CORRESPONDENCE.

THINGS WORTH NOTING.

To the Editor of the Assurance Magazine

SIR,—I regret to have observed of late a great falling off in that which I consider by no means the least important or least interesting department of the *Magazine*—I mean the Correspondence department. I feel sure that if those of us who are accustomed to derive information and instruction from your pages were duly to discharge the duty we owe to the *Magazine* in return, the department to which I refer would speedily show signs of renewed vitality. We all, in the course of our reading, meet from time to time with "Things worth noting," which although often apparently trivial in themselves, are yet frequently by no means destitute of interest, either from their suggestiveness, their throwing light on some point in the history of our science, their illustrating the peculiarities and comparative advantages of different methods of investigation, or the like. Now, why should we not offer, now and then, for editorial approval, if we have nothing better, a little "Budget" of such "Things" of the above or cognate character as have come under our notice? It is very conceivable that wholesome discussion might thereby not unfrequently be excited, and no small amount of instruction elicited.

The advantage to be derived by the adoption of the course I have ventured to suggest, would not, as all experience proves, be entirely, or even principally, on the side of the readers of the *Magazine*. The writers would largely participate. This is a trite point. Suffice it to say, that the information we acquire as to the amount and the completeness of our acquaintance with a subject when we try to write upon it, and the stimulus given to our endeavours to supply the deficiencies which then almost always come to light, furnish far more than a compensation for any amount of labour that the effort to put our ideas upon paper may have cost us.

"Example," we are often told, "is better than precept." I therefore propose here to exemplify, by a first instalment, what I mean by "Things worth noting."

1. The history of the celebrated formula, $a_x = vp_x(1+a_{x+1})$, which assigns the value of an annuity upon (x) in terms of that of the corresponding benefit upon $(x+1)$ is pretty well known, up to a certain point. Mr. Milne ("Introduction," pp. xv., xvi.) attributes its discovery to Thomas Simpson, who published it in 1742, and he further informs us that it was rediscovered by Euler, whose publication of it dates in 1760. Subsequent writers, as

Mr. Galloway (*Treatise on Probability*, p. 93), attribute the formula entirely to Euler. The late Mr. Farren, however, in his work on the *Rise and Early Progress of the Doctrine of Life Contingencies in England* (1844), satisfactorily shows that the formula really originated with De Moivre, in the first edition of whose *Treatise on Annuities*, published in 1725, it was given to the world. It was suppressed, however, in the subsequent editions; the author probably considering that it was unnecessary, as his celebrated "Hypothesis" enabled him to assign the value of any annuity independently.

So far, as I have said, the history of the formula is pretty well known. But I do not think it is so well known that it has also been attributed to the now famous Mr. George Barrett. In the Useful Knowledge Society's *Treatise on Probability* (written by Sir J. W. Lubbock and the late Mr. Drinkwater Bethune), on p. 36, after deducing the formula

$$a_{x+1} = \frac{a_x}{vp_x} - 1,$$

the authors say:—"By means of this expression, *which appears first to have been noticed by Mr. Barrett*, the value of any annuity may be deduced from that which precedes or follows it." And on p. 37, after a misdescription of Dr. Price's method of computing a table of the values of annuities, they further say:—"This labour, though diminished by means of *the equation noticed by Mr. Barrett*, is still unnecessary," &c.

The foregoing statements rest on entire misconception. The formula here given is De Moivre's, but inverted; that is, the equation is solved for a_{x+1}, instead of a_x, in which state it is useless. The value of the formula as given by De Moivre consists in this, that knowing, as we always do, the value of an annuity on the oldest tabular age, we are enabled thence to deduce in succession the values of annuities on all the younger ages. To acquire the like power in connexion with the inverted formula, we should require to know the value of an annuity on the youngest age, which we have no means of doing but by going through a laborious process, which De Moivre's formula was expressly devised to supersede. And it happens, oddly enough, that the relation noticed by Mr. Barrett—for he did notice *a* relation, as we all know—is one that enables us, if we please, to dispense altogether with the formula above attributed to him.

2. A circumstance that was pointed out to me some years ago by Mr. Welton, I consider quite deserving to be put on record here. It is, that Milne's Problems XVIII. and XXVII., pp. 204 and 222, although differently enunciated and symbolized, are in reality the same. A little consideration will show that they are so; and if confirmation is needed, it will be found in the identity of the forms given for their solution. Mr. Milne does not seem to have been aware of this, for there is no reference from the one to the other, and the paragraphs cited in the demonstrations are different in the two cases. Indeed, I believe Mr. Welton, by whom alone the identity of the two problems seems to have been observed, told me that on calling Mr. Milne's attention to the matter, that gentleman expressed surprise that it should be so. The two problems, or rather the two forms of the problem, seem to have been arrived at by following different routes; and Mr. Milne, it would appear, omitted to notice that the two routes conducted to the same point.

I purpose to send you hereafter some more "Things worth noting."

But, I confess, I shall be very much disappointed if I am suffered to monopolize this department of the *Magazine.*

I am Sir,
Your most obedient servant,
P. GRAY.

Camden Town,
21st February, 1865.

ON THE TABLES OF DEFERRED ANNUITIES PUBLISHED BY THE NATIONAL DEBT OFFICE.

To the Editor of the Assurance Magazine.

DEAR SIR,—In August, 1861, I drew the attention of your readers to the remarkable discrepancy which exists between the true premiums, as deduced from the Government Tables at 3 per cent, and those charged by the Government on the purchase of those deferred annuities in which the premiums are "returnable," either on death or at the option of the purchaser at any time prior to the commencement of the annuity, pursuant to 16 & 17 Vict., cap. 45.

I am now induced to revert to the subject, for two especial reasons; the first being, that these premiums are, as I am informed, computed at $3\frac{1}{4}$ per cent., and not at 3 per cent. as assumed in my last letter, whereby the difference is *greater* than I had then stated; the second, because I did not then give the very simple method by which those premiums may be deduced from the materials furnished by the tables themselves—nor, in fact, as far as I am aware, has any method of deducing premiums returnable at the option, as well as on the death, of a purchaser, been hitherto published in any work on life annuities.

The problem then is, to find the single premium for an annuity during the remainder of a life (x) after n years, with the condition that the premium is "returnable," without interest, on death or at the option of the purchaser at any time prior to the commencement of the annuity.

As the premium (P_x) is repayable at any time during the term (n), but without interest, it must be considered from two points of view; firstly, as a sum held on trust to be ready whenever called for; and, secondly, as a fund yielding an annual income which (not being repayable under any circumstances) is to be applied year by year, during the term, in the purchase of an annuity deferred for n years; but at the expiration of the term of n years, the condition as to the return of P_x having ceased, it must itself be applied to the purchase of an immediate annuity on the life at its then increased age of $(x+n)$ years.

Let P_x=single premium "returnable" for an annuity of £1;

i =interest on £1 for a year;

$p_{x_{\overline{n}|}}$=the annual premium payable at the end of the year for assuring to x a deferred annuity of £1 after n years;

a_{x+n}=annuity on a life aged $(x+n)$;

then $P_x\cdot\dfrac{i}{p_{x_{\overline{n}|}}}$=the amount of deferred annuity which can be assured by the conversion of the annual interest into an annual premium,

and $P_x\cdot\dfrac{1}{a_{x+n}}$=the amount of annuity which can be obtained by sinking P_x at the end of the term.