

The Schwarzian equation for completely replicable functions

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ABSTRACT

We describe the Schwarzian equations for the 328 completely replicable functions with integral q -coefficients [Ford *et al.*, ‘More on replicable functions’, *Comm. Algebra* 22 (1994) no. 13, 5175–5193].

1. Introduction

We assume familiarity with [1, 3, 7, 13, 14].

In 1877, Dedekind [4] gave the Schwarzian equation

$$\{\lambda, z\} + \frac{\lambda^2 - \lambda + 1}{\lambda^2(1 - \lambda)^2} \lambda'^2 = 0,$$

for the principal modulus, λ , on $\Gamma(2)$. He even *defined* the j -function by its Schwarzian equation

$$\{j, z\} + \frac{36j^2 - 41j + 32}{72j^2(1 - j)^2} j'^2 = 0,$$

which loses its mystery when expanded as a partial fraction that describes its fundamental domain (see (6)).

It is useful to define:

- the arithmetic j -function

$$j_{\text{arith}}(z) = \frac{1}{q} + 744 + 196884q + \dots \quad \text{for } q = e^{2\pi iz}, z \in \mathfrak{H}^+;$$

- the monster j -function

$$j_{\text{monst}}(z) = j_{\text{arith}}(z) - 744;$$

and

- the analytic j -function

$$j_{\text{anal}}(z) = \frac{1}{1728} j_{\text{arith}}(z).$$

From the Schwarzian equation

$$\{f, z\} + R(f)f'^2 = 0,$$

we list $R(f)$ for all completely replicable f with q -coefficients in \mathbb{Z} .

2. Preliminaries: the Schwarz derivative

2.1. Definition and properties

The Schwarz derivative is a differential operator defined for a meromorphic function f on a domain D of \mathbb{C} by

$$\{f, z\} = 2 \left(\frac{f''}{f'} \right)' - \left(\frac{f''}{f'} \right)^2 = 2 \frac{f'''}{f'} - 3 \frac{f''^2}{f'^2},$$

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where $f' = (1/2\pi i)(df/dz)$. This was used by Poincaré in his 1882 paper *Mémoire sur les fonctions fuchsienues* [12]. It also appears in [5, 8, 10, 11].

We give some properties of the Schwarz derivative (see [9]).

- For all $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \text{GL}_2^+(\mathbb{C})$, $\{(af + b)/(cf + d), z\} = \{f, z\}$.
- $\{f, z\} = \{g, z\}$ if and only if each is a linear fraction of the other.
- Consequently, $\{f, z\} = 0$ if and only if $f = (az + b)/(cz + d)$ for some $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \text{GL}_2^+(\mathbb{C})$.
- It is a quadratic differential. If u is a function of z , then

$$\{f, z\} dz^2 = \{f, u\} du^2 + \{u, z\} dz^2.$$

- In particular, for a, b, c, d , as above, and $u = (az + b)/(cz + d)$,

$$\{f, u\} = \frac{(cz + d)^4}{ad - bc} \{f, z\}.$$

- Inversion formula. If $f'(z_0) \neq 0$ for some point z_0 , then, in a neighborhood of z_0 ,

$$\{z, f\} = -(dz/df)^2 \{f, z\}.$$

2.2. Link with differential equations

One of our motivations for this paper is the role played by the Schwarz derivative in differential equations. In fact, there is a close relationship between the Schwarz derivative and second-order linear differential equations that can be illustrated as follows. Let $R(z)$ be a meromorphic function on a domain D of \mathbb{C} . Then the (local) solutions of the second-order linear differential equation

$$y''(z) + R(z)y(z) = 0 \tag{1}$$

are two-dimensional vector spaces, and if y_1 and y_2 are two linearly independent solutions, then the quotient $f = y_1/y_2$ is a solution of the Schwarzian equation

$$\{z, f\} = -R(z). \tag{2}$$

Conversely, if f is a solution to (2), then $y_1 = f/\sqrt{f'}$, $y_2 = 1/\sqrt{f'}$ are two linearly independent solutions to (1).

Furthermore, if we set $u = f''/f'$, then from (1) we obtain the Riccati differential equation

$$2u' - u^2 = R(z).$$

3. Modular Schwarzian equations

In this section, we establish the Schwarzian equation that we used to compute $R(f)$.

3.1. Schwarz derivative of modular functions

In [10], it was shown that the Schwarz derivative can have an important role in the theory of modular forms as well. The following proposition is an example.

PROPOSITION 3.1 [10]. (i) *If f is a modular function for a discrete subgroup G (of the first kind) of $\text{PSL}_2(\mathbb{R})$, then $\{f, z\}$ is a weight four modular form for G .*

(ii) *If Γ is of genus zero and f_Γ is a principal modulus for Γ , then $\{f_\Gamma, z\}$ is a weight four modular form for the normalizer of Γ inside $\text{PSL}_2(\mathbb{R})$.*

The first statement is an immediate consequence of the chain rule, while the second statement is, however, less trivial.

One result that is of importance to us is the following proposition from [10].

PROPOSITION 3.2. *Let Γ and f_Γ be as in Proposition 3.1. Then $\{f, z\}$ has a pole of order two at elliptic points of Γ and is holomorphic everywhere including at cusps.*

The following is a corollary of this.

COROLLARY 3.3. *If Γ has no elliptic points, then $\{f, z\}$ is a holomorphic weight four modular form for Γ .*

REMARKS 3.1.

- Note that $\{z, f\}$ is a modular function. However, its invariance group is, in general, not larger than Γ .
- The maximum number of elliptic points for moonshine groups is at most 26 (including ∞).

3.2. Schwarzian equations

Let f be a modular function for a Fuchsian group Γ of first kind.

THEOREM 3.4 [6]. *If f is non-constant, then the (multi-valued) inverse function $z = z(f)$ can be expressed as the quotient of two (linearly independent) solutions of the differential equation*

$$\frac{d^2y}{df^2} + R(f)y = 0,$$

where R is an algebraic function of f . Moreover, if f has a single pole in the fundamental domain of Γ , then R is a rational function of f .

Now, from the results of § 2.2, it follows that

$$\{z, f\} = R(f), \tag{3}$$

and using the inversion formula,

$$\{f, z\} = -R(f)f'^2. \tag{4}$$

For Γ of genus zero, we deduce a similar type of equation involving the principal modulus for Γ . Indeed, let f_Γ denote the principal modulus for Γ . Then, on the one hand, using the chain rule property for the Schwarz derivative, we see that $\{f, z\}$ is a weight four modular form for Γ . On the other hand, an easy computation shows that f' is a weight two modular form for Γ , so that f'^2 is of weight two. Hence the quotient $\{f, z\}/f'^2$ is a modular function for Γ which is of genus zero, and therefore it is a rational function in f_Γ .

In other words, f satisfies the differential equation

$$\{f, z\} + R(f_\Gamma)f'^2 = 0,$$

where $R(f_\Gamma)$ is a rational function in f_Γ . In particular, for $f = f_\Gamma$, every principal modulus satisfies the Schwarzian equation

$$\{f_\Gamma, z\} + R(f_\Gamma)f_\Gamma'^2 = 0$$

for some rational function R in f_Γ . The inversion formula for the Schwarz derivative gives (3).

REMARK 3.1.

- The above reasoning provides a direct proof in the case of genus zero groups only. In positive genus, this argument fails, simply because the field of modular functions is no longer generated by a single function.
- We conclude this subsection by noticing that the degree of $R(f_\Gamma)$, that is, the maximum degree of the numerator and denominator, is at most 52 by Proposition 3.1 as we are counting both elliptic points and ∞ as poles. This bound also holds for all completely replicable functions with integer coefficients, as our computation shows.

3.3. A fundamental example

In the introduction, we gave the example of the analytic j -function. We show the connection to Picard–Fuchs equations. We have seen that the j -function satisfies the Schwarzian equation

$$\{f, z\} + \frac{36j^2 - 41j + 32}{36j^2(1 - j)^2} f'^2 = 0, \tag{5}$$

from which we deduce that

$$R(j) = \frac{36j^2 - 41j + 32}{36j^2(1 - j)^2},$$

which has a partial fraction

$$R(j) = \frac{1 - \frac{1}{3^2}}{j^2} + \frac{1 - \frac{1}{2^2}}{(1 - j)^2} + \frac{1 - \frac{1}{3^2} - \frac{1}{2^2}}{j(1 - j)}. \tag{6}$$

We see from this expression that the poles are located at the elliptic points $z = \rho$ and $z = i$.

Equation (5) has an equivalent formulation of hypergeometric type given by the Picard–Fuchs equation

$$j(1 - j) \frac{d^2y}{dj^2} + \left(\frac{2}{3} - \frac{7}{6}j\right) \frac{dy}{dj} - \frac{1}{144}j = 0.$$

This example appears in [8], where Schwarz equations are used to find solutions of certain Halphen-type differential equations.

Appendix

In this appendix we list all rational modular functions for the class of completely replicable functions with integer coefficients as it appears in [7]. There are 328 such functions including the functions $1/q + cq$ with $c \in \{0, \pm 1\}$.

For the three series of finite type $f(z) = 1/q + cq, c \in \{0, \pm 1\}$, the corresponding rational functions are

$$R_f(Y) = Y^2 - 8c.$$

This can be established either by using our algorithm or simply by noticing that $f' = f'''$, $f'' = f$ and $f'^2 = f^2 - 4c$.

Our process for computing these rational functions relies on the Schwarz differential equation. For a completely replicable function f given as a q -series with $R(f) = P(f)/Q(f)$ as its corresponding rational function, (4),

$$(2f'f''' - 3f''^2)Q(f) + P(f)f'^4 = 0.$$

We then use the q -expansion of f to get a system of equations which we solve with respect to the coefficients of the polynomials P and Q . For this purpose, one has to generate as many

coefficients as needed and, for this reason, we use the Norton basis in [7]. One could also use the recurrence relations in [1]. However, these relations involve the knowledge of the second replicate of each function. [N.B. $f = f^{(2)}$ for odd level function f .] This is to be found in [2] and in character tables computed in GAP.

To illustrate how the algorithm works, let us consider the following example of the monster j -function. The degree of $R(j)$, the rational function, is six because the modular group has two classes of elliptic points, namely, the classes of $z = i$ and $z = \rho$. Let $R(j) = P(j)/Q(j)$ be the corresponding rational function. Then

$$(2j'j''' - 3j''^2)Q(j) + P(j)j'^4 = 0.$$

If we write $P(Y) = \sum_{n=0}^6 c_n Y^n, Q(Y) = \sum_{m=0}^6 d_m Y^m$, using the q -expansion of the j -function we get a system of equations in the c_i and d_l . The first 30 coefficients of the j -function are enough to solve for the corresponding rational function. We get infinitely many solutions $c_0 = 1743552c_2, c_1 = -480c_2, c_2 = c_2, d_0 = 1071929106432c_2, d_1 = 702812160c_2, d_2 = -2813184c_2, d_3 = -960c_2, d_4 = 2c_2, c_3 = c_4 = c_5 = c_6 = d_5 = d_6 = 0$, and, after the substitution $Y = 1/Y$, we find $R(1/j)$

$$\frac{(1743552Y^2 - 480Y + 1)Y^2}{(744Y + 1)^2(984Y - 1)^2}.$$

Table A.1 lists $R(f)$ for f being a moonshine function while Table A.2 completes the list for the remaining completely replicable functions. We used the class names as in [7] to label each function. Also notice that, in Table A.1, following [3], we listed the McKay–Thompson series 25z, 49z and 50z even though these are not related to conjugacy classes of the Monster group.

TABLE A.1. $R(f)$ for 171 moonshine functions.

Class	$R(f)$
1A	$\frac{(1743552t^2 - 480t + 1)t^2}{(744t + 1)^2(984t - 1)^2}$
2A	$\frac{(40640t^2 - 96t + 1)t^2}{(104t + 1)^2(152t - 1)^2}$
2B	$\frac{(2752t^2 + 32t + 1)t^2}{(40t + 1)^2(24t - 1)^2}$
3A	$\frac{(7560t^2 - 48t + 1)t^2}{(42t + 1)^2(66t - 1)^2}$
3B	$\frac{(513t^2 + 6t + 1)t^2}{(15t + 1)^2(12t - 1)^2}$
3C	$\frac{(9936t^3 + 1)t^2}{(12t - 1)^2(144t^2 + 12t + 1)^2}$
4A	$\frac{(2752t^2 - 32t + 1)t^2}{(24t + 1)^2(40t - 1)^2}$
4B	$\frac{(49152t^4 + 320t^2 + 1)t^2}{(16t - 1)^2(16t + 1)^2}$

Table A.1. (Continued.)

Class	$R(f)$
4C	$\frac{(1 + 192t^2)t^2}{(8t - 1)^2(8t + 1)^2}$
4D	$\frac{(4096t^4 - 64t + 1)t^2}{(64t^2 + 1)^2}$
5A	$\frac{(359872t^4 + 38096t^3 + 848t^2 + 8t + 1)t^2}{(16t + 1)^2(464t^2 + 12t - 1)^2}$
5B	$\frac{(2497t^4 + 1096t^3 + 98t^2 + 8t + 1)t^2}{(6t - 1)^2(29t^2 + 10t + 1)^2}$
6A	$\frac{(120608t + 16352t + 492t + 4t + 1)t^2}{(10t + 1)^2(14t + 1)^2(22t - 1)^2}$
6B	$\frac{(117841t^4 + 16804t^3 + 486t^2 + 4t + 1)t^2}{(12t + 1)^2(263t^2 + 10t - 1)^2}$
6C	$\frac{(2848t^4 - 800t^3 + 108t^2 + 4t + 1)t^2}{(10t + 1)^2(6t - 1)^2(2t - 1)^2}$
6D	$\frac{(1457t^4 + 1124t^3 + 6t^2 + 4t + 1)t^2}{(4t - 1)^2(41t^2 + 6t + 1)^2}$
6E	$\frac{(769t^4 + 388t^3 + 54t^2 + 4t + 1)t^2}{(3t + 1)^2(4t + 1)^2(5t - 1)^2}$
6F	$\frac{(4096t^6 - 304t^3 + 1)t^2}{(4t + 1)^2(16t^2 - 4t + 1)^2}$
7A	$\frac{(50328t^4 + 8904t^3 + 313t^2 + 2t + 1)t^2}{(10t + 1)^2(9t + 1)^2(18t - 1)^2}$
7B	$\frac{(201t^4 + 378t^3 + 19t^2 + 2t + 1)t^2}{(4t - 1)^2(13t^2 + 5t + 1)^2}$
8A	$\frac{(22784t^4 + 5120t^3 + 224t^2 + 1)t^2}{(8t + 1)^2(112t^2 + 8t - 1)^2}$
8B	$\frac{(64t^2 - 8t + 1)(64t^2 + 8t + 1)t^2}{(8t - 1)^2(8t + 1)^2}$
8C	$\frac{(2816t^4 + 1)t^2}{(4t - 1)^2(4t + 1)^2(16t^2 + 1)^2}$
8D	$\frac{(4t - 1)^2(4t + 1)^2t^2}{(16t^2 + 1)^2}$
8E	$\frac{(16t^2 + 1)^2t^2}{(4t - 1)^2(4t + 1)^2}$
8F	$\frac{(4096t^8 - 640t^4 + 1)t^2}{(8t^2 - 4t + 1)^2(8t^2 + 4t + 1)^2}$
9A	$\frac{t^2(1 + 14337t^4 + 3456t^3 + 162t^2)}{(6t + 1)^2(99t^2 + 6t - 1)^2}$
9B	$\frac{(6t + 1)(36t^2 - 6t + 1)t^2}{(3t - 1)^2(9t^2 + 3t + 1)^2}$

Table A.1. (Continued.)

Class	$R(f)$
10A	$\frac{(10560t^4 + 2256t^3 + 128t^2 + 1)t^2}{(12t - 1)^2(8t + 1)^2(4t + 1)^2}$
10B	$\frac{(3072t^6 - 2304t^5 + 1088t^4 - 304t^3 + 32t^2 + 1)t^2}{(4t - 1)^2(16t^2 - 4t - 1)^2}$
10C	$\frac{(37t^2 - 8t + 1)(5t^2 + 8t + 1)t^2}{(2t - 1)^2(17t^2 + 2t + 1)^2}$
10D	$\frac{(244897t^6 + 159706t^5 + 37683t^4 + 3716t^3 + 147t^2 + 10t + 1)t^2}{(5t + 1)^2(6t + 1)^2(71t^2 + 6t - 1)^2}$
10E	$\frac{(t^2 + 1)(13t^2 + 6t + 1)(61t^4 + 62t^3 + 14t^2 - 2t + 1)t^2}{(3t - 1)^2(2t + 1)^2(t + 1)^2(5t^2 + 2t + 1)^2}$
11A	$\frac{(115884t^6 + 86896t^5 + 23424t^4 + 2620t^3 + 112t^2 + 8t + 1)t^2}{(6t + 1)^2(212t^3 + 76t^2 + 2t - 1)^2}$
12A	$\frac{(2848t^4 + 800t^3 + 108t^2 - 4t + 1)t^2}{(2t + 1)^2(6t + 1)^2(10t - 1)^2}$
12B	$\frac{(769t^4 - 388t^3 + 54t^2 - 4t + 1)t^2}{(5t + 1)^2(4t - 1)^2(3t - 1)^2}$
12C	$\frac{(10368t^6 + 336t^4 + 32t^2 + 1)t^2}{(6t - 1)^2(2t + 1)^2(2t - 1)^2(6t + 1)^2}$
12D	$\frac{(4096t^6 + 304t^3 + 1)t^2}{(4t - 1)^2(16t^2 + 4t + 1)^2}$
12E	$\frac{(1152t^6 + 720t^4 + 1)t^2}{(2t + 1)^2(2t - 1)^2(12t^2 + 1)^2}$
12F	$\frac{(t^8 + 28t^6 + 1350t^4 + 28t^2 + 1)t^2}{(t^2 + 6t + 1)^2(t^2 - 6t + 1)^2}$
12G	$\frac{(6561t^8 - 324t^6 - 474t^4 - 4t^2 + 1)t^2}{(9t^2 + 2t + 1)^2(9t^2 - 2t + 1)^2}$
12H	$\frac{(54441t^6 + 47142t^5 + 14751t^4 + 1908t^3 + 87t^2 + 6t + 1)t^2}{(5t + 1)^2(4t + 1)^2(39t^2 + 6t - 1)^2}$
12I	$\frac{(3t^2 + 1)^4 t^2}{(t - 1)^2(3t - 1)^2(3t + 1)^2(t + 1)^2}$
12J	$\frac{(4096t^{12} - 1600t^6 + 1)t^2}{(4t^2 + 1)^2(16t^4 - 4t^2 + 1)^2}$
13A	$\frac{(41472t^6 + 36048t^5 + 11440t^4 + 1512t^3 + 73t^2 + 6t + 1)t^2}{(4t + 1)^2(3t + 1)^2(48t^2 + 4t - 1)^2}$
13B	$-\frac{(-1 + 447t^8 - 3342t^7 - 1345t^6 - 1302t^5 - 224t^4 - 74t^3 - t^2 - 2t)}{(2t - 1)^2(7t^2 + t + 1)^2(5t^2 + 2t + 1)^2}$
14A	$\frac{(17688t^6 + 18888t^5 + 7009t^4 + 1036t^3 + 62t^2 + 4t + 1)t^2}{(2t + 1)^2(6t + 1)^2(23t^2 + 6t - 1)^2}$
14B	$-\frac{(-1 + 1832t^8 - 1544t^7 - 2385t^6 + 2802t^5 - 1063t^4 + 180t^3 - 15t^2 + 2t)t^2}{(5t + 1)^2(3t - 1)^2(2t - 1)^2(2t^2 - t + 1)^2}$

Table A.1. (Continued.)

Class	$R(f)$
14C	$\frac{(285777t^8 + 454332t^7 + 293690t^6 + 98208t^5 + 17779t^4 + 1696t^3 + 106t^2 + 12t + 1)t^2}{(4t + 1)^2(391t^4 + 278t^3 + 53t^2 - 2t - 1)^2}$
15A	$\frac{(10540t^6 + 15628t^5 + 6841t^4 + 1040t^3 + 46t^2 + 4t + 1)t^2}{(8t - 1)^2(4t + 1)^2(13t^2 + 6t + 1)^2}$
15B	$-\frac{(-1 + 1175t^8 - 120t^7 - 1078t^6 - 488t^5 + 357t^4 - 152t^3 + 2t^2)t^2}{(2t - 1)^2(25t^4 - 10t^3 + 11t^2 + 2t + 1)^2}$
15C	$\frac{(125800t^8 + 220760t^7 + 158768t^6 + 59408t^5 + 12073t^4 + 1282t^3 + 83t^2 + 10t + 1)t^2}{(3t + 1)^2(25t^2 + 5t - 1)^2(11t^2 + 7t + 1)^2}$
15D	$\frac{(15625t^{12} - 8000t^9 - 1750t^6 - 64t^3 + 1)t^2}{(5t^2 - 2t + 1)^2(25t^4 + 10t^3 - t^2 + 2t + 1)^2}$
16A	$\frac{(256t^8 + 256t^6 + 608t^4 + 16t^2 + 1)t^2}{(4t^2 + 4t - 1)^2(4t^2 - 4t - 1)^2}$
16B	$\frac{(16t^4 - 16t^3 + 8t^2 + 4t + 1)(16t^4 + 16t^3 + 8t^2 - 4t + 1)t^2}{(2t - 1)^2(2t + 1)^2(4t^2 + 1)^2}$
16C	$\frac{(12864t^6 + 12864t^5 + 4912t^4 + 800t^3 + 44t^2 + 4t + 1)t^2}{(4t + 1)^2(2t + 1)^2(28t^2 + 4t - 1)^2}$
17A	$\frac{(41200t^8 + 84016t^7 + 70840t^6 + 30984t^5 + 7265t^4 + 864t^3 + 58t^2 + 8t + 1)t^2}{(2t + 1)^2(212t^4 + 176t^3 + 39t^2 - 2t - 1)^2}$
18A	$\frac{(2024t^8 + 3936t^7 - 176t^6 + 896t^5 - 156t^4 + 40t^3 - 4t^2 + 1)t^2}{(t - 1)^2(8t^2 + 1)^2(7t^2 + t + 1)^2}$
18B	$\frac{(3617t^6 + 4978t^5 + 2563t^4 + 468t^3 + 35t^2 + 2t + 1)t^2}{(5t + 1)^2(2t + 1)^2(7t - 1)^2(t + 1)^2}$
18C	$\frac{(1 - 2t + 6t^2 - 50t^3 + 85t^4)^2t^2}{(3t - 1)^2(2t - 1)^2(t + 1)^2(11t^2 - 2t - 1)^2}$
18D	$\frac{(2t^2 - 2t - 1)^2(4t^4 + 4t^3 + 6t^2 - 2t + 1)^2t^2}{(2t - 1)^2(t + 1)^2(t^2 - t + 1)^2(4t^2 + 2t + 1)^2}$
18E	$\frac{(30472t^8 + 68192t^7 + 60688t^6 + 27392t^5 + 6532t^4 + 776t^3 + 52t^2 + 8t + 1)t^2}{(3t + 1)^2(7t^2 + 5t + 1)^2(20t^2 + 4t - 1)^2}$
19A	$\frac{t^2(1 + 11340t^8 + 29064t^7 + 29212t^6 + 14908t^5 + 4120t^4 + 580t^3 + 41t^2 + 6t)}{2(4t + 1)^2(3t + 1)^2(12t^3 + 16t^2 + 4t - 1)^2}$
20A	$\frac{t^2(1 + 3072t^6 + 2304t^5 + 1088t^4 + 304t^3 + 32t^2)}{(4t + 1)^2(16t^2 + 4t - 1)^2}$
20B	$\frac{t^2(1 + 3072t^8 - 704t^6 + 848t^4 + 8t^2)}{(4t - 1)^2(4t + 1)^2(4t^2 + 1)^2}$
20C	$\frac{t^2(t^2 + 1)(13t^2 - 6t + 1)(61t^4 - 62t^3 + 14t^2 + 2t + 1)}{(t - 1)^2(2t - 1)^2(3t + 1)^2(5t^2 - 2t + 1)^2}$
20D	$\frac{t^2(1 + 4096t^{12} - 1024t^8 - 576t^6 - 64t^4)}{(4t^2 + 1)^2(16t^4 + 12t^2 + 1)^2}$
20E	$\frac{t^2(1 + t^{12} + 10t^{10} + 351t^8 + 44t^6 + 351t^4 + 10t^2)}{(t - 1)^2(t + 1)(t^2 + 4t^2 - 1)^2(t^2 - 4t - 1)^2}$

Table A.1. (Continued.)

Class	$R(f)$
20F	$\frac{t^2(1 + 48337t^{10} + 161530t^9 + 237105t^8 + 198000t^7 + 101870t^6 + 32700t^5 + 6350t^4 + 720t^3 + 65t^2 + 10t)}{(2t + 1)^2(3t + 1)^2(71t^4 + 80t^3 + 26t^2 - 1)^2}$
21A	$\frac{t^2(1 + 11340t^6 + 5916t^5 + 1705t^4 + 352t^3 + 30t^2 + 4t)}{(4t + 1)^2(27t^2 + 2t - 1)^2}$
21B	$\frac{t^2(1 + 504t^8 + 792t^7 + 576t^6 + 576t^5 + 9t^4 - 54t^3 - 5t^2 + 2t)}{(t - 1)^2(7t^2 - t + 1)^2(3t^2 + 3t + 1)^2}$
21C	$-\frac{t^2(3672t^9 - 1729t^6 - 110t^3 - 1)}{(3t - 1)^2(t + 1)^2(t - 1)^2(9t^2 + 3t + 1)^2}$
21D	$\frac{t^2(1 + 39321t^{10} + 130338t^9 + 190143t^8 + 158364t^7 + 81681t^6 + 26478t^5 + 5257t^4 + 628t^3 + 63t^2 + 10t)}{(2t + 1)^2(t + 1)^2(111t^4 + 114t^3 + 29t^2 - 2t - 1)^2}$
22A	$\frac{t^2(1 + 624t^8 + 5712t^7 + 9548t^6 + 6480t^5 + 2144t^4 + 332t^3 + 28t^2 + 4t)}{(6t - 1)^2(2t + 1)^2(4t^3 + 8t^2 + 6t + 1)^2}$
22B	$\frac{-t^2(-1 + 1280t^{12} - 3840t^{11} + 2240t^{10} + 3200t^9 - 8656t^8 + 9872t^7 - 6636t^6 + 2816t^5 - 736t^4 + 100t^3 - 12t^2 + 4t)}{(2t - 1)^2(4t^3 - 4t^2 + 2t - 1)^2(8t^3 - 4t^2 + 2t + 1)^2}$
23A	$\frac{t^2(1 + 8277t^{10} + 39090t^9 + 72287t^8 + 72694t^7 + 44173t^6 + 16546t^5 + 3686t^4 + 462t^3 + 44t^2 + 8t)}{(25t^3 + 17t^2 + 2t - 1)^2(7t^3 + 11t^2 + 6t + 1)^2}$
24A	$\frac{t^2(64t^4 - 8t^2 + 1)(1 + 8t^2)^2}{(4t + 1)^2(2t + 1)^2(2t - 1)^2(4t - 1)^2}$
24B	$\frac{t^2(1 + 5120t^8 + 10240t^7 + 10240t^6 + 6400t^5 + 2288t^4 + 368t^3 + 20t^2 + 4t)}{(2t + 1)^2(8t^2 + 4t + 1)^2(8t^2 + 4t - 1)^2}$
24C	$\frac{t^2(113t^8 - 248t^7 + 268t^6 - 248t^5 + 230t^4 - 136t^3 + 44t^2 - 8t + 1)(-1 - 2t + 5t^2)^2}{(t - 1)^2(2t - 1)^2(t + 1)^2(17t^4 - 16t^3 + 2t^2 + 1)^2}$
24D	$\frac{t^2(64t^6 + 48t^4 + 12t^2 - 1)(64t^6 - 48t^4 - 12t^2 - 1)}{(4t^2 + 1)^2(4t^2 - 2t + 1)^2(4t^2 + 2t + 1)^2}$
24E	$\frac{t^2(4096t^{12} + 1600t^6 + 1)}{(2t - 1)^2(2t + 1)^2(4t^2 + 2t + 1)^2(4t^2 - 2t + 1)^2}$
24F	$\frac{t^2(t^{16} + 316t^{12} + 1926t^8 + 316t^4 + 1)}{(t^2 + 2t - 1)^2(t^2 - 2t - 1)^2(t^4 + 6t^2 + 1)^2}$
24G	$\frac{t^2(6561t^{16} - 8100t^{12} - 2970t^8 - 100t^4 + 1)}{(3t^2 + 2t + 1)^2(3t^2 - 2t + 1)^2(9t^4 - 2t^2 + 1)^2}$
24H	$\frac{t^2(1 + t^{12} + 6t^{10} + 399t^8 - 236t^6 + 399t^4 + 6t^2)}{(t^2 + 1)^2(t^2 - 4t + 1)^2(t^2 + 4t + 1)^2}$
24I	$\frac{t^2(1 + 28513t^{12} + 151764t^{11} + 355470t^{10} + 482940t^9 + 422799t^8 + 249672t^7 + 100676t^6 + 27384t^5 + 4911t^4 + 612t^3 + 78t^2 + 12t)}{(2t + 1)^2(3t + 1)^2(t + 1)^2(23t^4 + 48t^3 + 22t^2 - 1)^2}$
24J	$\frac{t^2(4096t^{24} - 6784t^{12} + 1)}{(2t^2 - 2t + 1)^2(2t^2 + 2t + 1)^2(4t^4 + 4t^3 + 2t^2 + 2t + 1)^2(4t^4 - 4t^3 + 2t^2 - 2t + 1)^2}$
25A	$\frac{t^2(6t^2 + 3t + 1)(352t^6 + 852t^5 + 758t^4 + 223t^3 + 13t^2 + t + 1)}{(t + 1)^2(t^2 + 3t + 1)^2(19t^2 + 2t - 1)^2}$

Table A.1. (Continued.)

Class	$R(f)$
26A	$\frac{t^2(1 + 3840t^8 + 7376t^7 + 6624t^6 + 3888t^5 + 1345t^4 + 228t^3 + 22t^2 + 4t)}{(4t + 1)^2(16t^3 + 15t^2 + 2t - 1)^2}$
26B	$\frac{(t^2(1 + 83097t^{14} + 455190t^{13} + 1192717t^{12} + 1963740t^{11} + 2241876t^{10} + 1851100t^9 + 1122882t^8 + 500052t^7 + 161298t^6 + 36852t^5 + 5908t^4 + 740t^3 + 101t^2 + 14t))}{\sqrt{(2t + 1)^2(t + 1)^2(191t^6 + 384t^5 + 308t^4 + 114t^3 + 12t^2 - 4t - 1)^2}}$
27A; 27B	$\frac{t^2(1 + 5832t^9 + 17496t^8 + 23328t^7 + 17577t^6 + 7938t^5 + 2079t^4 + 288t^3 + 27t^2 + 6t)}{(3t + 1)^2(9t^3 + 9t^2 + 3t - 1)^2(3t^2 + 3t + 1)^2}$
28A	$\frac{t^2(1 + 31752t^{10} - 19583t^8 + 3820t^6 - 250t^4 + 4t^2)}{(2t + 1)^2(2t - 1)^2(7t^2 + 2t - 1)^2(7t^2 - 2t - 1)^2}$
28B	$\frac{-t^2(-1 + 1832t^8 + 1544t^7 - 2385t^6 - 2802t^5 - 1063t^4 - 180t^3 - 15t^2 - 2t)}{(2t + 1)^2(3t + 1)^2(5t - 1)^2(2t^2 + t + 1)^2}$
28C	$\frac{t^2(3t^2 + 1)(280t^{12} + 1715t^{10} + 1941t^8 + 94t^6 + 66t^4 - t^2 + 1)}{(t - 1)^2(t + 1)^2(7t^2 + 1)^2(2t^2 - t + 1)^2(2t^2 + t + 1)^2}$
28D	$\frac{t^2(1 + t^{16} + 4t^{14} + 170t^{12} + 80t^{10} - 909t^8 + 80t^6 + 170t^4 + 4t^2)}{(t^4 + 2t^3 - 5t^2 + 2t + 1)^2(t^4 - 2t^3 - 5t^2 - 2t + 1)^2}$
29A	$\frac{t^2(1 + 624t^{12} + 7248t^{11} + 28472t^{10} + 53152t^9 + 59625t^8 + 45520t^7 + 24526t^6 + 9000t^5 + 2091t^4 + 296t^3 + 38t^2 + 8t)}{(2t + 1)^2(4t^6 + 32t^5 + 83t^4 + 66t^3 + 17t^2 - 2t - 1)^2}$
30A	$\frac{t^2(1 + 368536t^{12} - 708784t^{11} + 633016t^{10} - 318416t^9 + 61972t^8 + 23264t^7 - 15664t^6 + 2432t^5 + 289t^4 - 158t^3 + 43t^2 - 10t)}{(3t - 1)^2(t^2 - 3t + 1)^2(11t^2 - t - 1)^2(8t^2 - 1)^2}$
30B	$\frac{t^2(1 + 2700t^8 - 2340t^7 + 2613t^6 + 1386t^5 + 311t^4 + 112t^3 + 15t^2 + 2t)}{(t - 1)^2(3t + 1)^2(4t + 1)^2(5t - 1)^2}$
30C	$\frac{t^2(1 + 2700t^{12} - 5580t^{11} + 7077t^{10} - 2658t^9 + 4169t^8 - 3368t^7 + 2710t^6 - 1244t^5 + 362t^4 - 76t^3 + 5t^2 - 2t)}{(t - 1)^2(3t + 1)^2(4t^2 - t + 1)^2(5t^2 - 2t + 1)^2}$
30D	$\frac{t^2(1 + 46521t^{12} + 155646t^{11} + 231861t^{10} + 209214t^9 + 135662t^8 + 71734t^7 + 31681t^6 + 10642t^5 + 2414t^4 + 362t^3 + 53t^2 + 10t)}{(2t + 1)^2(7t^2 + 5t + 1)^2(9t^2 + 3t - 1)^2(t^2 - t - 1)^2}$
30E	$\frac{t^2(t^{18} + 70t^{15} + 291t^{12} + 1004t^9 + 291t^6 + 70t^3 + 1)}{(t - 1)^2(t^2 + t + 1)^2(t^2 - 3t + 1)^2(t^4 + 3t^3 + 8t^2 + 3t + 1)^2}$
30F	$\frac{-t^2(-1 + 40t^{12} + 240t^{11} - 1080t^{10} - 7600t^9 - 17172t^8 - 20448t^7 - 14360t^6 - 6072t^5 - 1497t^4 - 210t^3 - 27t^2 - 6t)}{(t + 1)^2(t^2 + t + 1)^2(5t^2 + 5t + 1)^2(4t^2 + 4t - 1)^2}$
30G	$\frac{(t^2(1 + 1024t^{20} - 5120t^{19} + 12032t^{18} - 22784t^{17} + 54592t^{16} - 132736t^{15} + 238528t^{14} - 293696t^{13} + 249824t^{12} - 159232t^{11} + 99968t^{10} - 79616t^9 + 62456t^8 - 36712t^7 + 14908t^6 - 4148t^5 + 853t^4 - 178t^3 + 47t^2 - 10t))}{\sqrt{((t - 1)^2(2t - 1)^2(4t^2 - 2t - 1)^2(t^2 + t - 1)^2(4t^4 - 6t^3 + 5t^2 - 3t + 1)^2)}}$
31A	$\frac{t^2(1 + 648t^{12} + 3864t^{11} + 10189t^{10} + 22122t^9 + 35047t^8 + 33086t^7 + 18981t^6 + 6930t^5 + 1622t^4 + 246t^3 + 36t^2 + 8t)}{(27t^3 + 17t^2 - 1)^2(t^3 + 3t^2 + 4t + 1)^2}$
32A	$\frac{-t^2(-1 + 2496t^{12} - 768t^{11} - 25664t^{10} - 60800t^9 - 74096t^8 - 56576t^7 - 28640t^6 - 9600t^5 - 2044t^4 - 272t^3 - 36t^2 - 8t)}{(2t + 1)^2(2t^2 + 2t + 1)^2(28t^4 + 32t^3 + 12t^2 - 1)^2}$
32B	$\frac{t^2(16t^8 + 56t^4 + 1)(1 + 2t^2)^2}{(2t^2 - 1)^2(2t^2 + 4t + 1)^2(2t^2 - 4t + 1)^2}$

Table A.1. (Continued.)

Class	$R(f)$
33A	$-t^2(-1 + 1584t^{16} - 4312t^{15} + 5368t^{14} - 10912t^{13} + 11295t^{12} - 13318t^{11} + 9919t^{10} - 9038t^9 + 5586t^8 - 3966t^7 + 1843t^6 - 790t^5 + 214t^4 - 30t^3 - 5t^2 + 2t) / ((t-1)^2(3t^2 - t + 1)^2(11t^6 - 11t^5 + 20t^4 - 3t^3 + 8t^2 + t + 1)^2)$
33B	$\frac{t^2(1 + 1452t^{10} + 5104t^9 + 9200t^8 + 8092t^7 + 4976t^6 + 2756t^5 + 1081t^4 + 188t^3 + 14t^2 + 4t)}{(11t^2 + 2t - 1)^2(4t^3 + 8t^2 + 4t + 1)^2}$
34A	$\frac{t^2(1 + 432t^{10} - 912t^9 - 664t^8 + 752t^7 + 1417t^6 + 898t^5 + 411t^4 + 100t^3 + 11t^2 + 2t)}{(2t + 1)^2(t + 1)^2(2t^2 - 3t - 1)^2(2t^2 - 5t + 1)^2}$
35A	$\frac{-t^2(-1 + 6900t^{10} + 3360t^9 - 12112t^8 - 15964t^7 - 11632t^6 - 4780t^5 - 1273t^4 - 180t^3 - 14t^2 - 4t)}{(2t + 1)^2(5t^2 + 2t + 1)^2(20t^3 + 4t^2 + 2t - 1)^2}$
35B	$t^2(1 + 1224t^{16} + 9048t^{15} + 33272t^{14} + 83320t^{14} + 160073t^{12} + 243226t^{11} + 290017t^{10} + 267514t^9 + 188542t^8 + 100130t^7 + 39221t^6 + 10970t^5 + 2130t^4 + 314t^3 + 53t^2 + 10t) / ((t+1)^2(t^2 + 3t + 1)^2(19t^6 + 51t^5 + 62t^4 + 39t^3 + 10t^2 - t - 1)^2)$
36A	$\frac{t^2(1 + 2t + 6t^2 + 50t^3 + 85t^4)^2}{(t - 1)^2(2t + 1)^2(3t + 1)^2(11t^2 + 2t - 1)^2}$
36B	$\frac{t^2(2t^2 + 2t - 1)^2(4t^4 - 4t^3 + 6t^2 + 2t + 1)^2}{(t - 1)^2(2t + 1)^2(t^2 + t + 1)^2(4t^2 - 2t + 1)^2}$
36C	$\frac{t^2(1 + 729t^{12} + 162t^{10} + 2511t^8 - 612t^6 + 279t^4 + 2t^2)}{(t - 1)^2(3t - 1)^2(3t + 1)^2(t + 1)^2(3t^2 + 1)^2}$
36D	$(t^2(1 + 5184t^{16} + 41472t^{15} + 155520t^{14} + 362880t^{13} + 591840t^{12} + 720576t^{11} + 682560t^{10} + 514080t^9 + 308376t^8 + 144864t^7 + 51696t^6 + 13536t^5 + 2580t^4 + 408t^3 + 72t^2 + 12t)) / ((2t+1)^2(t+1)^2(3t^2 + 3t + 1)^2(12t^4 + 24t^3 + 12t^2 - 1)^2)$
38A	$\frac{t^2(1 + 768t^{12} + 2464t^{11} + 3308t^{10} + 3328t^9 + 3976t^8 + 5260t^7 + 4532t^6 + 2440t^5 + 769t^4 + 112t^3 + 14t^2 + 4t)}{(4t^3 + 4t^2 + 4t + 1)^2(8t^3 + 7t^2 + 2t - 1)^2}$
39A	$\frac{t^2(1 + 648t^{10} - 3048t^9 + 5608t^8 - 4104t^7 + 161t^6 + 792t^5 + 42t^3 + 8t^2)}{(t - 1)^2(3t + 1)^2(3t^2 - 5t + 1)^2(t^2 - 3t - 1)^2}$
39B	$\frac{-t^2(4104t^{15} + 2519t^{12} - 828t^9 - 1006t^6 - 36t^3 - 1)}{(t + 1)^2(t^2 - t + 1)^2(3t^2 + t - 1)^2(9t^4 - 3t^3 + 4t^2 + t + 1)^2}$
39C	$(-t^2(-1 + 180t^{18} + 1620t^{17} + 6435t^{16} + 14760t^{15} + 19146t^{14} + 4242t^{13} - 40861t^{12} - 106092t^{11} - 158719t^{10} - 166144t^9 - 126450t^8 - 70044t^7 - 27877t^6 - 7836t^5 - 1567t^4 - 260t^3 - 51t^2 - 10t)) / ((2t+1)^2(t+1)^2(3t^4 + 6t^3 + 8t^2 + 5t + 1)^2(t^4 + 2t^3 + 4t^2 + 3t - 1)^2)$
40A	$\frac{-t^2(7424t^{12} - 3344t^8 - 104t^4 - 1)}{(2t + 1)^2(2t - 1)^2(4t^2 + 1)^2(2t^2 + 2t + 1)^2(2t^2 - 2t + 1)^2}$
40B	$\frac{t^2(1 + 4096t^{12} - 1024t^8 + 576t^6 - 64t^4)}{(2t + 1)^2(2t - 1)^2(4t^2 - 2t - 1)^2(4t^2 + 2t - 1)^2}$
40C	$\frac{t^2(t^{16} + 4t^{14} + 180t^{12} + 636t^{10} + 1174t^8 + 636t^6 + 180t^4 + 4t^2 + 1)(t - 1)^2(t + 1)^2}{(t^2 + 1)^2(t^4 + 2t^3 - 2t^2 + 2t + 1)^2(t^4 - 2t^3 - 2t^2 - 2t + 1)^2}$
41A	$(t^2(1 + 192t^{16} - 624t^{15} + 1184t^{14} + 848t^{13} - 8839t^{12} - 8504t^{11} + 11872t^{10} + 28440t^9 + 27584t^8 + 18616t^7 + 9914t^6 + 3880t^5 + 992t^4 + 168t^3 + 32t^2 + 8t)) / (-1 - 4t + 8t^2 + 66t^3 + 120t^4 + 56t^5 - 53t^6 - 36t^7 + 16t^8)^2)$

Table A.1. (Continued.)

Class	$R(f)$
42A	$\frac{t^2(1+1728t^{10}-288t^9+1020t^8+3108t^7+781t^6+122t^5+343t^4+88t^3+7t^2+2t)}{(t-1)^2(3t+1)^2(4t-1)^2(4t^2+3t+1)^2}$
42B	$\frac{(t^2(1+58257t^{16}-183726t^{15}+117675t^{14}+277146t^{13}-576411t^{12}+407916t^{11}-39382t^{10}-131528t^9+97282t^8-31616t^7+4730t^6-260t^5+133t^4-106t^3+43t^2-10t))}{\sqrt{((2t-1)^2(5t^2-t-1)^2(11t^4-5t^3-2t^2-t+1)^2(3t^2-3t+1)^2)}}$
42C	$\frac{t^2(4096t^{24}-13312t^{21}-33216t^{18}-35520t^{15}-18368t^{12}-4440t^9-519t^6-26t^3+1)}{(2t+1)^2(t+1)^2(t^2-t+1)^2(2t^2-t+1)^2(4t^2-2t+1)^2(4t^4+2t^3-t^2+t+1)^2}$
42D	$\frac{(-t^2(-1+72t^{16}+576t^{15}-288t^{14}-12096t^{13}-46788t^{12}-97272t^{11}-132692t^{10}-128728t^9-92701t^8-50668t^7-21074t^6-6508t^5-1411t^4-212t^3-34t^2-8t))}{\sqrt{((t+1)^2(3t^2+3t-1)^2(8t^4+16t^3+13t^2+5t+1)^2(t^2+t+1)^2)}}$
44A	$\frac{-t^2(-1+1280t^{12}+3840t^{11}+2240t^{10}-3200t^9-8656t^8-9872t^7-6636t^6-2816t^5-736t^4-100t^3-12t^2-4t)}{(2t+1)^2(8t^3+4t^2+2t-1)^2(4t^3+4t^2+2t+1)^2}$
45A	$\frac{t^2(1+592t^{12}-8t^{11}-3776t^{10}-7232t^9-3543t^8+1524t^7+2850t^6+1416t^5+363t^4+64t^3+10t^2+4t)}{(t+1)^2(t^2+3t+1)^2(t^2-t+1)^2(11t^2+t-1)^2}$
46A	$\frac{(t^2(1-6t+23t^2-94t^3+431t^4-1834t^5+6210t^6-16564t^7+35555t^8-63012t^9+96486t^{10}-132532t^{11}+171542t^{12}-206124t^{13}+221548t^{14}-189264t^{15}+106711t^{16}+1034t^{17}-77957t^{18}+96226t^{19}-66669t^{20}+28278t^{21}-6694t^{22}+564t^{23}+237t^{24}))}{\sqrt{((t-1)^2(7t^3-t^2+t+1)^2(t^3-3t^2+2t-1)^2(t^6+2t^5+2t^4-t^3+4t^2-t+1)^2)}}$
46C	$\frac{t^2(1+45t^{14}+414t^{13}-863t^{12}-3378t^{11}+1612t^{10}+2520t^9-609t^8+484t^7+1837t^6+1174t^5+359t^4+62t^3+10t^2+4t)}{(7t^2+2t-1)^2(t^3+t^2+2t+1)^2(t^3-3t^2+2t+1)^2}$
47A	$\frac{(-t^2(-1+708t^{18}+648t^{17}-6488t^{16}-27640t^{15}-66397t^{14}-120520t^{13}-177346t^{12}-213722t^{11}-209397t^{10}-165234t^9-103965t^8-51414t^7-19584t^6-5570t^5-1158t^4-188t^3-36t^2-8t))}{\sqrt{((19t^5+24t^4+20t^3+5t^2-1)^2(t^5+4t^4+8t^3+7t^2+4t+1)^2)}}$
48A	$\frac{t^2(1+256t^{16}+1024t^{12}+2560t^{10}-2592t^8+640t^6+64t^4)}{(2t^2+2t+1)^2(2t^2-2t+1)^2(2t^2+2t-1)^2(2t^2-2t-1)^2}$
50A	$\frac{t^2(1+2280t^{12}+1536t^{11}-4096t^{10}-3040t^9+2380t^8+2840t^7+1124t^6+496t^5+245t^4+60t^3+10t^2+4t)}{(t-1)^2(3t+1)^2(t^2-t-1)^2(8t^3+8t^2+t-1)^2}$
51A	$\frac{-t^2(-1+116t^{14}+1536t^{13}+4288t^{12}+2628t^{11}-2024t^{10}-5212t^9-5905t^8-4388t^7-2598t^6-1308t^5-499t^4-92t^3-10t^2-4t)}{(2t+1)^2(4t^3+4t^2+2t-1)^2(t^4-2t^3+3t^2+2t+1)^2}$
52A	$\frac{t^2(1+3072t^{16}+320t^{14}+336t^{12}+5832t^{10}+5089t^8+1644t^6+214t^4+4t^2)}{(4t^2+1)^2(4t^3+t^2+2t+1)^2(4t^3-t^2+2t-1)^2}$
52B	$\frac{t^2(t^{28}-2t^{26}+65t^{24}+108t^{22}-1224t^{20}+3308t^{18}-4890t^{16}+6036t^{14}-4890t^{12}+3308t^{10}-1224t^8+108t^6+65t^4-2t^2+1)}{(t-1)^2(t+1)^2(t^6+2t^5-2t^4+2t^2+2t-1)^2(t^6-2t^5-2t^4+2t^2-2t-1)^2}$
54A	$\frac{(-t^2(-1+712t^{16}+2848t^{15}+5712t^{14}+8128t^{13}+6076t^{12}+696t^{11}-6404t^{10}-10880t^9-11169t^8-8272t^7-4540t^6-1788t^5-478t^4-64t^3-12t^2-4t))}{\sqrt{((t+1)^2(t^2+t+1)^2(4t^3+3t-1)^2(5t^3+3t^2+3t+1)^2)}}$
55A	$\frac{t^2(1+7640t^{14}+17960t^{13}+12400t^{12}-8320t^{11}-15911t^{10}-6500t^9+3154t^8+3114t^7+201t^6-160t^5+105t^4+46t^3+6t^2+4t)}{(t+1)^2(t^2+3t+1)^2(5t^2-5t+1)^2(7t^3+t^2-3t-1)^2}$
56A	$\frac{(t^2(1+588t^{16}+3108t^{15}+8693t^{14}+15798t^{13}+22963t^{12}+29448t^{11}+29657t^{10}+20058t^9+9735t^8+4900t^7+3167t^6+1706t^5+569t^4+112t^3+19t^2+6t))}{\sqrt{((t-1)^2(t+1)^2(4t^2+3t+1)^2(7t^4+16t^3+6t^2-1)^2)}}$
56B	$\frac{t^2(t^{32}+100t^{28}-22t^{24}+2000t^{20}-5901t^{16}+2000t^{12}-22t^8+100t^4+1)}{(t^4+2t^3+t^2+2t+1)^2(t^4-2t^3+t^2-2t+1)^2(t^8+2t^6-5t^4+2t^2+1)^2}$

Table A.1. (Continued.)

Class	$R(f)$
57A	$\frac{-t^2(216t^{21} - 13753t^{18} + 17630t^{15} - 6691t^{12} + 728t^9 + t^6 - 30t^3 - 1)}{(t+1)^2(t^2 - t + 1)^2(3t^3 + t^2 + t - 1)^2(9t^6 - 3t^5 - 2t^4 - 7t^3 + 2t^2 + t + 1)^2}$
59A	$(-t^2(-1 + 536t^{22} + 2936t^{21} + 9496t^{20} + 22808t^{19} + 42791t^{18} + 65192t^{17} + 80728t^{16} + 78750t^{15} + 53200t^{14} + 9016t^{13} - 39939t^{12} - 75264t^{11} - 85432t^{10} - 73000t^9 - 48984t^8 - 25672t^7 - 10303t^6 - 3064t^5 - 664t^4 - 126t^3 - 32t^2 - 8t)) / ((t^3 + 2t + 1)^2(11t^9 + 24t^8 + 46t^7 + 61t^6 + 60t^5 + 44t^4 + 21t^3 + 4t^2 - 2t - 1))$
60A	$\frac{t^2(1 + 68040t^{14} - 71655t^{12} + 43978t^{10} - 15153t^8 + 2612t^6 - 169t^4 - 6t^2)}{(t-1)^2(3t-1)^2(2t+1)^2(2t-1)^2(3t+1)^2(t+1)^2(5t^2-1)^2}$
60B	$\frac{t^2(1 + 2700t^{12} + 5580t^{11} + 7077t^{10} + 2658t^9 + 4169t^8 + 3368t^7 + 2710t^6 + 1244t^5 + 362t^4 + 76t^3 + 5t^2 + 2t)}{(3t-1)^2(t+1)^2(4t^2+t+1)^2(5t^2+2t+1)^2}$
60C	$(t^2(1 + 1024t^{20} + 5120t^{19} + 12032t^{18} + 22784t^{17} + 54592t^{16} + 132736t^{15} + 238528t^{14} + 293696t^{13} + 249824t^{12} + 159232t^{11} + 99968t^{10} + 79616t^9 + 62456t^8 + 36712t^7 + 14908t^6 + 4148t^5 + 853t^4 + 178t^3 + 47t^2 + 10t)) / ((2t+1)^2(t+1)^2(4t^2+2t-1)^2(t^2-t-1)^2(4t^4+6t^3+5t^2+3t+1)^2)$
60D	$(t^2(1 + 256t^{22} - 2816t^{21} + 14656t^{20} - 48000t^{19} + 111040t^{18} - 193152t^{17} + 261152t^{16} - 274432t^{15} + 211328t^{14} - 93056t^{13} - 21144t^{12} + 80727t^{11} - 81608t^{10} + 56192t^9 - 30476t^8 + 13408t^7 - 4576t^6 + 1120t^5 - 167t^4 - 10t^3 + 19t^2 - 6t)) / ((t-1)^2(t^2-t+1)^2(4t^4-8t^3+4t^2+1)^2(4t^4-8t^3+5t^2-t+1)^2)$
60E	$\frac{t^2(t^{24} - 2t^{22} + 61t^{20} - 294t^{18} + 122t^{16} + 3446t^{14} - 5003t^{12} + 3446t^{10} + 122t^8 - 294t^6 + 61t^4 - 2t^2 + 1)}{(t^2+t+1)^2(t^2-t+1)^2(t^2+3t+1)^2(t^2-3t+1)^2(t^2+t-1)^2(t^2-t-1)^2}$
60F	$\frac{t^2(t^{36} + 394t^{30} - 33t^{24} + 6188t^{18} - 33t^{12} + 394t^6 + 1)}{(t-1)^2(t+1)^2(t^2+t+1)^2(t^2-t+1)^2(t^2+t-1)^2(t^2-t-1)^2(t^4-t^3+2t^2+t+1)^2(t^4+t^3+2t^2-t+1)^2}$
62A	$(t^2(1 + 336t^{18} + 1536t^{17} + 304t^{16} - 3184t^{15} - 7211t^{14} - 4002t^{13} + 4081t^{12} + 9862t^{11} + 8436t^{10} + 4360t^9 + 2271t^8 + 2084t^7 + 1821t^6 + 998t^5 + 343t^4 + 54t^3 + 10t^2 + 4t)) / ((3t^3 + 5t^2 + 4t + 1)^2(t^3 - t^2 - 1)^2(4t^4 - 4t^3 - 3t^2 - 2t + 1)^2)$
66A	$\frac{t^2(1 + 6912t^{14} - 17568t^{13} + 20748t^{12} - 17208t^{11} + 3836t^{10} + 12476t^9 - 7216t^8 - 3072t^7 + 1577t^6 + 334t^5 - 57t^4 + 4t^3 - t^2 + 2t)}{(t-1)^2(3t+1)^2(8t^2+t-1)^2(4t^3-4t^2+1)^2}$
66B	$(t^2(1 + 12t + 68t^2 + 280t^3 + 1130t^4 + 4660t^5 + 17136t^6 + 51832t^7 + 126635t^8 + 247932t^9 + 375764t^{10} + 387976t^{11} + 108730t^{12} - 532496t^{13} - 1313972t^{14} - 1791528t^{15} - 1680263t^{16} - 1126632t^{17} - 526284t^{18} - 148488t^{19} - 5116t^{20} + 16544t^{21} + 7760t^{23} + 1632t^{23} + 136t^{24})) / ((t+1)^2(t^2+t+1)^2(8t^4+16t^3+7t^2-t-1)^2(t^6+3t^5+2t^4-t^3+2t^2+3t+1)^2)$
68A	$\frac{t^2(3072t^{20} - 2240t^{18} + 21008t^{16} - 47384t^{14} + 47017t^{12} - 24118t^{10} + 6591t^8 - 940t^6 + 71t^4 - 6t^2 + 1)}{(t-1)^2(t+1)^2(4t^4+t^2-1)^2(4t^2+t-1)^2(4t^2-t-1)^2}$
69A	$(t^2(1 + 117t^{18} - 2394t^{17} + 1233t^{16} + 3582t^{15} - 4794t^{14} - 6276t^{13} + 5291t^{12} + 4220t^{11} - 7386t^{10} - 5504t^9 + 1952t^8 + 2344t^7 + 719t^6 + 398t^5 + 217t^4 + 42t^3 + 6t^2 + 4t)) / ((t^3 - t^2 + 1)^2(5t^3 + 7t^2 + 4t + 1)^2(3t^4 - 6t^3 + 5t^2 + 2t - 1)^2)$
70A	$\frac{-t^2(-1 + 528t^{14} + 8976t^{13} + 3652t^{12} + 2392t^{11} + 4468t^{10} + 420t^9 - 2004t^8 - 676t^7 - 965t^6 - 806t^5 - 179t^4 - 8t^3 - 3t^2 - 2t)}{(3t-1)^2(2t+1)^2(t+1)^2(2t^2-t+1)^2(4t^3+2t+1)^2}$
70B	$(t^2(1 - 8t + 32t^2 - 116t^3 + 518t^4 - 2076t^5 + 5912t^6 - 10196t^7 + 3463t^8 + 35596t^9 - 112764t^{10} + 168492t^{11} - 58834t^{12} - 356588t^{13} + 1048384t^{14} - 1754936t^{15} + 2145569t^{16} - 2062200t^{17} + 1601820t^{18} - 1007240t^{19} + 502972t^{20} - 191840t^{21} + 52400t^{22} - 9120t^{23} + 760t^{24})) / ((t-1)^2(t^2-t-1)^2(4t^4-8t^3+7t^2-3t+1)^2(5t^6-15t^5+16t^4-7t^3+t-1)^2)$

Table A.1. (Continued.)

Class	$R(f)$
71A	$(t^2(1 + 8t + 28t^2 + 90t^3 + 410t^4 + 1458t^5 + 3073t^6 + 3798t^7 + 4021t^8 + 8108t^9 + 13349t^{10} + 88t^{11} - 40168t^{12} - 63932t^{13} - 13304t^{14} + 79626t^{15} + 94791t^{16} - 5466t^{17} - 91159t^{18} - 52578t^{19} + 31090t^{20} + 41224t^{21} + 1849t^{22} - 12328t^{23} - 3112t^{24} + 1320t^{25} + 364t^{26}))$ $\div ((t^7 - 2t^5 - 3t^4 + t^3 + 5t^2 + 4t + 1)^2(11t^7 - 4t^6 - 18t^5 - 5t^4 + 11t^3 + 7t^2 - 1)^2)$
78A	$(t^2(1 + 648t^{16} + 5904t^{15} + 14424t^{14} + 5648t^{13} - 10660t^{12} - 12752t^{11} - 2400t^{10} + 3656t^9 + 2909t^8 + 398t^7 - 591t^6 - 154t^5 + 132t^4 + 34t^3 + t^2 + 2t)) \div ((3t - 1)^2(t + 1)^2(t^2 + t + 1)^2(3t^2 - t - 1)^2(4t^3 - t - 1)^2)$
78B	$(t^2(1 - 10t + 49t^2 - 156t^3 + 364t^4 - 890t^5 + 3626t^6 - 16788t^7 + 62992t^8 - 185826t^9 + 442843t^{10} - 871628t^{11} + 1438554t^{12} - 2020966t^{13} + 2462186t^{14} - 2646524t^{15} + 2513284t^{16} - 2051814t^{17} + 1362881t^{18} - 692340t^{19} + 283888t^{20} - 171710t^{21} + 188800t^{22} - 176220t^{23} + 111405t^{24} - 46800t^{25} + 12672t^{26} - 2016t^{27} + 144t^{28})) \div ((t - 1)^2(t^4 - 2t^3 + 2t^2 - t + 1)^2(3t^4 - 6t^3 + 2t^2 + t - 1)^2(4t^6 - 12t^5 + 13t^4 - 6t^3 + 3t^2 - 2t + 1)^2)$
84A	$\frac{t^2(1152t^{18} + 12240t^{16} + 14784t^{14} - 311t^{12} - 2014t^{10} + 1087t^8 + 604t^6 + 103t^4 + 2t^2 + 1)}{(t - 1)^2(2t + 1)^2(2t - 1)^2(t + 1)^2(3t^2 + 1)^2(2t^2 - t + 1)^2(2t^2 + t + 1)^2}$
84B	$(t^2(t^{32} + 10t^{30} - 5t^{28} - 506t^{26} - 3191t^{24} - 12204t^{22} - 37490t^{20} - 82088t^{18} - 108062t^{16} - 82088t^{14} - 37490t^{12} - 12204t^{10} - 3191t^8 - 506t^6 - 5t^4 + 10t^2 + 1)) \div ((t^4 + 5t^2 + 1)^2(t^4 + t^3 + 4t^2 + t + 1)^2(t^4 - t^3 + 4t^2 - t + 1)^2(t^2 + t + 1)^2(t^2 - t + 1)^2)$
84C	$\frac{t^2(4096t^{24} + 13312t^{21} - 33216t^{18} + 35520t^{15} - 18368t^{12} + 4440t^9 - 519t^6 + 26t^3 + 1)}{(t - 1)^2(2t - 1)^2(t^2 + t + 1)^2(2t^2 + t + 1)^2(4t^2 + 2t + 1)^2(4t^4 - 2t^3 - t^2 - t + 1)^2}$
87A	$(-t^2(-1 + 2124t^{22} + 5040t^{21} + 14220t^{20} + 29628t^{19} + 45531t^{18} + 55542t^{17} + 39367t^{16} + 6562t^{15} - 44982t^{14} - 80524t^{13} - 101547t^{12} - 91948t^{11} - 72122t^{10} - 44920t^9 - 24748t^8 - 10852t^7 - 4151t^6 - 1226t^5 - 305t^4 - 66t^3 - 14t^2 - 4t)) \div ((3t^3 + 3t^2 + 2t + 1)^2(t^3 + t^2 + 2t - 1)^2(8t^4 + 4t^5 + 13t^4 + 6t^3 + 7t^2 + 2t + 1)^2)$
88A	$\frac{t^2(1024t^{20} - 1536t^{18} - 2048t^{16} + 12800t^{14} - 18944t^{12} + 13120t^{10} - 4736t^8 + 800t^6 - 32t^4 - 6t^2 + 1)(4t^4 - 2t^2 + 1)}{(2t^3 - 2t - 1)^2(2t^3 - 2t + 1)^2(4t^3 + 4t^2 - 1)^2(4t^3 - 4t^2 + 1)^2}$
92A	$(t^2(1 + 6t + 23t^2 + 94t^3 + 431t^4 + 1834t^5 + 6210t^6 + 16564t^7 + 35555t^8 + 63012t^9 + 96486t^{10} + 132532t^{11} + 171542t^{12} + 206124t^{13} + 221548t^{14} + 189264t^{15} + 106711t^{16} - 1034t^{17} - 77957t^{18} - 96226t^{19} - 66669t^{20} - 28278t^{21} - 6694t^{22} - 564t^{23} + 237t^{24})) \div ((t + 1)^2(t^3 + 3t^2 + 2t + 1)^2(7t^3 + t^2 + t - 1)^2(t^6 - 2t^5 + 2t^4 + t^3 + 4t^2 + t + 1)^2)$
93A	$\frac{-t^2(7560t^{33} + 53819t^{30} + 90822t^{27} + 40265t^{24} - 38414t^{21} - 71229t^{18} - 53058t^{15} - 21590t^{12} - 4614t^9 - 420t^6 - 8t^3 - 1)}{(3t^3 - 4t^2 + 3t - 1)^2(9t^6 + 12t^5 + 7t^4 + 6t^3 + 5t^2 + 3t + 1)^2(t^3 + t + 1)^2(t^6 - t^4 + 2t^3 + t^2 - t + 1)^2}$
94A	$(-t^2(-1 - 4t - 2t^2 + 4t^3 - 71t^4 - 486t^5 - 844t^6 + 1026t^7 + 2089t^8 - 5456t^9 - 2155t^{10} + 18040t^{11} - 9269t^{12} - 28398t^{13} + 45506t^{14} + 7458t^{15} - 68336t^{16} + 55696t^{17} + 26471t^{18} - 69872t^{19} + 43972t^{20} + 9720t^{21} - 29632t^{22} + 22032t^{23} - 7216t^{24} + 992t^{25} + 232t^{26})) \div ((t^5 - 2t^4 + 2t^3 - t^2 + 1)^2(5t^5 + 2t^4 - 2t^3 + 3t^2 + 4t + 1)^2(4t^4 - 4t^3 + 3t^2 + 2t - 1)^2)$
95A	$(t^2(1 + 165t^{22} - 930t^{21} + 3245t^{20} + 6050t^{19} - 18870t^{18} + 10646t^{17} + 9653t^{16} - 19472t^{15} + 6564t^{14} + 7508t^{13} - 6511t^{12} + 348t^{11} + 7379t^{10} + 1542t^9 - 3347t^8 - 1034t^7 + 848t^6 + 478t^5 + 67t^4 - 4t^3 + 2t^2 + 4t)) \div ((t - 1)^2(5t^4 + 10t^3 + 6t^2 - t - 1)^2(t^4 - 2t^3 + 2t^2 - t - 1)^2(3t^3 - t^2 + t + 1)^2)$
104A	$\frac{-t^2(27904t^{28} + 24816t^{24} - 13608t^{20} - 20065t^{16} - 7308t^{12} - 982t^8 - 4t^4 - 1)}{(2t^2 + 2t + 1)^2(2t^2 - 2t + 1)^2(2t^3 + t^2 - 1)^2(2t^3 - t^2 + 1)^2(4t^6 + t^4 + 2t^2 + 1)^2}$

Table A.1. (Continued.)

Class	$R(f)$
105A	$(t^2(1 + 18000t^{18} + 30600t^{17} + 11880t^{16} - 44064t^{15} - 57447t^{14} + 17382t^{13} + 57793t^{12} + 9344t^{11} - 26620t^{10} - 10512t^9 + 5926t^8 + 3432t^7 - 566t^6 - 496t^5 + 24t^4 + 24t^3 - 3t^2 + 2t))$ $\sphericalangle((t-1)^2(3t^2+3t+1)^2(t^2+t-1)^2(5t^3+t^2-t-1)^2(5t^2+t-1)^2)$
110A	$(t^2(1 + 5016t^{20} - 14736t^{19} + 4568t^{18} + 18128t^{18} + 15300t^{16} - 35328t^{15} + 2440t^{14} - 16792t^{13} + 29593t^{12} + 1742t^{11} + 10043t^{10} - 3578t^9 + 1091t^8 + 44t^7 + 1002t^6 + 440t^5 + 179t^4 + 38t^3 + 7t^2 + 2t))$ $\sphericalangle((t-1)^2(3t^2+t+1)^2(t^2-t-1)^2(8t^3+t-1)^2(t^3-3t^2-t-1)^2)$
119A	$(t^2(1 + 4t + 14t^2 + 40t^3 + 193t^4 + 814t^5 + 3046t^6 + 9970t^7 + 27348t^8 + 65380t^9 + 137087t^{10} + 251940t^{11} + 411609t^{12} + 591020t^{13} + 748046t^{14} + 828256t^{15} + 778011t^{16} + 618670t^{17} + 394924t^{18} + 179710t^{19} + 62284t^{20} + 36940t^{21} + 41961t^{22} + 43232t^{23} + 34104t^{24} + 12824t^{25} + 868t^{26}))$ $\sphericalangle((5t^4+6t^3+3t^2+2t+1)^2(t^5+4t^4+6t^3+3t^2+2t+1)^2(7t^5+6t^3-3t^2+2t-1)^2)$
25z	$\frac{t^2(1 + 532t^{12} + 5880t^{11} + 296t^{10} + 7020t^9 - 375t^8 + 2316t^7 - 230t^6 + 228t^5 - 45t^4 + 2t^2)}{(t-1)^2(4t^2+1)^2(11t^4+6t^3+6t^2+t+1)^2}$
49z	$\frac{t^2(1 + 300t^{14} - 1380t^{13} + 925t^{12} + 1100t^{11} + 1064t^{10} - 1880t^9 - 1780t^8 + 992t^7 + 1346t^6 + 628t^5 + 236t^4 + 56t^3 + 8t^2 + 4t)}{(2t+1)^2(t^3+t^2-2t-1)^2(3t^4-10t^3+9t^2+2t-1)^2}$
50z	$(t^2(2t^2+2t+1)(96t^{20}+960t^{19}+4210t^{18}+10530t^{17}+18365t^{16}+32272t^{15}+71480t^{14}+150820t^{13}+247030t^{12}+305680t^{11}+291554t^{10}+218910t^9+130775t^8+61920t^7+22800t^6+6348t^5+1330t^4+240t^3+50t^2+10t+1))\sphericalangle((t+1)^2(t^4+2t^3+4t^2+3t+1)^2(16t^6+48t^5+55t^4+30t^3+5t^2-2t-1)^2)$

TABLE A.2. $R(f)$ for the remaining completely replicable functions.

Class	$R(f)$
2a	$-\frac{(4416t^2-1)t^2}{2(1728t^2+1)^2}$
4a	$-\frac{(-1+704t^2)t^2}{(256t^2+1)^2}$
5a	$-\frac{(48t^4-936t^3+47t^2+2t-1)t^2}{(4t-1)^2(36t^2+3t+1)^2}$
6a	$-\frac{(-1+312t^2)t^2}{(108t^2+1)^2}$
6b	$\frac{(10368t^4+120t^2+1)t^2}{(108t^2-1)^2}$
6c	$\frac{(729t^4-42t^2+1)t^2}{(27t^2+1)^2}$
6d	$\frac{(5504t-1456t+144t-8t+1)t^2}{(8t-1)^2(8t+1)^2(4t-1)^2}$
8a	$-\frac{(-1+192t^2)t^2}{(64t^2+1)^2}$
8b	$\frac{(40960t^6-2560t^4+32t^2+1)t^2}{(4t+1)^2(4t-1)^2(32t^2-1)^2}$

Table A.2. (Continued.)

Class	$R(f)$
$8c$	$-\frac{(40960t^6 + 2560t^4 + 32t^2 - 1)t^2}{2(32t^2 + 1)^2(16t^2 + 1)^2}$
$9a$	$\frac{(8748t^6 + 540t^3 + 1)t^2}{(108t^3 - 1)^2}$
$9b$	$\frac{(1944t^4 - 648t^3 + 81t^2 - 6t + 1)t^2}{(6t - 1)^2(3t - 1)^2(6t + 1)^2}$
$9c$	$\frac{(729t^6 - 162t^3 + 1)t^2}{(3t + 1)^2(9t^2 - 3t + 1)^2}$
$9d$	$\frac{(3888t^8 + 19440t^7 - 648t^6 + 2808t^5 - 351t^4 + 72t^3 - 6t^2 + 1)t^2}{(12t^2 + 1)^2(18t^3 - 9t^2 - 1)^2}$
$10a$	$-\frac{(-1 + 1344t^6 - 2288t^4 - 40t^2)t^2}{(-1 + 44t^2 + 16t^4)^2}$
$10b$	$\frac{(80t^6 - 1896t^5 + 1553t^4 - 236t^3 + 22t^2 - 4t + 1)t^2}{(4t + 1)^2(28t^3 - 17t^2 + 6t - 1)^2}$
$10c$	$-\frac{(-1 + 1344t^6 + 647t^4 + 10t^2)t^2}{(1 + 11t^2 + 64t^4)^2}$
$12a$	$-\frac{(-1 + 124416t^6 + 6144t^4 + 40t^2)t^2}{(32t^2 + 1)^2(36t^2 + 1)^2}$
$12b$	$-\frac{(-1 + 6656t^6 - 2304t^4 - 24t^2)t^2}{(4t^2 + 1)^2(32t^2 - 1)^2}$
$12c$	$\frac{(9216t^6 - 1152t^4 + 24t^2 + 1)t^2}{(4t + 1)^2(4t - 1)^2(12t^2 - 1)^2}$
$12d$	$\frac{(64t^4 + 8t^2 + 1)(-1 + 8t^2)^2}{(1 + 4t^2)^2(16t^2 + 1)^2}$
$12e$	$\frac{(27648t^8 + 2880t^6 - 624t^4 + 8t^2 + 1)t^2}{(4t + 1)^2(4t - 1)^2(12t^2 - 1)^2}$
$12f$	$-\frac{(-1 + 2240t^6 + 240t^4 + 24t^2)t^2}{(1 + 4t^2)^2(16t^2 + 1)^2}$
$14a$	$-\frac{(-1 + 67704t^6 + 4151t^4 + 34t^2)t^2}{(27t^2 + 1)^2(28t^2 + 1)^2}$
$14b$	$\frac{(2401t^8 - 294t^6 - 245t^4 - 6t^2 + 1)t^2}{(7t^2 + t + 1)^2(7t^2 - t + 1)^2}$
$14c$	$\frac{(1848t^6 + 721t^4 + 22t^2 + 1)t^2}{(t - 1)^2(t + 1)^2(28t^2 - 1)^2}$
$15a$	$\frac{(69040t^8 - 58800t^7 + 7000t^6 + 4880t^5 - 975t^4 - 64t^3 + 10t^2 + 1)t^2}{(2t - 1)^2(11t^2 - 2t - 1)^2(20t^2 - 1)^2}$
$15b$	$\frac{(19980t^8 + 16488t^7 + 1500t^6 - 2484t^5 - 488t^4 + 92t^3 + 9t^2 + 2t + 1)t^2}{(3t - 1)^2(2t + 1)^2(36t^3 + 12t^2 - 2t - 1)^2}$
$16a$	$\frac{(4096t^8 + 640t^4 + 1)t^2}{(8t^2 - 1)^2(8t^2 + 1)^2}$

Table A.2. (Continued.)

Class	$R(f)$
16b	$\frac{(6144t^6 - 704t^4 + 16t^2 + 1)t^2}{(4t+1)^2(4t-1)^2(8t^2-1)^2}$
16d	$\frac{(16t^4 - 16t^2 + 1)(16t^4 + 16t^2 + 1)t^2}{(16t^4 + 1)^2}$
16e	$\frac{(32768t^{10} - 25600t^8 + 5632t^6 - 384t^4 + 1)t^2}{(8t^2 - 1)^2(32t^4 - 8t^2 + 1)^2}$
16f	$-\frac{(32768t^{10} + 25600t^8 + 5632t^6 - 384t^4 - 1)t^2}{(8t^2 + 1)^2(32t^4 + 8t^2 + 1)^2}$
16g	$-\frac{(-1 + 12288t^{10} - 22784t^8 + 3328t^6 - 32t^4)t^2}{(8t^2 - 1)^2(16t^4 + 8t^2 - 1)^2}$
16h	$\frac{(12288t^{10} + 22784t^8 + 3328t^6 + 32t^4 + 1)t^2}{(8t^2 + 1)^2(16t^4 - 8t^2 - 1)^2}$
18a	$\frac{(7520t^8 + 4224t^7 + 1660t^6 - 1852t^5 - 39t^4 + 148t^3 + 2t^2 + 1)t^2}{(2t+1)^2(2t-1)^2(34t^3 + 3t^2 - 1)^2}$
18b	$\frac{(2808t^6 + 1)t^2}{(108t^6 - 1)^2}$
18c	$\frac{(729t^8 - 324t^6 + 702t^4 + 12t^2 + 1)t^2}{(-1 + 18t^2 + 27t^4)^2}$
18d	$\frac{(t^{12} + 148t^9 + 1590t^6 + 148t^3 + 1)t^2}{(t^6 - 34t^3 + 1)^2}$
18e	$\frac{(4096t^{12} - 4352t^9 - 660t^6 - 68t^3 + 1)t^2}{(4t^3 + 1)^2(16t^3 + 1)^2}$
18f	$-\frac{(-1 + 1080t^6 + 135t^4 + 18t^2)t^2}{(12t^2 + 1)^2(3t^2 + 1)^2}$
18g	$\frac{(10368t^8 + 1080t^6 - 351t^4 + 6t^2 + 1)t^2}{(3t-1)^2(3t+1)^2(12t^2-1)^2}$
18h	$\frac{(31808t^8 - 36096t^7 + 3820t^6 + 3824t^5 - 624t^4 - 68t^3 + 8t^2 + 1)t^2}{(4t-1)^2(4t+1)^2(20t^3 - 12t^2 + 1)^2}$
18i	$\frac{(729t^{12} - 810t^6 + 1)t^2}{(3t^2 + 1)^2(3t^2 + 3t + 1)^2(3t^2 - 3t + 1)^2}$
18j	$\frac{t^2(1 + 1136t^8 - 1968t^7 + 376t^6 + 896t^5 - 39t^4 - 80t^3 + 2t^2)}{(2t+1)^2(2t-1)^2(14ts - 3t^2 + 1)^2}$
20a	$-\frac{t^2(-1 + 20160t^6 + 1840t^4 + 24t^2)}{(16t^2 + 1)^2(20t^2 + 1)^2}$
20b	$\frac{t^2(1 + 4160t^6 - 160t^4 + 16t^2)}{(2t+1)^2(2t-1)^2(20t^2-1)^2}$
20c	$\frac{t^2(1 + 625t^8 - 100t^6 - 250t^4 - 4t^2)}{(5t^2 - 2t + 1)^2(5t^2 + 2t + 1)^2}$
20d	$\frac{t^2(3072t^{10} + 1280t^8 - 1600t^6 + 320t^4 + 1)}{(2t+1)^2(2t-1)^2(16t^4 - 4t^2 - 1)^2}$

Table A.2. (Continued.)

Class	$R(f)$
20e	$\frac{t^2(1 + 49152t^{12} + 3392t^{10} - 14367t^8 + 3708t^6 - 282t^4 - 4t^2)}{(16t^3 + 7t^2 - 2t - 1)^2(16t^3 - 7t^2 - 2t + 1)^2}$
22a	$\frac{t^2(1 + 3520t^{10} - 4224t^8 + 952t^6 + 112t^4 + 8t^2)}{(-1 + 20t^2 - 56t^4 + 44t^6)^2}$
24a	$\frac{-t^2(-1 + 9216t^6 + 1152t^4 + 24t^2)}{(12t^2 + 1)^2(16t^2 + 1)^2}$
24b	$\frac{-t^2(-1 + 1152t^6 - 720t^4)}{(4t^2 + 1)^2(12t^2 - 1)^2}$
24c	$\frac{t^2(3t^2 - 1)^2}{(9t^2 + 1)^2(1 + t^2)^2}$
24d	$\frac{-t^2(-1 + 2048t^{10} - 10752t^8 - 3200t^6 - 272t^4)}{(2t + 1)^2(2t - 1)^2(4t^2 + 1)^2(8t^2 + 1)^2}$
24e	$\frac{t^2(1 + 2048t^{10} + 10752t^8 - 3200t^6 + 272t^4)}{(2t + 1)^2(2t - 1)^2(4t^2 + 1)^2(8t^2 - 1)^2}$
24f	$\frac{-t^2(-1 + 92160t^{10} + 41472t^8 + 7296t^6 + 496t^4)}{(4t^2 + 1)^2(12t^2 + 1)^2(8t^2 + 1)^2}$
24g	$\frac{t^2(1 + 92160t^{10} - 41472t^8 + 7296t^6 - 496t^4)}{(2t + 1)^2(2t - 1)^2(8t^2 - 1)^2(12t^2 - 1)^2}$
24h	$\frac{t^2(64t^6 - 48t^4 + 12t^2 + 1)(64t^6 + 48t^4 - 12t^2 + 1)}{(2t + 1)^2(2t - 1)^2(16t^4 - 4t^2 + 1)^2}$
24i	$\frac{t^2(1 + 59392t^{14} - 25088t^{12} + 25728t^{10} - 12656t^8 + 2560t^6 - 168t^4 - 8t^2)}{(2t + 1)^2(2t - 1)^2(8t^2 - 1)^2(2t^2 + 2t - 1)^2(2t^2 - 2t - 1)^2}$
24j	$\frac{-t^2(-1 + 59392t^{14} + 25088t^{12} + 25728t^{10} + 12656t^8 + 2560t^6 + 168t^4 - 8t^2)}{(4t^2 + 1)^2(8t^2 + 1)^2(4t^4 + 8t^2 + 1)^2}$
26a	$\frac{t^2(1 + 192t^{10} + 2416t^8 - 312t^6 + 289t^4 + 6t^2)}{(t - 1)^2(t + 1)^2(16t^4 + 12t^2 - 1)^2}$
27a	$\frac{t^2(1 + 10692t^8 - 9720t^7 + 1296t^6 + 1404t^5 - 351t^4 - 36t^3 + 6t^2)}{(12t^2 - 1)^2(9t^3 - 9t^2 + 1)^2}$
27b	$\frac{t^2(-1 - 36t^3 + 27t^6)^2}{(-1 + 18t^3 + 27t^6)^2}$
27c	$\frac{t^2(729t^{12} - 972t^9 - 594t^6 - 36t^3 + 1)}{(1 + 9t^3 + 27t^6)^2}$
27d	$\frac{t^2(8748t^{12} + 8100t^9 - 1611t^6 + 66t^3 + 1)}{(12t^3 - 1)^2(9t^3 - 1)^2}$
27e	$\frac{t^2(648t^9 + 873t^6 - 42t^3 + 1)}{(9t^3 + 1)^2(3t^3 - 1)^2}$
28a	$\frac{-t^2(-1 + 2184t^{10} - 1169t^8 - 636t^6 - 502t^4 - 4t^2)}{(4t^2 + 1)^2(7t^4 + 10t^2 - 1)^2}$
30a	$\frac{-t^2(-1 + 711000t^{10} + 234423t^8 + 27108t^6 + 1138t^4 - 4t^2)}{(12t^2 + 1)^2(125t^4 + 22t^2 + 1)^2}$

Table A.2. (Continued.)

Class	$R(f)$
30b	$\frac{t^2(1+5616t^{10}+8208t^9+6984t^8+2232t^7-1407t^6-942t^5+107t^4+76t^3+3t^2+2t)}{(3t-1)^2(2t+1)^2(2t-1)^2(t+1)^2(6t^2+3t+1)^2}$
30c	$\frac{t^2(1+6561t^{16}+2916t^{14}-11502t^{12}-9504t^{10}-3861t^8-1056t^6-142t^4+4t^2)}{(1+10t^2+47t^4+90t^6+81t^8)^2}$
30d	$\frac{t^2(1+2520t^{10}-7335t^8+1452t^6+62t^4+4t^2)}{(12t^2-1)^2(5t^4-2t^2+1)^2}$
30e	$\frac{t^2(1+18800t^{12}-15760t^{11}+27180t^{10}-7840t^9-3992t^8-660t^7-648t^6+796t^5+57t^4+64t^3+2t^2)}{(2t-1)^2(10t-5t^2-1)^2(4t^3+8t^2+2t+1)^2}$
30f	$\frac{-t^2(-1+2880t^{14}+6912t^{12}-13176t^{10}+512t^8+704t^6-57t^4+2t^2)}{(5t^2-1)^2(12t^6+8t^4+4t^2-1)^2}$
32b	$\frac{t^2(256t^{16}+3328t^{12}+608t^8+208t^4+1)}{(4t^4+4t^2-1)^2(4t^4-4t^2-1)^2}$
32c	$\frac{t^2(1+9216t^{10}-3008t^8+1152t^6-176t^4)}{(8t^2-1)^2(8t^4-8t^2+1)^2}$
32d	$\frac{-t^2(-1+9216t^{10}+3008t^8+1152t^6+176t^4)}{(8t^2+1)^2(8t^4+8t^2+1)^2}$
32e	$\frac{t^2(16t^8-32t^4+1)(16t^8+32t^4+1)}{(16t^8+1)^2}$
34a	$\frac{-t^2(-1+576t^{14}+2896t^{12}+1784t^{10}-841t^8-764t^6-286t^4-4t^2)}{(-1+6t^2+27t^4+28t^6+16t^8)^2}$
35a	$(t^2(1+8925t^{14}-23520t^{13}+24010t^{12}-10780t^{11}+4463t^{10}-9746t^9+7196t^8+692t^7-2221t^6+472t^5+119t^4-28t^3-t^2-2t))/((7t^3-7t^2+4t-1)^2(5t^3-5t^2+1)^2(t^2+3t+1)^2)$
36a	$\frac{t^2(1+5632t^{14}+40192t^{14}+30360t^{10}+7985t^8+476t^6-42t^4+4t^2)}{(4t^2+1)^2(32t^6-9t^4-6t^2-1)^2}$
36b	$\frac{t^2(4096t^{12}+4352t^9-660t^6+68t^3+1)}{(16t^3-1)^2(4t^3-1)^3}$
36c	$\frac{-t^2(-1+17024t^{14}-131200t^{14}+37536t^{10}+3856t^8-2168t^6+168t^4+8t^2)}{(8t^2-1)^2(4t^6+36t^4-12t^2+1)^2}$
36d	$\frac{t^2(1+10368t^{14}-34992t^{12}+31968t^{10}-9567t^8+1116t^6-42t^4-4t^2)}{(2t+1)(2t-1)^2(6t^3-3t^2+1)^2(6t^3+3t^2-1)^2}$
36e	$\frac{t^2(4096t^{24}-25088t^{18}-4224t^{12}-392t^6+1)}{(4t^6+1)^2(16t^6+1)^2}$
36f	$\frac{t^2(81t^{16}+540t^{12}-1728t^{10}+1494t^8+576t^6+60t^4+1)}{(t-1)^2(t+1)^2(3t^2+1)^2(3t^4-6t^2-1)^2}$
36g	$\frac{t^2(t^{24}+796t^{18}+2886t^{12}+796t^6+1)}{(t^6-6t^3+1)^2(t^6+6t^3+1)^2}$
36h	$\frac{-t^2(-1+114048t^{14}+176256t^{12}+88992t^{10}+21456t^8+2952t^6+168t^4-8t^2)}{(8t^2+1)^2(36t^6+36t^4+12t^2+1)^2}$

Table A.2. (Continued.)

Class	$R(f)$
36i	$\frac{-t^2(-1+3776t^{14}+18032t^{12}+16152t^{10}+5167t^8+724t^6+42t^4-4t^2)}{(4t^2+1)^2(16t^6+9t^4+6t^2+1)^2}$
38a	$\frac{t^2(1+14592t^{14}-24320t^{12}+17848t^{10}-6576t^8+1112t^6-63t^4-2t^2)}{(t-1)^2(t+1)^2(76t^6-64t^4+16t^2-1)^2}$
40a	$\frac{t^2(-1+3072t^{10}-1280t^8-1600t^6-320t^4)}{(4t^2+1)^2(16t^4+4t^2-1)^2}$
40b	$\frac{-t^2(-1+337920t^{14}+401920t^{12}+192640t^{10}+46320t^8+5632t^6+264t^4-8t^2)}{(8t^2+1)^2(4t^2+1)^2(20t^4+8t^2+1)^2}$
40c	$\frac{t^2(1+337920t^{14}-401920t^{12}+192640t^{10}-46320t^8+5632t^6-264t^4-8t^2)}{(2t+1)^2(2t-1)^2(8t^2-1)^2(20t^4-8t^2+1)^2}$
40d	$\frac{t^2(4096t^{24}-24576t^{20}-22528t^{16}-8832t^{12}-1408t^8-96t^4+1)}{(2t^2+2t+1)^2(2t^2-2t+1)^2(16t^8+12t^4+1)^2}$
40e	$\frac{t^2(187136t^{20}-123423t^{16}+28572t^{12}-2778t^8+92t^4+1)}{(4t^3+3t^2+2t+1)^2(4t^3-3t^2+2t-1)^2(16t^6-7t^4-2t^2+1)^2}$
42a	$\frac{t^2(1+10368t^{12}+984t^{10}+4441t^8+2476t^6+318t^4+4t^2)}{(4t^2+1)^2(27t^4+2t^2-1)^2}$
42b	$\frac{-t^2(-1+792t^{10}-3465t^8+36t^6+50t^4-4t^2)}{(2t+1)^2(2t-1)^2(3t^4+10t^2-1)^2}$
42c	$\frac{(t^2(1+128088t^{14}-364608t^{13}+395856t^{12}-106416t^{11}-134364t^{10}+96840t^9+5356t^8-18648t^7+2041t^6+1348t^5-168t^4-26t^3-8t^2))/((t-1)^2(3t+1)^2(5t^2-t-1)^2(8t^2-1)^2(3t^2-3t+1)^2)}{(t-1)^2(t+1)^2(t^8-6t^6-17t^4-6t^2+1)^2}$
42d	$\frac{t^2(t^{20}+2t^{18}+171t^{16}-128t^{14}+701t^{12}+1098t^{10}+701t^8-128t^6+171t^4+2t^2+1)}{(t-1)^2(t+1)^2(t^8-6t^6-17t^4-6t^2+1)^2}$
44a	$\frac{-t^2(-1+7480704t^{14}+5443456t^{12}+1614816t^{10}+245392t^8+19048t^6+536t^4-16t^2)}{(8t^2+1)^2(484t^6+188t^4+24t^2+1)^2}$
44b	$\frac{(t^2(1024t^{20}+1536t^{18}-2048t^{16}-12800t^{14}-18944t^{12}-13120t^{10}-4736t^8-800t^6-32t^4+6t^2+1)(4t^4+2t^2+1))/((16t^6+16t^4+8t^2+1)^2(4t^6+8t^4+4t^2+1)^2)}{(8t^2-1)^2(4t^6-4t^4+1)^2}$
44c	$\frac{t^2(1+1408t^{14}+896t^{12}-6368t^{10}+3888t^8-872t^6+168t^4)}{(8t^2-1)^2(4t^6-4t^4+1)^2}$
45a	$\frac{(t^2(1+13396t^{14}-15648t^{13}+14620t^{12}+3004t^{11}+11457t^{10}+10426t^9+3651t^8+4302t^7+213t^6+682t^5-42t^4+34t^3+4t^2))/((4t^2+1)^2(t^3+3t^2+1)^2(11t^3-3t^2-1)^2)}{(t^6+11t^3-1)^2(t^6-t^3-1)^2}$
45b	$\frac{t^2(t^{24}-34t^{21}+533t^{18}+746t^{15}+2644t^{12}-746t^9+533t^6+34t^3+1)}{(t^6+11t^3-1)^2(t^6-t^3-1)^2}$
45c	$\frac{t^2(87480t^{21}+256473t^{18}+109566t^{15}+7723t^9-3424t^9-633t^6-22t^3+1)}{(5t^3+1)^2(27t^9-27t^6-11t^3-1)^2}$
48a	$\frac{t^2(1+4608t^{14}+23616t^{12}-22848t^{10}+8176t^8-1216t^6+60t^4-4t^2)}{(2t+1)^2(2t-1)^2(6t^2-1)^2(2t^2+1)^2(2t^2-1)^2}$
48b	$\frac{-t^2(-1+4608t^{14}-23616t^{12}-22848t^{10}-8176t^8-1216t^6-60t^4-4t^2)}{(4t^2+1)^2(2t^2+1)^2(2t^2-1)^2(6t^2+1)^2}$

Table A.2. (Continued.)

Class	$R(f)$
48c	$\frac{t^2(1+32768t^{18}+52224t^{16}-80896t^{14}-13824t^{12}+26112t^{10}+10832t^8+1440t^6+72t^4+8t^2)}{(4t^2+1)^2(4t^4-4t^2-1)^2(8t^4-4t^2-1)^2}$
48d	$\frac{-t^2(-1+32768t^{18}-52224t^{16}-80896t^{14}+13824t^{12}+26112t^{10}-10832t^8+1440t^6-72t^2+8t^2)}{(2t+1)^2(2t-1)^2(4t^4+4t^2-1)^2(8t^4+4t^2-1)^2}$
48e	$\frac{-t^2(-1+147456t^{18}+377856t^{16}+377856t^{14}+216064t^{12}+78336t^{10}+17968t^8+2400t^6+120t^4-8t^2)}{(4t^2+1)^2(8t^4+4t^2+1)^2(12t^4+4t^2+1)}$
48f	$\frac{t^2(1+147456t^{18}-377856t^{16}+377856t^{14}-216064t^{12}+78336t^{10}-17968t^8+2400t^6-120t^4-8t^2)}{(2t+1)^2(2t-1)^2(8t^4-4t^2+1)^2(12t^2-4t^2+1)^2}$
48g	$\frac{t^2(4096t^{24}+6784t^{12}+1)}{(2t^2-1)^2(2t^2+1)^2(4t^4+2t^2+1)^2(4t^4-2t^2+1)^2}$
48h	$\frac{t^2(t^{24}+102t^{20}+1167t^{16}-1964t^{12}+1167t^8+102t^4+1)}{(t^4+1)^2(t^4-4t^2+1)^2(t^4+4t^2+1)^2}$
52a	$\frac{t^2(1+57408t^{14}-90272t^{12}+55440t^{10}-17007t^8+2668t^6-170t^4-4t^2)}{(2t+1)^2(2t-1)^2(52t^6-49t^4+14t^2-1)^2}$
54a	$(t^2(1+13328t^{14}+16944t^{13}+10036t^{12}+2680t^{11}-7524t^{10}-10696t^9+309t^8+4392t^7+384t^6-680t^5-42t^4+32t^3-4t^2))/((2t+1)^2(2t-1)^2(10t^3+3t^2-1)^2(t^3+3t^2-1)^2)$
54b	$\frac{t^2(t^{24}+32t^{21}+64t^{18}+848t^{15}-2906t^{12}+848t^9+64t^6+32t^3+1)}{(t^6-10t^3+1)^2(t^6-t^3+1)^2}$
54c	$\frac{t^2(729t^{24}-10692t^{18}+4158t^{12}+396t^6+1)}{(-1+18t^6+27t^{12})^2}$
54d	$\frac{t^2(46008t^{18}-8415t^{12}+390t^6+1)}{(12t^6-1)^2(3t^3-1)^2(3t^3+1)^2}$
56a	$\frac{-t^2(3t^2-1)(280t^{12}-1715t^{10}+1941t^8-94t^6+66t^4+t^2+1)}{(7t^2-1)^2(t^2+1)^2(4t^4-3t^2+1)^2}$
56b	$\frac{t^2(1+7560t^{18}+36785t^{16}-42704t^{14}+66284t^{12}-43264t^{10}+12054t^8-1456t^6+76t^4-8t^2)}{(2t+1)^2(2t-1)^2(t^2+2t-1)^2(t^2-2t-1)^2(7t^4+2t^2-1)^2}$
56c	$\frac{-t^2(-1+7560t^{18}-36785t^{16}-42704t^{14}-66284t^{12}-43264t^{10}-12054t^8-1456t^6-76t^4-8t^2)}{(4t^2+1)^2(t^4+6t^2+1)^2(7t^4-2t^2-1)^2}$
58a	$\frac{t^2(192t^{22}-1168t^{20}-4696t^{18}+18545t^{16}-20972t^{14}+12466t^{12}-5304t^{10}+1755t^8-416t^6+66t^4-4t^2+1)}{(-1+10t^2-23t^4+10t^6+15t^8-20t^{10}+16t^{12})^2}$
60a	$\frac{t^2(6120t^{22}-4455t^{20}+14334t^{18}+4301t^{16}-10568t^{14}+11170t^{12}-5900t^{10}+1778t^8-400t^6+5t^4-2t^2+1)}{(t-1)^2(t+1)^2(3t^2+1)^2(4t^4-t^2+1)^2(5t^4-2t^2+1)^2}$
60b	$\frac{t^2(3960t^{14}+15345t^{12}-366t^{10}-769t^8+820t^6+207t^4+2t^2+1)}{(t^2-1)(t+1)^2(3t^2+1)^2(4t^2+1)^2(5t^2-1)^2}$
60c	$\frac{t^2(27648t^{20}+35136t^{18}+193488t^{16}+253288t^{14}+140953t^{12}+37146t^{10}+3679t^8-140t^6-9t^4+10t^2+1)}{(t-1)^2(t+1)^2(3t^2+1)^2(4t^2+1)^2(4t^2-t+1)^2(4t^2+t+1)^2}$
60d	$(-t^2(279936t^{22}+608640t^{20}+931552t^{18}+1025296t^{16}+747240t^{14}+354136t^{14}+109952t^{10}+21759t^8+2444t^6+82t^4-12t^2-1))/((8t^6+17t^4+6t^2+1)^2(36t^6+28t^4+8t^2+1)^2)$

Table A.2. (Continued.)

Class	$R(f)$
60e	$\frac{t^2(1 + 11840t^{18} - 28320t^{16} + 58320t^{14} - 56575t^{12} + 27546t^{10} - 6657t^8 + 796t^6 - 33t^4 - 6t^2)}{(t-1)^2(2t+1)^2(2t-1)^2(t+1)^2(5t^2-1)^2(4t^4-t^2+1)^2}$
63a	$\frac{t^2(8748t^{18} - 7236t^{15} + 9001t^{12} + 6016t^6 + 798t^6 + 4t^3 + 1)}{(4t^3 + 1)^2(27t^6 + 2t^3 - 1)^2}$
64a	$\frac{t^2(256t^{32} + 15616t^{24} + 608t^{16} + 976t^8 + 1)}{(4t^8 + 4t^4 - 1)^2(4t^8 - 4t^4 - 1)^2}$
66a	$\frac{-t^2(4928t^{18} + 5632t^{16} + 2792t^{14} - 4688t^{12} - 8744t^{10} - 5209t^8 - 1220t^6 - 110t^4 - 4t^2 - 1)}{(11t^4 + 2t^2 - 1)^2(4t^6 + 8t^4 + 4t^2 + 1)^2}$
70a	$\frac{t^2(1 + 17920t^{18} - 76160t^{16} + 81816t^{14} - 57376t^{12} + 24608t^{10} - 6951t^8 + 1108t^6 - 34t^4 - 4t^2)}{(5t^4 - 2t^2 + 1)^2(28t^6 - 16t^4 + 8t^2 - 1)^2}$
72a	$\frac{t^2(1 + 81t^{16} + 540t^{12} + 1728t^{10} + 1494t^8 - 576t^6 + 60t^4)}{(3t^2 - 1)^2(t^2 + 1)^2(3t^4 + 6t^2 - 1)^2}$
72b	$\frac{t^2(4096t^{24} + 25088t^{18} - 4224t^{12} + 392t^6 + 1)}{(4t^3 - 1)^2(4t^3 + 1)^2(2t^3 - 1)^2(2t^3 + 1)^3}$
72c	$\frac{t^2(47104t^{26} + 530944t^{24} + 1617024t^{22} + 1902608t^{20} + 1008896t^{18} + 155025t^{16} - 115536t^{14} - 89364t^{12} - 31824t^{10} - 6346t^8 - 496t^6 + 60t^4 + 16t^2 + 1)}{((4t^2 + 1)^2(8t^6 + 9t^4 + 6t^2 + 1)^2(2t^3 - 3t^2 - 1)^2(2t^3 + 3t^2 + 1)^2)}$
72d	$\frac{-t^2(47104t^{26} - 530944t^{24} + 1617024t^{22} - 1902608t^{20} + 1008896t^{18} - 155025t^{16} - 115536t^{14} + 89364t^{12} - 31824t^{10} + 6346t^8 - 496t^6 - 60t^4 + 16t^2 - 1)}{((2t + 1)^2(2t - 1)^2(8t^6 - 9t^4 + 6t^2 - 1)^2(4t^6 + 9t^4 - 6t^2 + 1)^2)}$
72e	$\frac{(t^{48} + 3388t^{36} + 8070t^{24} + 3388t^{12} + 1)t^2}{(t^6 + 2t^3 - 1)^2(t^6 - 2t^3 - 1)^2(t^{12} + 6t^6 + 1)^2}$
76a	$\frac{-t^2(-1 + 1664t^{22} + 10624t^{20} + 18848t^{18} + 14480t^{16} - 1144t^{14} - 9752t^{12} - 8296t^{10} - 3601t^8 - 820t^6 - 110t^4 - 4t^2)}{(4t^6 + 4t^4 + 4t^2 + 1)^2(8t^6 + 7t^4 + 2t^2 - 1)^2}$
80a	$\frac{t^2(4096t^{24} + 24576t^{20} - 22528t^{16} + 8832t^{12} - 1408t^8 + 96t^4 + 1)}{(2t^2 - 1)^2(2t^2 + 1)^2(4t^4 - 2t^2 - 1)^2(4t^4 + 2t^2 - 1)^2}$
82a	$\frac{t^2(960t^{30} + 5936t^{28} - 13432t^{26} - 43207t^{24} + 3376t^{22} + 89728t^{20} + 96540t^{18} + 46736t^{16} + 22768t^{14} + 21854t^{12} + 15568t^{10} + 6080t^8 + 1260t^6 + 128t^4 + 8t^2 + 1)}{((-1 - 4t^2 + 8t^4 + 66t^6 + 120t^8 + 56t^{10} - 53t^{12} - 36t^{14} + 16t^{16})^2)}$
84a	$\frac{-t^2(-1 + 6935040t^{18} + 10886400t^{16} + 7469544t^{14} + 2901823t^{12} + 688094t^{10} + 98913t^8 + 7692t^6 + 161t^4 - 18t^2)}{(4t^2 + 1)^2(7t^2 + 1)^2(3t^2 + 1)^2(32t^4 + 11t^2 + 1)^2}$
90a	$\frac{t^2(1 + 162000t^{20} - 921456t^{19} + 1686420t^{18} - 1088640t^{17} - 641304t^{16} + 1442448t^{15} - 585855t^{14} - 414558t^{13} + 450801t^{12} - 39510t^{11} - 116700t^{10} + 43596t^9 + 12071t^8 - 9060t^7 - 95t^6 + 802t^5 - 9t^4 - 22t^3 - 10t^2)}{((2t + 1)^2(2t - 1)^2(7t^3 - 3t^2 + 1)^2(3t^3 - 3t^2 + 1)^2(6t^3 - 3t^2 + 1)^2)}$
90b	$\frac{t^2(t^{36} - 22t^{33} - 629t^{30} - 4934t^{27} - 17393t^{24} - 30244t^{21} - 26398t^{18} - 30244t^{15} - 17393t^{12} - 4934t^9 - 629t^6 - 22t^3 + 1)}{((t^6 + 3t^3 + 1)^2(t^6 + 6t^3 + 1)^2(t^6 + 7t^3 + 1)^2)}$
96a	$\frac{(4096t^{48} + 27520t^{24} + 1)t^2}{(2t^4 - 1)^2(2t^4 + 1)^2(4t^8 + 2t^4 + 1)^2(4t^8 - 2t^4 + 1)^2}$
102a	$\frac{t^2(78336t^{26} - 450432t^{24} + 1163832t^{22} - 1829344t^{20} + 1930304t^{18} - 1420191t^{16} + 738188t^{14} - 269838t^{12} + 67584t^{10} - 10837t^8 + 888t^6 + 18t^4 - 12t^2 + 1)}{((12t^6 - 16t^4 + 8t^2 - 1)^2(17t^8 - 22t^6 + 15t^4 - 6t^2 + 1)^2)}$

Table A.2. (Continued.)

Class	$R(f)$
117 α	$\frac{t^2(8748t^{30} - 136620t^{27} + 751213t^{24} - 642942t^{21} + 139865t^{18} + 52758t^{15} - 34680t^{12} + 7506t^9 - 715t^6 + 18t^3 + 1)}{(4t^3 - 1)^2(27t^6 - 11t^3 + 1)^2(t^6 + 3t^3 - 1)^2}$
120 α	$\frac{-t^2(6120t^{22} + 4455t^{20} + 14334t^{18} - 4301t^{16} - 10568t^{14} - 11170t^{12} - 5900t^{10} - 1778t^8 - 400t^6 - 5t^4 - 2t^2 - 1)}{(3t^2 - 1)^2(t^2 + 1)^2(4t^4 + t^2 + 1)^2(5t^4 + 2t^2 + 1)^2}$
126 α	$\frac{-t^2(51624t^{30} - 33625t^{24} - 25132t^{18} - 3390t^{12} - 4t^6 - 1)}{(4t^6 + 1)^2(27t^{12} + 2t^6 - 1)^2}$
132 α	$(t^2(14976t^{26} + 17280t^{24} - 114912t^{22} + 93872t^{20} + 48152t^{18} - 63736t^{16} - 5736t^{12} + 14849t^{10} - 470t^{10} - 1497t^8 + 196t^6 + 95t^4 + 2t^2 + 1)) / ((t-1)^2(t+1)^2(3t^2+1)^2(8t^4+t^2-1)^2(4t^2-4t^4+1)^2)$
140 α	$(-t^2(13440t^{26} - 503680t^{24} + 1587104t^{22} - 2412976t^{20} + 2149192t^{18} - 1169016t^{16} + 369280t^{14} - 47561t^{12} - 8778t^{10} + 4313t^8 - 492t^6 - 39t^4 + 14t^2 - 1)) / ((t-1)^2(t+1)^2(5t^2-1)^2(8t^4-5t^2+1)^2(4t^6+4t^4-4t^2+1)^2)$

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