a linguistic representation or a physically embodied representation.
(2) Propositions are, by hypothesis, non-linguistic and non-physical representations.
(3) Therefore, propositions contain no non-representational parts. (by 1 and 2)
(4) Therefore, propositions are quasi-pictorial representations. (by definition and 3)

Premise 1 is the most controversial premise in the above argument. Why should we accept it? We typically think of representations in two distinct but related senses, either as the informational content that is being represented, or as the act or form of representing. At the risk of stating the obvious: a representation presents (act) a presentation (information). Abstract from the form the presentation takes and the mode by which it is presented, and all that remains is information about the object represented. Thus, each portion of that informational content must present itself as information about the object itself, as a representation of a corresponding portion of the object. This means that the

half of the right half of Fernando also qualifies as a portion of the original representation of Fernando, namely the top righthand quarter of L. Hence, by our definition of QPRs, this portion will also be a representation of that portion of Fernando. This will be true in general for every part of a part of L, because every part of a part of L, by the nature of the parthood relation in this case, will also be a part of L itself. We may, therefore, conclude that every part of a QPR is also a QPR, since every part of these parts is also a representation of the corresponding portion of the object represented by the original QPR.

Not only does every part of an object that has the property of being a QPR also have the property being a QPR, but the property itself determines no intrinsic limit of the division of a QPR into ultimate and indivisible QPRs. Suppose Fernando has no atoms. That is to say, he is infinitely complex. Then every part of L would also be divisible into parts, and, by the same reasoning, each of these parts would be a QPR. The point here isn't that any QPR is infinitely divisible into QPRs, for as long as the object represented has ultimate parts, then its QPR must have ultimate parts. My point is that there is nothing in the possession of the property of being a QPR that prevents this division from continuing on indefinitely. Instead, it stands in need of the assistance of principles of individuation that are external to the property itself, principles found in the objects represented. Thus, the property of being a quasi-pictorial representation fails to provide distinct and ultimate units of measurement for the entities it qualifies. Moreover, this failure repeats the same pattern of indeterminacy of individuation found in mass properties of physical pedigree.

This does not yet show that the property of being a proposition is mass-like. This is because we have only demonstrated that every proposition is indefinitely divisible into QPRs. For all we know, not every QPR portion of a proposition qualifies as a proposition. So, in order to complete the proof, we must show that every QPR is intrinsically truth apt and thus by TB's lights a proposition. But why should we consider each QPR portion of a proposition intrinsically truth apt? Keller, among others, has claimed otherwise. She claims that there are representations that are not truth apt, namely, representations that do not determine truth conditions (Keller (2018), 14). Indeed, she adduces pictures and maps as paradigm examples of non-truth apt representations. If her assessment is correct, then this could call my entire account of propositions as quasi-pictures into question, as pictures are definitely a species of quasi-picture, and perhaps their lack of truth aptness is a function of their quasi-pictorial nature rather than their specific visual nature as pictures. Keller, in this particular paper, does not provide an argument for this claim. She, presumably, takes it as an obvious fact about pictures. But she could be relying on a common view about the representational content of pictures. This is the view that pictures do not express propositional content. As representations they possess content, but this content is non-propositional and hence not something that is truth apt.[15] The argument for pictures being non-propositional relies on the limitations of what pictures can represent by way of resemblance. Propositions represent logical operations such as negation, and disjunction. In other words, not only can propositions be asserted, denied, and disjoined, but there are propositions that represent the negation and disjunction of propositions. But it makes no sense to talk about a picture or photo of the non-obtaining of a state of affairs. What does a picture of disjunction look like? Thus, if propositions can and do represent logical or Boolean relations, but pictorial representations cannot, it follows that propositions are not pictorial.

Strictly speaking, the original argument is not directed against propositions being pictorial but rather against physical pictures having propositional content. I have reformulated the argument to show its consequences with respect to the representational nature of propositions, for if it is the pictorial nature of the content of physical pictures that precludes it from being propositional, then it follows that propositions are not pictorial

representations. But even as an argument against pictures expressing propositional content it leaves a lot to be desired. Let us assume for the sake of argument that sensory pictures cannot represent certain logical relations like negation and disjunction. It does not follow from this fact that pictures express no propositional content, only that they are limited in the propositional content they can represent. To see this, consider a language that lacks symbols for negation and disjunction. Imagine that it is a truncated form of English, only without the means of expressing these relations. So, it can state things like 'snow is white' and 'roses are red' but it cannot express what we express in English when we state, 'either snow is white or roses are not red'. Certainly this limitation on the representational content of this imaginary dialect does not ban any of its declarative sentences from expressing propositions. Obviously, a sentence like 'snow is white' still expresses the same proposition as its English counterpart. So why should the analogous representational limitations of pictures preclude them from representing propositions?[16]

Moreover, it is not clear that any of these limitations apply to QPRs in general. To think so is to confuse representing by resemblance with 'looking like' or 'sounding like'. Those who attempt to model propositions as sets of possible worlds capitalize precisely on the notion of resemblance of logical form. They are not merely attempting to construct an interesting abstract structure that can be isomorphically correlated with the truth functional behaviour of the logical operators; they are claiming that this isomorphism illuminates and hence represents the actual logical structure of these operators. They are claiming that a resemblance relation holds between their model and the logical reality it attempts to model. Hence, in this view, the logical structure of a proposition like 'P or Q' has the same logical form as the union of the set of all worlds where P is true and the set of all worlds where Q is true.

So how then do we decide whether some entity is a bearer of truth? The simple answer is: it presents something *as* something or *as* such and such. It thus makes a claim about that something that can either be asserted or denied. Certainly, pictures and quasi-pictures do this. I can point to the painting *Washington Crossing the Delaware* and say, 'That's not how it happened! The flag it depicts didn't have that design until at least a year after the event.' In a QPR, since every part represents something as having certain characteristics or being in certain relations, then every part may either be affirmed or denied. Hence, every part of a QPR is a truth bearer, and a primary one at that. This is because, neither being linguistic nor physical representations, they possess their content essentially and intrinsically, and given that this content is truth apt, it follows that they are primary bearers of truth. This entails that every QPR is a proposition. We may, therefore, conclude that every proposition, being arbitrarily divisible into QPRs, is arbitrarily divisible into propositions and that the property of being a proposition fails to provide a principle by which propositions are individuated into distinct and ultimate units. The property of being a proposition is mass-like.

Quasi-picturing omniscience

I will now describe a representation of the totality of reality that is such that any being that grasps the truth of this representation knows all truth and is thereby omniscient. Let us consider the actual world, A, and let us define the actual world as simply the totality of what is real or exists. The only thing I'm assuming in this definition is that A is an individual thing that may or may not have an enormous amount of complexity, that may or may not be a composite individual. An entity E is *complex* just in case E has parts. All I mean by *part* is a distinctly conceivable aspect of E. E is *composite* if and only if E has parts that could exist apart from E and each other. Every E will have a determinate *depth* of complexity, defined inductively as follows:

D1: E has a depth of 1 if and only if E has parts.
D2: E has a depth of $n+1$ if and only if some part of E at level n has parts.

Let's move on to consider a QPR of E. We raised the fact above that a QPR may represent E to different levels of resolution. We can now provide a more precise formulation of this characteristic of representations. A QPR of E has a *resolution* of n just in case the QPR represents E, E has at least a depth of n, and, for every $i \leqslant n$, the QPR represents all of E's i-th level parts and their interrelations. A QPR of E, therefore, has *maximal resolution* when the resolution of the QPR equals the depth of E. We are now ready to describe a QPR, R, of A with the following features:

R1: R represents A with maximal resolution.
R2: R represents A with perfect accuracy.
R3: For every S that is a part of A, R represents S without entailing that there is a separate QPR individual that represents S.
R4: R represents nothing else besides A.

R1, R2, and R4 are what endow R with the quality of being *the Truth* about A. To say that a QPR represents E with *perfect accuracy* is merely to say that every part of E that it does represent is exactly as it represents it to be. R3 effectively states that R is not a composite representation and that it is possible for a representation to depict a complex multiplicity without it itself being the conjunction or aggregate of multiple representations. Let's now ratify this possibility.

When we examine *Washington Crossing the Delaware*, we see that it cannot be the representation of this event without also being a representation of George Washington as the major constituent of this event. It must also represent the Delaware river, the boats used in the crossing, and so on. But, for all of this multiplicity in its representational objects, it is still just a single representation. It is not a composite representation formed out of these more restricted representations. Indeed, these smaller representations do not exist apart from their occurrence within the larger picture. What is true for physical pictures is true in general for QPRs. Owing to the mass-like nature of QPRs, just because something is logically or conceptually distinguishable as a distinct portion of the QPR does not mean that it exists as a distinct individual. And yet, due to the pictorial nature of QPRs, every portion of a QPR serves as a representation of a corresponding portion of the object of the representation. Hence, QPRs that represent complex objects can represent multiple things without multiplying representations.

If we take the view that propositions are platonic abstract objects whose existence does not depend on the representational activity of minds, then the existence of R will entail that, apart from R, there are no other separately existing truths. To see this, consider the fact that a proposition as a QPR will now amount to nothing more nor less than its representational content. Any other truth T will either represent the whole of A or some more restricted segment. But, since R represents the entirety of A with maximal specificity, it follows that T will either contain the same representational content as R or it will contain the same representational content as some segment of R. If the former is the case, then T = R; if the latter is the case, then T will be identical to that segment. Either way T will not exist as a separate individual.

Of course, we could view propositions as products of the activity of representing, or as inextricably tied to this activity as the content of any specific act of representing. Then the propositional situation would be analogous to what occurs with physical masses like marble. R would be a product of the representational activity of a divine mind, thereby constituting the totality of Truth, and all other truths would only exist as

products of other specific representational activities. They would be more restricted 'cuts' out of this primordial representational mass, just as statues and blocks are more restricted cuts out of the primordial marble mass. This would not multiply propositions beyond the bounds of cardinality because propositions would require some propositional attitude, some act of representation, in order to exist. We couldn't simply assert that every member of the power set of the set of truths corresponds to some truth, for such truths would have to be the products of actual acts of representation. At best we could assert that every member of the power set of truths corresponds to a *potential* truth, a potentially accurate representation of reality. But just because a representation can be constructed doesn't mean it actually is. In the same manner, just because a particular hunk of marble has the potential to be carved into a particular number of statues, that does not imply that the statues actually exist to be numbered.[17]

One might worry that envisaging a potential truth for each member of the power set of truths would still imply an actual truth corresponding to each member.[18] For suppose that P is a potential truth corresponding to some member *s* of the power set of truths. Then it will be true that *P is a potentially accurate representation of s*, thus multiplying the number of actual truths to the size of the power set of truths and throwing us back into the arms of our Cantorian cardinality conundrums. Let us, however, unpack the claim that there are potential truths corresponding to every member of the power set and see whether it really does have this consequence. I could mean one of two things by this claim:

P1: For every $s \in P(T)$, it is possible that there exists a truth that corresponds to *s*,

or

P2: For every $s \in P(T)$, there exists a possible truth that corresponds to *s*,

where P(T) stands for the power set of all truths. P1 is a *de dicto* statement which does not imply the existence of any representation, let alone ones that are potentially accurate representations of reality.[19] Thus, if there are no such representations to start with, it will follow that they cannot have the additional quality of being potentially true. P2 does imply the existence of each of these truths, but only because it is already committed to the existence of representations that are at least possibly true. And yet, the activist position I outline above explicitly denies the existence of any representation of reality that has not been actually constructed. So, while the activist may be committed to P1, she must, on pain of contradicting the constructive nature of propositions, deny P2.

We are now in a position to formulate a robust reinterpretation of omniscience, at least from the perspective of the extent of the things it knows. We say that a being is omniscient if and only if that being knows R. Since R represents the total truth about reality, to know R is to know everything. This definition will suffice on both a platonic and an activist notion of propositions, whether R is the only propositional individual or not. The platonic case should be obvious; since R encompasses the totality of truth, to know R is straightforwardly to know every truth. In the activist scenario where there might be other truths besides R, what distinguishes other truths from R is not their representational content but rather the fact that they are results of distinct acts of representation. Hence to know R is to know everything that is known in knowing those other propositions, to know the whole truth and nothing but the truth.

Given our linguistic and perceptual ways of grasping truth, it is easy to see why we would be tempted to believe that Truth comes pre-packaged into thought sized truths. We cannot grasp the whole of Truth all at once; we must always come at it through finite, piecemeal representations and then attempt to fit these all together into a coherent but

compositional depiction of the real. It is no wonder, then, why different sentences, grabbing hold of different segments of the same propositional whole, will communicate distinct propositional content. But we would do well not to mistake the limits of our sentences with the limits of the Truth they seek to express. Otherwise, we risk ensconcing ourselves in the same epistemic prison as the frog of Chinese lore, who, looking up at the sky from the bottom of a well, mistook the shape of the well for the shape of the sky itself.

Notes

1. For his initial set theoretic argument that there is no set of all truths, see Grim (1984). For his applications against the coherence of the traditional notion of omniscience, see Grim (1988), (1990); Plantinga and Grim (1993). See also Grim (2000) for his attempt to free the original argument from its seeming commitment to Zermelo-Fraenkel Set Theory.

2. This is not to say that there aren't count nouns that have vague extensions. Such nouns and their corresponding concepts would just fail to have extensions with a definite cardinality.

3. By 'autonomously' I mean: the possession of the property is a sufficient condition for being a unit and thereby grounding truths about cardinality.

4. Frege only makes this claim relative to finite numbers, as we see in the quote above.

5. Even if we give up the idea of a best of all possible worlds, certainly God will be rejecting worlds that do not actualize a minimal amount of perfection, goodness, or harmony, and this rejection will be based on synoptic judgements about each maximal plurality as a determinate and distinct whole.

6. That is to say, even the attenuated form of the law of exclude middle with regard to membership is denied. Thus, it will not be the case, at any level of iteration for determinacy, that *either determinately $x \in I$ or not determinately $x \in I$.*

7. Rescher and Grim seem to hesitate to commit to a fully intensional account of collectivity individuation. See Rescher and Grim (2008, 424) where they allow for both extensional and intensional criteria. In Rescher and Grim (2011, 1) they hedge their bets and state, 'membership in a collectivity *standardly* corresponds to possession of some specified features encompassed in a qualifying condition' (my italics).

8. It will not do to divide the work and say that determinate collectivities are extensionally individuated, and indeterminate ones are intensionally individuated, for then collectivities as a kind or sort would not have uniform principles of individuation, which would cast suspicion on such things as an ontological category *tout court*.

9. I think this result holds for CO only if God does exist. If God doesn't, then the generalization is easily falsifiable despite the indeterminacy of the scope of the quantifier. This is because all of the determinate truths within *T* will not possess the property of being known by God and thus act as counterexamples to the universal claim being made by CO.

10. See Dickinson (2019) for an excellent critique of the non-propositional approach of Alston.

11. For a separate diagonalization argument against there being a set of all obtaining states of affairs, namely, facts, see Chihara (2001). Of course, one could resist the above proof of equinumerosity by claiming that the principles that individuate facts are not as fine-grained as those that individuate propositions, but such a response just confirms the claim that the cardinality problems that infect propositions infect facts and their individuation as well.

12. 'TB' is short for 'Truth Bearer'.

13. There might be pictures that represent things symbolically rather than through resemblance, but I would argue that all such pictures involve some kind of linguistic element and hence are not purely pictorial representations. See Grzankowski (2015) for more discussion on the ways that visual pictures represent.

14. Here I am using the term 'picture' as short for quasi-picture. So, this could be a mental image or representation.

15. See, for example, Crane (2009), Heck (2000), Kulvicki (2006), Sainsbury (2005), Schier (1986), and Sober (1976).

16. This argument is a reformulation of a similar line of reasoning used by Grzankowski (2015).

17. Menzel (2018) employs a similar strategy for restricting set existence. Only, whereas he argues that sets are the products of aggregation out of indeterminate pluralities, I am arguing that propositions could be construed as the products of representational cuts out of a comprehensive mass-like totality.

18. I owe this objection to one of the referees.

19. Except by assuming the truth of the Barcan Formula, which would then imply P2, a very big assumption indeed. This would also follow if propositions existed necessarily, but the activist explicitly writes this out of her theory by making propositions the products of contingent acts of representation.

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