Complex Planetary Systems II: Latest Methods for an Interdisciplinary Approach
Proceedings IAU Symposium No. 382, 2024
A. Lemaitre & A.-S. Libert, eds.
doi:10.1017/S1743921323003927

Chaotic diffusion in the GJ 876 exoplanetary system

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Abstract. This is a study about the dynamical stability of the GJ 876 exoplanetary system. The phase space of initial conditions (ICs) is characterized using the MEGNO indicator of chaos and the Shannon entropy approach for estimations of diffusivity. The results are compared to analyze correlations between the chaotic layers and the instability timescales. The long-term dynamical behavior of the system is reminiscent of the *stable chaos*, at least within the system's lifetime.

Keywords. Shannon entropy, planetary instability, chaotic diffusion, Laplace resonance

1. Chaotic diffusion and instability times

Chaos and (macroscopic) *instability* constitute two different concepts that sometimes present intrinsic correlations among general dynamical problems. This may be the reason why there are some occasions in which both are wrongly used as synonyms. One can find systems where the existence of the first one does not implicate the occurrence of the second one, a scenario that has been reported in the case of the GJ 876 system (Batygin et al. 2015).

In our work, we applied the orbital parameters established in the work of Millholland et al. (2018) as the nominal condition of the real system (see Table 4, p. 7 therein), disregarding the dynamical contribution of the innermost planet due to both its reduced mass with respect to the others, and its proximity to the host star (Batygin et al. 2015; Martí et al. 2016). Hence, we modeled the system's architecture as a 4-body problem, with the Hamiltonian written using Jacobi angle-action variables (Alves Silva et al. 2021).

In Fig. 1, we plotted the results of our analysis. The first panel (left-hand frame) exhibits a MEGNO dynamical map, obtained from integrations of a grid of 300×300 ICs in the (a_3, e_3)-plane of the outermost mass. Each solution was integrated for a maximum timespan of $T_1 = 10^4$ yr (the yellow region indicates unstable orbits within times $t < T_1$). For each solution of such a map, the respective values of the MEGNO $\langle Y \rangle$ (Cincotta & Simó 2000) were computed numerically. We used them to obtain the respective Lyapunov times T_L of the trajectories, as it is indicated in the color bar.

The middle panel exhibits an instability dynamical map constructed from a grid of 200×200 ICs integrated within the same timespan T_1. At this opportunity, each solution was solved together with another five shadow trajectories to enhance statistically the outcomes. Then, for each one of this set of six trajectories, our numerical routine was able to compute the normalized Shannon entropy S associated with that orbital solution

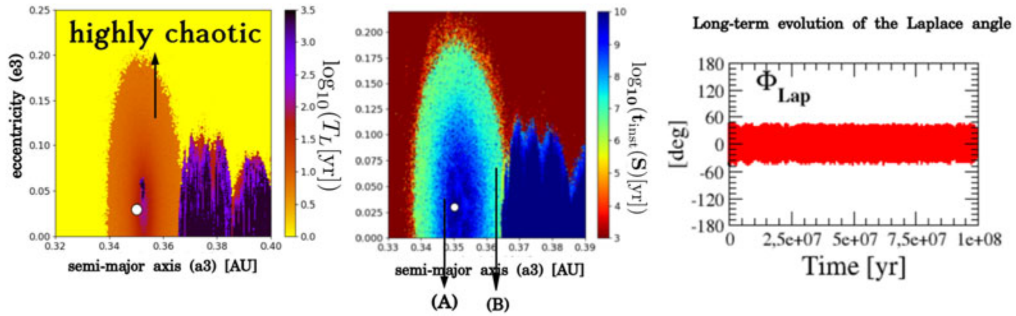

Figure 1. (*Left frame*): MEGNO map, with each solution associated to its respective Lyapunov time T_L. The nominal condition appears near the inner core of the resonance, where 10 yr $\lesssim T_L \lesssim$ 500 yr. (*Center frame*): instability time map, given in terms of the logarithm of $t_{\rm inst}(S)$: dark red regions are unstable in fast times. Dark blue region (A) is a domain of low diffusivity, with instability times of orders of tens of Gyr; (B) indicates domains where the instabilities manifest within tens of Myr. (*Right frame*): long-term integration of the nominal condition, displaying a stable evolution of the Laplace angle $\Phi_{\rm Lap}$ for 0.1 Gyr.

(see Cincotta et al. (2020a) and Cincotta et al. (2020b) for detailed explanations of the Shannon entropy approach). Using ergodic arguments, from the time derivative of S one can estimate a diffusion coefficient D_S, to which is possible to associate a macroscopic instability time† $t_{\rm inst}(S) = \frac{\Delta^2}{D_S}$, where Δ^2 denotes a given mean-square displacement, the squared distance between the initial and boundary values of actions-like variables (see Cincotta et al. (2020a,b); Alves Silva et al. (2021)).

In the right-hand panel, we show a long-term integration of the nominal condition (identified by the white dot in both previous frames) run for $T_2 = 0.1$ Gyr. The plot presents the evolution of the Laplace angle $\Phi_{\rm Lap} = \lambda_1 - 3\lambda_2 + 2\lambda_3$ (λ_i designates the mean longitude of the i-th body), since it is known that the GJ 876 system is currently inside a three-body resonant chain among its most massive planets (Batygin et al. 2015; Martí et al. 2016; Millholland et al. 2018). The system displayed stability within T_2, with $\Phi_{\rm Lap}$ oscillating around zero with an amplitude of ≈ 96 deg.

2. Discussion

Using numerical methods to solve the equations of motion, we obtained a chaotic profile of the phase space surrounding the nominal condition in terms of $\langle Y \rangle$, noticing that the system is immersed in a highly chaotic domain, for which T_L is no greater than hundreds of years. Despite the strong chaoticity, we verified that the system is expected to last over billions of years since it is located in a region of slow diffusion according to our Shannon entropy estimations. The most exciting characteristic of such measures is that they have run within the same timespan T_1 as the Lyapunov time simulations. The long-term behavior of the system was checked with integration of 0.1 Gyr, which is one-tenth of the instability times $t_{\rm inst}$ estimated for the small vicinity (A) of the nominal solution. In the current stage of our work, we are carefully analyzing the stable chaos scenario of the system.

3. Acknowledgements

This work was supported by grants from FAPESP (2016/13750-6) and CAPES.

† The instability time $t_{\rm inst}$ shall be interpreted as an expectation of the time within which dynamical instabilities would manifest on a large-scale, making the disruption of the original system likely.

Supplementary material

To view supplementary material for this article, please visit http://doi.org/10.1017/S1743921323003927

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https://doi.org/10.1017/S1743921323003927 Published online by Cambridge University Press