Correspondence

DEAR EDITOR,

In pursuing the early history of trigonometric tables, it is useful to know for what angles sines can be calculated fairly easily.

The sines of the angles 15°, 18°, 30°, 45°, 54°, 60° and 75° are all of the form $a + b\sqrt{B}$ or $a + b\sqrt{B} + c\sqrt{C}$, where a, b and c are rational numbers and B and C are integers. Do readers know any other angles (between 0° and 90°), measurable as a whole number of minutes, whose sines are of this or similar form?

Yours sincerely,

BOB BURN

Sunnyside, Barrack Road, Exeter EX2 6AB

DEAR EDITOR,

Bill Richardson solves the old chestnut 'How many presents did my true love send to me?' in the November 2002 *Gazette* (p. 468) by summing the triangular numbers 1, 3, 6, ... corresponding to each day's haul. There is a direct route to his answer: notate each present by an integer triple (r, s, t), where this stands for the rth present of type s received on day t. So, for example, (3, 5, 8) stands for the 3rd of the 5 gold rings received on day 8.

We now have to count the triples (r, s, t) with $1 \leqslant r \leqslant s \leqslant t \leqslant 12$, or (equivalently), if we put $R = r$, $S = s + 1$ and $T = t + 2$, we must count the triples (R, S, T) with $1 \leqslant R < S < T \leqslant 14$, and plainly the answer is the number of ways of choosing 3 objects from 14, or $\binom{14}{3}$.

For n days, just replace 12 by n, and the answer is $\binom{n+2}{3}$. These numbers are sometimes called tetrahedral numbers, and it should be clear that if we want to sum the first n tetrahedral numbers, a similar technique involving quadruples of integers will yield the result. Indeed, triangular numbers can be produced this way also: for

$$1 + 2 + 3 + \dots + n = 1 + (1 + 1) + (1 + 1 + 1) + \dots,$$

and if we let (r, s) stand for the rth 1 in the sth bracket, then we are counting solutions of $1 \leqslant r \leqslant s \leqslant n$, or $1 \leqslant R < S \leqslant n + 1$, which gives the number of ways of choosing 2 objects from $n + 1$, or $\binom{n+1}{2}$.

Yours sincerely,

JOHN SILVESTER

Dept of Mathematics, King's College, Strand, London WC2R 2LS

https://doi.org/10.1017/S002555720017295X Published online by Cambridge University Press

DEAR EDITOR,

Like Doug French, I was flabbergasted by the figures in the Gleaning on p. 389 of the November issue. The obvious questions are: (a) what do they mean? and (b) how were they arrived at? After some experimental prodding of the buttons on my calculator I arrived at an answer to (b): in 3 cases out of the 4, the figure quoted is the number of standers expressed as a percentage of the excess of sitters over standers, i.e. if there are m sitters and n standers, then this is $100n/(m - n)$. In the case of the Central Line, this formula gives 600% (not 578), so presumably there is a mistake (!) here. We obtain the answer claimed if $n = 176$ rather than 180. But I am at a loss to explain (a)! It is worthy of note that, if $n > m$, the result is negative and if $n = m$, infinite! Comments from London Transport (or whatever it calls itself nowadays) would be interesting.

Yours sincerely,

A. ROBERT PARGETER

10 Turnpike, Sampford Peverell, Tiverton EX16 7BN

DEAR EDITOR,

Readers who have not looked at the cumulative on-line index of the *Gazette* recently may wish to know of its current status. The simple listing of items now begins in 1930 (and there are also some of the very earliest ones – but in a very different style). There is also a searchable index. Both these can be found by going to the MA site (www.m-a.org.uk) and following link to periodicals and then to the *Gazette*.

Yours sincerely,

BILL RICHARDSON

Kintail, Longmorn, Elgin IV30 8RJ

https://doi.org/10.1017/S002555720017295X Published online by Cambridge University Press