

Throughout the book, theory is motivated by many worked examples in the text, and there are many exercises provided for the reader to do. For example, before Morita equivalence is developed, there is a thorough discussion of the relation between the category of modules over a ring R and the category of modules over the ring of matrices $M_n(R)$. This approach makes the book well-suited for individual study. Also, the author's writing style makes the book easy to read. The reviewer strongly recommends this book for anyone with an interest in studying this area of algebra.

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DIMASSI, M. AND SJÖSTRAND, J. *Spectral asymptotics in the semi-classical limit* (LMS Lecture Note Series no. 268, Cambridge, 1999), xi + 227 pp., 0 521 66544 2 (paperback), \$24.95.

Semi-classical analysis is concerned with relationships between quantum mechanics and classical mechanics when Planck's constant h goes to zero. In the Schrödinger operator $-h\Delta + V$, describing the motion of an electron under the influence of an electric field with potential V , the main quantum mechanical objects of study are the wave functions (eigenfunctions) and energy levels (eigenvalues), while the classical ones are given by the trajectories of the associated classical Hamiltonian $p(x, \xi) = \xi^2 + V(x)$, that is, the integral curves of the corresponding Hamiltonian field $H_p = 2\xi \cdot (\partial/\partial x) - V'(x) \cdot (\partial/\partial \xi)$. In recent years considerable progress has been made in the understanding of many of the spectral problems within semi-classical analysis through the application of the results and techniques of microlocal analysis; a good example of this is the work of Hörmander, Duistermaat-Guillemin, Ivrii and others on the asymptotic distribution of eigenvalues of elliptic operators on compact manifolds and bounded domains, in which the error terms in the asymptotic formulae depend on the dynamical properties of the Hamiltonian flow. However, while microlocal analysis has had a profound influence on semi-classical analysis, the benefits have not flowed in one direction only, and the relationship between the two subjects is a symbiotic one. Microlocal analysis was originally motivated by problems in partial differential equations, but there are features which are reminiscent of quantum mechanics. For instance, the uncertainty principle has a vital role in both theories. In microlocal analysis the analogues of the objects of quantum mechanics are the pseudo-differential and Fourier integral operators, while the classical ones are those of symplectic geometry such as Poisson brackets and canonical transformations.

These lecture notes give an up-to-date account of recent developments in semi-classical analysis, many of them by the authors and their collaborators, and include necessary background material from microlocal analysis. The following selection of chapter headings gives the flavour: local symplectic geometry, the WKB method, the method of stationary phase, the tunnel effect and interaction matrix, h -pseudodifferential operators, trace class operators and applications of the functional calculus, spectral theory for perturbed periodic systems.

Researchers and graduate students in mathematical analysis are the intended readership, but anyone with an interest in the current state of the mathematics of quantum mechanics will get a lot out of this book. The pace of delivery is brisk, but this has made it possible for a lot of material to be compressed into a relatively short space.

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