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Vladimir Abramovich Rokhlin (23 August 1919 to 3 December 1984): Introductory note

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The Ergodic Theory and Dynamical Systems journal pays tribute to V. A. Rokhlin, one of the founders of ergodic theory, a world-renowned topologist and geometer, and a man of tragic fate and exceptional courage. Rokhlin's mathematical heritage splits rather sharply into the ergodic theory – measure theory and topology – geometry parts. This fact has to do with a natural evolution of his interests but also with the keen sense of style in mathematics that Rokhlin possessed to an unusual degree. Naturally, we will concentrate on Rokhlin's contributions to ergodic theory and measure theory, his influence on other mathematicians working in those fields, and the development of some of his ideas. Fortunately, the topology part of Rokhlin's heritage has been superbly presented in Part I of the book 'A la recherche de la topologie perdue' published by Birkhäuser in *Progress in Mathematics* series (v. 62, 1986). The same cannot be said about the work on real algebraic geometry, Rokhlin's last big achievement. The impact of that work, carried out by his students, however, is very much felt now, and is very unlikely to be forgotten or neglected. On the other hand, his work in ergodic theory and measure theory, was restricted primarily to two relatively short time periods, 1947–1950 and 1959–1964, and for the most part was not followed by Rokhlin's immediate students. Hence, it runs a certain risk of being underestimated. We hope that the articles by A. Vershik; S. Yuzvinsky and B. Weiss, published in this issue, will put Rokhlin's work and his influence in ergodic theory into the right perspective.

An obituary for Rokhlin was published in *Uspehi Math. Nauk* v. 41, no. 3, 1986, pp. 159–163. It gives a balanced assessment of Rokhlin's work but a rather blatantly inadequate account of his life. The Birkhäuser book is preceded by a short biography, which puts the main facts of his life straight, but still does not quite convey its drama. We feel that Rokhlin, whose life was greatly affected by World War II, Stalin tyranny, and later misfortunes of his country, and who met the greatest challenges in a superbly courageous way, deserves more. For this reason the article by Vershik contains a more detailed account of Rokhlin's life than is customary in scientific biographies.

The most complete bibliography of Rokhlin's work (62 items in Russian) can be found in three successive pieces in *Matematika v SSSR za* 40 *let*, v. 2, Moscow

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1959, pp. 606–604; *Matematika v SSSR* 1958–1967 v. 2, Moscow 1970, pp. 1142–1143 and the Uspehi obituary. The Birkhäuser book contains a 58-title bibliography in English. Naturally, all of Rokhlin's papers on ergodic theory and measure theory are contained in the bibliographies of the articles by Vershik and Yuzvinsky.

In the rest of this note I am taking the liberty to say a few words as a colleague, and to a certain extent, an indirect student of Rokhlin.

My first encounter with Rokhlin's mathematics came in 1963 when I was a fourth year undergraduate student at Moscow University and started to study ergodic theory with Ya. G. Sinai. One of the first articles Sinai gave me for reading was the 1960 Uspehi survey 'New progress in ergodic theory' by Rokhlin. I was quite impressed by the lucid and laconic style of the work, and it certainly influenced my view of the subject for a number of years to come. Somewhat later, in a process of a more systematic study of ergodic theory, I worked through 'On the fundamental ideas of measure theory', a masterpiece that presented central measure-theoretic results in a very geometric way, which is extremely stimulating for thinking in ergodic theory. Needless to say I later read virtually all of the Rokhlin papers in ergodic theory, operator theory and measure theory. The single most important influence, however, came from the famous approximation lemma (see B. Weiss's article) that I found in the Halmos book on ergodic theory. For a while we (young Moscow ergodic theorists of that time) did not realize that the result originally belonged to Rokhlin, and called it the 'Halmos lemma'. Anyway, this lemma and related categorical results formed the original impulse for V. I. Oseledec, A. M. Stepin, and myself for the development of what became known as the theory of periodic approximations of dynamical systems, and what was in fact the first serious advance in the combinatorial approach to ergodic theory as opposed to probabilistic methods that dominated the field at that time.

In the mid-sixties, I became personally acquainted with Rokhlin. The dominating feeling from my encounters with him was his extreme sharpness and his quest for beauty and clarity. It looked to me that he almost detested the mixed structures that are inevitable in geometric dynamics, and that was probably the reason he missed opportunities arising from combining measure-theoretic and geometric points of view for the study of dynamical systems as was suggested by Kolmogorov and E. Hopf. I still do not quite understand why Rokhlin, being a superb topologist and ergodic theorist, did not work in geometric dynamics and smooth ergodic theory. Nevertheless, he was thinking about certain problems in the area, and I remember his comment on my work with D. V. Anosov which in particular contained the first examples of C^∞ ergodic diffeomorphisms on many manifolds including such simple ones as the 2-disc. He said that analytic functions were much more natural than differentiable, and however nice our construction was, there was a certain artificiality in it that prevented it from being realized in the real-analytic category. Well, he was right. That particular construction is still not known to produce analytic examples; it took me about ten years to find another one that does.

In general, questions which Rokhlin asked and could not answer tended to be simple in form but very tricky. The multiple mixing problem is an outstanding

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example. It is still not answered, but among the developments related to it are, on one hand, the great body of work by Furstenberg and others tying together ergodic theory, number theory and combinatorics, and, on the other, extremely deep special results by Kalikov. Rokhlin was acutely aware of outstanding open problems in the field. Once he told me that he had his own file for each of those problems. I am sure those files contained a number of deep and interesting insights. But he was too proud to publish partial results, he only cared for complete solutions.

I remember another interesting episode. Once, shortly after several successful works using periodic approximations and similar methods were completed by young Moscow mathematicians, I had a discussion with Rokhlin about the current state of ergodic theory. Naturally, being enthusiastic about our recent successes, I tended to take a rather narrow view, in particular, separating rather sharply the study of zero-entropy transformations, where our results mostly belonged, from the positive entropy case. Rokhlin rebuked rather ironically, asking whether ergodic theory had become so specialized that separate people must work on zero-entropy and positive entropy transformations. Again, his insight was correct. Soon afterwards, Ornstein developed his isomorphism theory that represents a triumph of the combinatorial approach applied to positive entropy transformations based not to a small extent on the Rokhlin lemma. Several years later, an even closer synthesis between the approximations and entropy was achieved via the development of Kakutani equivalence theory. Needless to say, the separation between zero-entropy and positive entropy theory never really materialized.

During the period I knew Rokhlin personally (from the mid-sixties to the late seventies), Moscow and Leningrad stood as the two principal but unequal centers of mathematics in the Soviet Union. Moscow counted several dozen mathematicians with world class reputations, seminars teeming with brilliant students, including such grand events as the weekly citywide Mathematical Society colloquium, and the even more famous Monday evening Gelfand seminar. Leningrad mathematicians, on the other hand, were fewer in number, not so broadly spread among modern fields, and aware of the danger of becoming too parochial. Only very few Leningrad mathematicians had unquestionable status among the Moscow community. Their visits were considered important events. This elite group included A. D. Alexandrov, father and son Faddeevs, Ladyzhenskaya, Rokhlin, maybe several others, and later Gromov. It would be fair to say that Rokhlin, not necessarily being considered the number one among these people, commanded the most respect and excitement among the sophisticated Moscow audiences. His grasp of newest developments, fresh ideas of his own, and quiet but brilliant style of presentation, encouraged many young people to deeply admire him.

Gelfand was known for his rather demanding treatment of both speakers and participants in his seminar. It was not uncommon for him to stop a speaker and ask one of the advanced participants to continue (!?) with either a proof or a discussion. His own interruptions and comments, often lengthy, were also common. This was one of the reasons the seminar was not entirely to my taste. However, around 1969 I came to a Gelfand seminar to listen to Rokhlin who spoke about

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embeddings of Riemannian manifolds. The whole event turned out to be quite exemplary by the strictest standards. The speaker was in charge of his allotted time, and questions were polite and to the point. Rokhlin knew how to command respect even from the greatest people.

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