PRIMITIVE SKEW LAURENT POLYNOMIAL RINGS

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Introduction. In [**8**] the author studied the question of the primitivity of an Ore extension $R[x, \delta]$, where δ is a derivation of the ring R. If α is an automorphism of R then it can be shown that $R[x, \alpha]$ is primitive if the following conditions are satisfied: (i) no power $\alpha^s, s \geq 1$, of α is inner; (ii) the only ideals of R invariant under α are 0 and R. These conditions are also known to be necessary and sufficient for the skew Laurent polynomial ring $R[x, x^{-1}, \alpha]$ to be simple [**9**]. The object of this paper is to find conditions which are sufficient for $R[x, x^{-1}, \alpha]$ to be primitive. The results obtained are remarkably similar to those of [**8**]. Two logically independent conditions are each found to be sufficient for the primitivity of $R[x, x^{-1}, \alpha]$. Of these, one is also shown to be sufficient for $R[x, \alpha]$ to be primitive. Included in the examples illustrating these results are some applications to the theory of primitive group rings. The basic techniques involved are also applied to produce a counterexample to the converse of a theorem of Goldie and Michler [**3**] on when $R[x, x^{-1}, \alpha]$ is a Jacobson ring.

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1. Throughout, R will denote a ring with identity, α will be an automorphism of R and $R[x, x^{-1}, \alpha]$ will denote the skew Laurent polynomial ring, i.e. the ring of polynomials over R in an indeterminate x and its inverse, with multiplication subject to the relation

$$xr = \alpha(r)x \quad \text{for all} \quad r \in R.$$

An ideal I of R is said to be an α-*ideal* of R if $\alpha(I) = I$. An α-ideal I of R is said to be α-*prime* if for all α-ideals A,B of R, $AB \subseteq I$ implies $A \subseteq I$ or $B \subseteq I$. R is said to be α-*prime* if the ideal 0 is α-prime.

The following result is easily proved.

Proposition 1 (cf. [**3**, Lemmas 1.1, 1.3, 1.7], [**7**, Lemmas 1.3, 1.4]). *Let* $S = R[x, x^{-1}, \alpha]$, *let* I *be an ideal of* S *and let* J *be an* α-*ideal of* R. *Then*

(i) $I \cap R$ *is an* α-*ideal of* R *and* JS *is an ideal of* S;

(ii) $\dfrac{S}{(I \cap R)S} \simeq \dfrac{R}{I \cap R}[x, x^{-1}, \alpha]$;

(iii) *if* I *is prime then* $I \cap R$ *is* α-*prime and if* J *is* α-*prime then* JS *is prime.*

For any ring T the Jacobson radical of T is denoted $J(T)$. If I is an ideal of T, we denote by $J(I)$ the ideal of T such that $J(I)/I = J(T/I)$. $\mathscr{C}_T(I)$ will denote the set $\{c \in T : [c + I] \text{ is a regular element of } T/I\}$.

Proposition 2. *Let* $S = R[x, x^{-1}, \alpha]$. *If* R *is right noetherian and* α-*prime then* $J(S) = 0$.

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Proof. This follows from [6, Theorem 2].

2. α-primitive rings and αG rings.

DEFINITION. Let $S=R[x, x^{-1}, \alpha]$ and let $f(x)=\sum_{i=m}^{n} a_i x^i \in S$, with $a_n \neq 0$ and $a_m \neq 0$. Then the *length* of $f(x)$ is the non-negative integer $n-m$.

DEFINITION. The automorphism α is said to be *stiff* on R if for all non-zero ideals I of $R[x, x^{-1}, \alpha]$, $I \cap R \neq 0$.

DEFINITION. The automorphism α is said to be *rigid* on R if the mapping θ from the set of ideals of $R[x, x^{-1}, \alpha]$ to the set of α-ideals of R defined by $\theta(I)=I \cap R$, for all ideals I of $R[x, x^{-1}, \alpha]$, is a bijection.

LEMMA 1. (i) *If there exists a central element z of R such that $(\alpha^n(z)-z) \in \mathscr{C}_R(0)$ for all $n>0$ then α is stiff on R;*

(ii) *if there exists a central element z of R such that $(\alpha^n(z)-z)$ is a unit for all $n>0$ then α is rigid on R.*

Proof. (i) Let I be a non-zero ideal of S. Let $f(x)$ be a non-zero element of I of minimal length, $f(x)=\sum_{i=m}^{n} a_i x^i$, $a_m \neq 0$, $a_n \neq 0$. Since $f(x)x^r \in I$ for all integers r, it is clear that m may be assumed to be 0. Suppose that $n>0$. Let $g(x)=f(x)z-zf(x)=\sum_{i=0}^{n} a_i(\alpha^i(z)-z)x^i$. By the choice of $z, g(x) \neq 0$ but $g(x) \in I$ and the length of $g(x)$ is less than the length of $f(x)$, which contradicts the choice of $f(x)$. Hence $n=0$ and $0 \neq f(x)=a_0 \in R$. Thus α is stiff on R.

(ii) It is sufficient to show that $I=(I \cap R)S$ for all ideals I of S, where $S=R[x, x^{-1}, \alpha]$. If I is an ideal of S then, by (i), α is stiff on $R/(I \cap R)$. It follows from Proposition 1(ii) that $I=(I \cap R)S$.

DEFINITION. R is said to be *α-primitive* if there exists a maximal right ideal M of R such that M contains no non-zero α-ideals of R.

THEOREM 1 (cf. [8, Theorem 1]. *If R is α-primitive and α is stiff on R then $R[x, x^{-1}, \alpha]$ is primitive.*

Proof. The proof is a precise analogue of that of [8, Theorem 1].

DEFINITION. R is said to be αG if it is α-prime and the intersection of the non-zero a-prime ideals of R is non-zero.

THEOREM 2 (cf. [8, Theorem 2]). *If R is right noetherian, R is αG and α is stiff on R then $R[x, x^{-1}, \alpha]$ is primitive.*

Proof. Let I denote the intersection of the non-zero α-prime ideals of R and let P be a non-zero primitive ideal of S, where $S=R[x, x^{-1}, \alpha]$. Since α is stiff, it follows from

Proposition 1 that $I \subseteq P \cap R$. Consequently either S is primitive or $I \subseteq J(S)$. But $I \neq 0$ and $J(S) = 0$ by Proposition 2. Hence S is primitive.

If D is a division ring which is not algebraic over its centre then the ordinary Laurent polynomial ring $D[x, x^{-1}]$ is primitive. Consequently the condition that α is stiff on R is not necessary for $R[x, x^{-1}, \alpha]$ to be primitive. Example 1 (resp. Example 2) below is of a ring R with automorphism α satisfying the conditions of Theorem 2 (resp. Theorem 1) but not those of Theorem 1 (resp. Theorem 2). Thus the statements "R is αG" and "R is α-primitive" are logically independent and neither is necessary for $R[x, x^{-1}, \alpha]$ to be primitive.

Example 1 (cf. [**8**, Example 1]). Let $R = k[[y]]$ be the power series ring over a field k of characteristic 0. Let α be the k-automorphism of R such that $\alpha(y) = 2y$. The non-zero ideals of R are of the form $y^r R$, $r > 0$, and are all α-invariant. It follows that the only non-zero α-prime ideal of R is yR and hence that R is αG. For $n > 0$, $\alpha^n(y) - y = (2^n - 1)y \in \mathscr{C}_R(0)$ and so, by Lemma 1(i), α is stiff on R. It follows from Theorem 2 that $R[x, x^{-1}, \alpha]$ is primitive. However, the only maximal ideal of the commutative ring R is an α-ideal, so that R is not α-primitive.

Example 2 (cf. [**8**, Example 2]). Let $R = k(t)[y]$ be the polynomial ring in an indeterminate y over the field of rational functions in an indeterminate t over a field k of characteristic 0. Let α be the k-automorphism such that $\alpha(t) = 2t$ and $\alpha(y) = 2y$. Let M be the maximal ideal $(y-1)R$. Then for all integers i, $\alpha^i(M) = (2^i y - 1)R$, so that $\bigcap_{i=-\infty}^{\infty} \alpha^i(M) = 0$ and M contains no non-zero α-ideals. Thus R is α-primitive. For $n > 0$, $\alpha^n(y) - y = (2^n - 1)y \in \mathscr{C}_R(0)$ and so, by Lemma 1(i), α is stiff on R. It follows from Theorem 1 that $R[x, x^{-1}, \alpha]$ is primitive. R is not αG because for all $\lambda \in k$, $(y - \lambda t)R$ is a non-zero α-prime ideal and $\bigcap_{\lambda \in k} (y - \lambda t)R = 0$ since k is infinite.

Example 3 (cf. [**8**, Example 3]). Let $R = k[y]$ be the polynomial ring over a field k of characteristic 0 and α the k-automorphism of R such that $\alpha(y) = 2y$. R is α-primitive because $\bigcap_{i=-\infty}^{\infty} \alpha^i((y-1)R) = 0$ as in Example 2. R is αG because the only non-zero α-ideals of R are those of the form $y^r R$, $r > 0$, so that yR is the only non-zero α-prime ideal of R. As in the previous examples α is stiff on R, so that, by Theorem 1 or Theorem 2, $R[x, x^{-1}, \alpha]$ is primitive.

3. Primitivity of $R[x, \alpha]$**.** The object of this section is to show that $R[x, \alpha]$ is primitive whenever the conditions of Theorem 2 hold. By adapting the argument given in [**5**, p. 22] for the case where R is a field it is easy to prove the following.

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PROPOSITION 3 [**9**]. *If the only α-ideals of R are* 0 *and R and if α^n is an outer automorphism for all $n>0$, then*:

(i) *every ideal of $R[x,\alpha]$ contains a power of x;*
(ii) *$R[x,\alpha]$ is primitive;*
(iii) *$R[x,x^{-1},\alpha]$ is simple.*

LEMMA 2. *If there exists a central element z of R such that $\alpha^n(z)-z\in\mathscr{C}_R(0)$ for all $n>0$ then for all non-zero prime ideals P of $R[x,\alpha]$ either $x\in P$ or $P\cap R$ is a non-zero α-prime ideal of R.*

Proof. Let P be a non-zero prime ideal of $R[x,\alpha]$ such that $x\notin P$ and let $f(x)=a_nx^n+\ldots+a_1x+a_0$ be a non-zero element of minimal degree in P. By [**3**, Lemma 1.2], $x\in\mathscr{C}_{R[x,\alpha]}(P)$ so that $a_0\neq 0$. The argument of Lemma 1(i) now shows that $f(x)=a_0\in P\cap R$ so that $P\cap R\neq 0$. Finally $P\cap R$ is α-prime by [**3**, Lemmas 1.2, 1.3].

THEOREM 3. *If R is right noetherian, R is αG, and $(\alpha^n(z)-z)\in\mathscr{C}_R(0)$ for some central $z\in R$ and all $n>0$, then $R[x,\alpha]$ is primitive.*

Proof. Let I denote the intersection of the non-zero α-prime ideals of R. Suppose that $R[x,\alpha]$ is not primitive and let P be a primitive ideal of $R[x,\alpha]$. If $x\notin P$ then, by Lemma 2, $P\cap R$ is a non-zero α-prime ideal of R and hence $I\subseteq P\cap R$. It follows that $0\neq Ix\subseteq J(R[x,\alpha])$. But by [**6**, Theorem 2], $J(R[x,\alpha])=0$. Thus $R[x,\alpha]$ is primitive.

If R and α are as in Example 2 or Example 3 then the hypotheses of Theorem 3 are satisfied and $R[x,\alpha]$ is primitive. In neither of these examples are the conditions of Proposition 3 satisfied. The next example is of a ring R and automorphism α satisfying the conditions of Proposition 3 but not those of Theorem 3.

EXAMPLE 4. Let $K=k(t)$ be the field of rational functions over a field k of characteristic 0. Let δ be the derivation d/dt and let R be the Ore extension $K[y,\delta]$. R is known to be simple, see e.g. [**2**, Theorem 3.2], and clearly $K\setminus\{0\}$ is the set of units of R. Let α be the k-automorphism of K such that $\alpha(t)=t+1$. Extend the action of δ and α to R by setting $\delta(y)=0$ and $\alpha(y)=y$. Since $\delta\alpha=\alpha\delta$, α is then an automorphism of R. To see that α^n is outer for all $n>0$ let $c\in K\setminus\{0\}$ and let β be the inner automorphism, $\beta(r)=c^{-1}rc$ for all $r\in R$. In particular $\beta(y)=c^{-1}yc=y+c^{-1}\delta(c)$. Since for $n>0$, $\alpha^n(y)=y+n$ and there does not exist $c\in K\setminus\{0\}$ such that $\delta(c)=nc$, it follows that α^n is outer for all $n>0$. R is simple and so, by Proposition 3 (ii), $R[x,\alpha]$ is primitive. However, the conditions of Theorem 3 are not satisfied since the centre of R is k and α acts as the identity on k. By Proposition 3(ii), $R[x,x^{-1},\alpha]$ is simple and hence α is stiff on R. It follows that the converse of Lemma 1.1 (i) is false.

4. Application to group rings. Let k be a field and G a group having a normal subgroup H such that G/H is an infinite cyclic group, generated by xH, say. Let α be the k-automorphism of the group ring kH defined by setting $\alpha(h)=xhx^{-1}$ for all $h\in H$. Then $kG\simeq kH[x,x^{-1},\alpha]$ and the results of §2 apply. In particular we have the following result, where for $h\in G$, $C_G(h)=\{g\in G: hg=gh\}$.

THEOREM 4. *Let k, G, H and α be as above. If kH is prime and α-primitive, and there exists $h \in H$ such that $C_G(h) = H$ then kG is primitive.*

Proof. For $n \geq 1$, $x \notin C_G(h)$, so $\alpha^n(h) - h \neq 0$. Since h is central in H and α is an automorphism, $\alpha^n(h) - h$ is central and hence, since kH is prime, $\alpha^n(h) - h \in \mathscr{C}_{kH}(0)$. By Lemma 1(i), α is stiff on kH. It follows by Theorem 1 and the above remarks that kG is primitive.

EXAMPLE 5. Let $G = C_\infty \sim C_\infty$ be the restricted wreath product of two infinite cyclic groups. G is a cyclic extension of H where H is the restricted direct product of a countable number of infinite cyclic groups. For a given field k, kH is then the Laurent polynomial ring over k in a countable set of commuting indeterminates $\{x_i\}_{i \in \mathbb{Z}}$. If α is the k-automorphism of kH such that $\alpha(x_i) = x_{i+1}$ for all $i \in \mathbb{Z}$ then $kG = kH[x, x^{-1}, \alpha]$. We claim that kG is primitive for all fields k.

Consider first the case where k is countable, possibly finite. It is clear that any non-zero α-ideal I of kH has non-zero intersection with $k[x_1, x_2, \ldots, x_n]$ for some $n = n(I)$. Let $\hat{k}$ denote the algebraic closure of k. Then for each $r \geq 1$ the set of r-tuples $(\lambda_1, \lambda_2, \ldots, \lambda_r)$, $\lambda_i \in \hat{k} \backslash \{0\}$ is countable. The union of these sets, taken over all $r \geq 1$, is countable and hence there exists a sequence $(\mu)_{i \geq 1}$, $\mu_i \in \hat{k} \backslash \{0\}$, such that for every positive integer r and r-tuple $(\lambda_1, \lambda_2, \ldots, \lambda_r)$, $\lambda_i \in \hat{k} \backslash \{0\}$, there exists $l \geq 1$ such that $\lambda_1 = \mu_l, \lambda_2 = \mu_{l+1}, \ldots, \lambda_r = \mu_{l+r+1}$. For $i \leq 0$ let $\mu_i = 1$. Let $M = \sum_{i \leq j} M_{i,j} kH$, where for the integers i,j such that $i \leq j$, $M_{i,j} = \{f \in k[x_i, \ldots, x_j] : f(\mu_i, \ldots, \mu_j) = 0\}$. By Hilbert's Nullstellensatz [**1**, Proposition 2, p. 351], each $M_{i,j}$ is a maximal ideal of $k[x_i, \ldots, x_j]$ and it follows that M is a maximal ideal of kH. Suppose that there exists a non-zero α-ideal I of kH such that $I \subseteq M$. Then for some positive integer n there exists non-zero $f = f(x_1, x_2, \ldots, x_n) \in I \cap k[x_1, x_2, \ldots, x_n]$. Since $\alpha^i(f) \in M$ for all i, $\alpha^i(f) \in M_{i,n+i-1}$ for all i and hence

$$\alpha^i(f)(\mu_i, \ldots, \mu_{n+i-1}) = 0 \quad \text{for all } i.$$

Equivalently,

$$f(\mu_i, \ldots, \mu_{n+i-1}) = 0 \quad \text{for all } i. \tag{1}$$

Now let N be any maximal ideal of $k[x_1, x_2, \ldots, x_n]$. By the Nullstellensatz, either $x_1 x_2 \ldots x_n \in N$ or there exists an n-tuple $(\lambda_1, \lambda_2, \ldots, \lambda_n)$, $\lambda_i \in \hat{k} \backslash \{0\}$, such that, for $g \in k[x_1, x_2, \ldots, x_n]$, $g \in N$ iff $g(\lambda_1, \lambda_2, \ldots, \lambda_n) = 0$. In the latter case there exists $l \geq 1$ such that $\lambda_1 = \mu_l$, $\lambda_2 = \mu_{l+1}, \ldots, \lambda_n = \mu_{l+n-1}$ so that by (1), $f \in N$. It follows that $0 \neq x_1 x_2 \ldots x_n f \in N$ for every maximal ideal N of $k[x_1, x_2, \ldots, x_n]$. But the Jacobson radical of $K[x_1, x_2, \ldots, x_n]$ is zero, which gives a contradiction. Thus M contains no non-zero α-ideal of kH and kH is α-primitive. Since kH is a commutative domain and $C_G(x_1) = H$, it follows from Theorem 4 that kG is primitive

Now let k be an arbitrary field and l the prime subfield of k. Then lG is primitive by the above. It follows from [**10**, Theorem 2] that kG is primitive. We note that for the case

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where the transcendence degree of k over l is infinite kG is known to be primitive by [**10**, Corollary 13].

Example 6. Let $H=\langle x_1, x_2\rangle$ be a free abelian group of rank 2 and let α be the automorphism of H such that $\alpha(x_1)=x_2$ and $\alpha(x_2)=x_1x_2$. Let G be the semidirect product $H\times_\alpha\langle x\rangle$, where $\langle x\rangle$ is infinite cyclic. We claim that if k is a field of characteristic zero then kH is α-primitive and hence that kG is primitive.

Consider first the case where $k=\mathbb{Q}$, the field of rational numbers. Let $M=(x_1-2)\mathbb{Q}H+(x_2-2)\mathbb{Q}H$, a maximal ideal of $\mathbb{Q}H$. Suppose that there exists a non-zero α-ideal I of $\mathbb{Q}H$ such that $I\subseteq M$. Then there exists non-zero $f(x_1, x_2)\in\mathbb{Q}[x_1, x_2]$ such that $\alpha^n(f(x_1, x_2))\in M$ for all $n\geq 0$, i.e. $f(\alpha^n(x_1), \alpha^n(x_2))=0$ for all $n\geq 0$. In general, $\alpha^n(x_1)=x_1^{u(n-1)}x_2^{u(n)}$ and $\alpha^n(x_2)=x_1^{u(n)}x_2^{u(n+1)}$, where $u(0)=0$ and, for $i\geq 1$, $u(i)$ is the ith Fibonacci number. Thus $f(2^{u(n-1)}2^{u(n)}, 2^{u(n)}2^{u(n+1)})=0$ for all $n\geq 0$, i.e. $f(2^{u(n+1)}, 2^{u(n+2)})=0$ for all $n\geq 0$. It follows from Lemma 3 below that $f=0$, which gives a contradiction. Thus $\mathbb{Q}H$ is α-primitive, and since $\mathbb{Q}H$ is a commutative domain and $C_G(x_1)=H$ it follows from Theorem 3 that $\mathbb{Q}G$ is primitive. Since $\{g\in G:\{y^{-1}gy : y\in G\}$ is finite$\}=\{1\}$ it follows from [**10**, Theorem 2] that kG is primitive for all fields k of characteristic 0.

Lemma 3. *For $i\geq 1$ let $u(i)$ denote the i-th Fibonacci number and let $f=f(x_1, x_2)\in\mathbb{Q}[x_1, x_2]$ be such that $f(2^{u(n)}, 2^{u(n+1)})=0$ for all $n\geq 1$. Then $f=0$.*

Proof. For $n\geq 1$ let $\lambda(n)=u(n+1)/u(n)$. It is known that $\lambda(n)\rightarrow\lambda=(1+\sqrt{5})/2$ as $n\rightarrow\infty$ (see e.g. [**4**, Chapter X]). Define an order $>$ on the set of monomials $x_1^px_2^q$, $p,q\geq 0$ as follows:

$$x_1^px_2^q>x_1^rx_2^s \quad \text{iff} \quad p+\lambda q>r+\lambda s.$$

Since λ is irrational, $>$ is a total order. Suppose $f\neq 0$. Then for some integer $t>1$

$$f(x_1, x_2)=\sum_{i=1}^{t} f_i x_1^{p(i)}x_2^{q(i)},$$

where $f_i\in\mathbb{Q}\setminus\{0\}$ for $1\leq i\leq t$ and $x_1^{p(i)}x_2^{q(i)}<x_1^{p(j)}x_2^{q(j)}$ whenever $1\leq i<j\leq t$. For $n\geq 1$,

$$\begin{aligned} f(2^{u(n)}, 2^{u(n+1)}) &= \sum_{i=1}^{t} f_i 2^{p(i)u(n)+q(i)u(n+1)} \\ &= \sum_{i=1}^{t} f_i 2^{u(n)(p(i)+\lambda(n)q(i))} \\ &= f_t 2^{u(n)(p(t)+\lambda(n)q(t))}\left(1+\sum_{i=1}^{t-1}(f_i/f_t)2^{u(n)(p(i)+\lambda(n)q(i)-(p(t)+\lambda(n)q(t)))}\right). \end{aligned} \tag{2}$$

But $p(i)+\lambda q(i)<p(t)+\lambda q(t)$ for $1\leq i\leq t-1$ and, as $n\rightarrow\infty$, $\lambda(n)\rightarrow\lambda$ and $u(n)\rightarrow\infty$. Hence

$$\sum_{i=1}^{t-1}(f_i/f_t)2^{u(n)(p(i)+\lambda(n)q(i)-(p(t)+\lambda(n)q(t)))}\rightarrow 0 \quad \text{as} \quad n\rightarrow\infty.$$

It follows from (2) that there exists N such that for all $n\geq N$, $f(2^{u(n)}, 2^{u(n+1)})\neq 0$.

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5. Jacobson rings. A ring R is said to be a *Jacobson ring* if every prime ideal is the intersection of primitive ideals. Goldie and Michler [**3**, Theorem 1.12] have shown that if α is an automorphism of a right noetherian Jacobson ring R then $R[x, x^{-1}, \alpha]$ is also a right noetherian Jacobson ring. Using the ideas of §2 we prove the following result, providing counterexamples to the converse of [**3**, Theorem 1.12].

THEOREM 5. *Let R be right noetherian. If α is rigid on R then $S = R[x, x^{-1}, \alpha]$ is a Jacobson ring.*

Proof. Let P be a prime ideal of S. Then by Proposition 1, $P \cap R$ is α-prime and $(P \cap R)S$ is an ideal of S. But α is rigid and hence $P = (P \cap R)S$. By Proposition 1 (ii),

$$\frac{S}{P} = \frac{S}{(P \cap R)S} \simeq \frac{R}{P \cap R}[x, x^{-1}, \alpha].$$

It follows by Proposition 2 that $J(P) = P$, i.e. that P is the intersection of primitive ideals. Thus S is a Jacobson ring.

EXAMPLE 7. Let $R = k(t)[[y]]$ be the power series ring in an indeterminate y over the field of rational functions in an indeterminate t over a field k of characteristic 0. Let α be the k-automorphism of R such that $\alpha(t) = 2t$ and $\alpha(y) = y$. Then for $n \geq 1$, $\alpha^n(t) - t = (2^n - 1)t$ is a unit and so, by Lemma 1 (ii), α is rigid on R. It follows from Theorem 4 that $R[x, x^{-1}, \alpha]$ is Jacobson. However, since R is a local domain, it is certainly not a Jacobson ring.

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