

C A Murray, P M Corben and R W Argyle  
Royal Greenwich Observatory

ABSTRACT

In continuation of a long term programme of measurement of parallaxes and proper motions in the South Galactic Cap from plates taken with the UK Schmidt Telescope and measured in GALAXY at RGO, data for some 800 red stars brighter than  $B = 17.5$  have been derived from two independent plate series. The parallaxes obtained from unfiltered IIa0 plates show a large systematic error depending on magnitude compared with those obtained from IIaD plates with GG 495 filter. After correction, the combined trigonometric parallaxes show very satisfactory agreement with photometric parallaxes, and the average r.m.s. error of a combined parallax is found to be about  $\pm 0''.012$  both from internal and external evidence.

1. INTRODUCTION.

First results from an extensive parallax and proper motion survey in the region of the South Galactic Cap have been reported elsewhere by two of us (Murray and Corben 1979: Paper I). These results were derived from measurements made with the GALAXY machine at the Royal Greenwich Observatory on a series of plates obtained with the UK Schmidt Telescope at Siding Spring. The field has an area of  $4\frac{1}{2}^\circ \times 4\frac{1}{2}^\circ$ , centred at  $\alpha_{1950} = 0^h 53^m 30^s$ ,  $\delta_{1950} = -28^\circ 2'8''$  (SAO star no 166686). The discussion in Paper I was confined to some 900 stars brighter than  $B = 14$ , and was based on the measurements of about fifty plates; most of these were unfiltered IIa0 plates, but the solutions included a few plates with IIIaJ and also filtered IIa0 (+GG 385) and IIaD (+GG 495) emulsions. An important lesson which was learned in that preliminary investigation was the need to have accurate colours of stars so that proper allowance could be made for the effects of atmospheric dispersion as a function of emulsion/filter combination.

The data reductions for Paper I were carried out by means of a central overlap program using the ICL 1903T and ICL 1906A computers

which were then in use at the Royal Greenwich Observatory and the Rutherford Laboratory respectively. This program made extensive use of the special ICL Matrix Scheme package, and was not readily adaptable to other computer systems. Accordingly it has been necessary to develop new programs which we describe briefly below. These have been written in FORTRAN 77 code and implemented on the VAX 11/750 computer at the Royal Greenwich Observatory.

## 2. ASTROMETRIC DATA REDUCTION

Data reduction is carried out in two stages. Firstly, the measurements on all plates for each star are converted to standard coordinates (in units of  $0^{\circ}01$ ) referred to B1950.0 on a common field centre, and are collated in a single file. Full corrections for aberration and refraction are applied.

The second main stage of data reduction is the central overlap adjustment. This follows the Eichhorn-Jefferys technique in which plate constants are obtained from reduced normal equations after elimination of star constants, which are then derived by back-substitution. The singularity is removed by forcing linear constraints on the star constants by means of Lagrangian multipliers. Only stars appearing on all plates in a solution are used in deriving the plate constants and Lagrangian multipliers. Only a limited number of stars and plates can be processed at one time; for the present implementation on the VAX 11/750 these limits are 1000 stars on 50 plates. FORTRAN 77 codes for all these programmes and local library subroutines can be made available on request.

## 3. PHOTOMETRY

It will be recalled from Paper I that photographic B and V magnitudes were derived from one plate in each colour; these were calibrated by means of about fifty photo-electric standards obtained from various published sources. In order to improve the calibration, one of us (PMC) obtained new observations for a selection of stars uniformly distributed over the field, using the 40-inch and 20-inch telescopes at the South African Astronomical Observatory. Including these new observations, we now have 152 standard stars in the ranges  $9 < B < 17.4$  and  $8 < V < 16.7$ .

Many plates in each colour were calibrated using all the available standard stars, although some showed unacceptably large r.m.s residuals from a fourth order polynomial calibration curve. It was therefore decided to adopt the averages of three good plates in each colour; the B magnitudes were obtained from unfiltered IIA0 plates. The average r.m.s. residual per plate is  $\pm 0.05$  mag in B and  $\pm 0.07$  mag in V. Systematic corrections depending on B-V were applied in forming the

final photometric master file; this now contains over 6400 stars out of the original measuring list of more than 15000 objects.

#### 4. PARALLAXES AND PROPER MOTIONS OF RED STARS

Since the publication of Paper I, a further series of IIaD plates with GG 495 filter have been obtained; these extend in time from 1978.6 to 1980.9 and were all taken through the new achromatic corrector plate which was installed on the telescope in 1977. The present report describes the reduction of 29 of these plates, and of 40 unfiltered IIaO plates taken through the original corrector plate, for some 800 stars with B-V greater than 1.2. The apparent magnitude limit for this sample is about  $B = 17.5$ .

##### 4.1 Central overlap solutions

Because of the different sensitivities, and the fact that a star is included in the adjustment only if it appears on all plates in a series, it was not practicable to use exactly the same subset of stars in the adjustments and constraints for the reduction of the IIaD and IIaO plate series. Characteristics of the solutions for the two series are as follows:-

##### Series A

40 unfiltered IIaO plates: 1975.0 - 1977.0  
 503 stars used in adjustment  
 32 stars with  $12 < V < 14$ ;  $1.2 < B-V < 1.4$  used in constraints  
 s.e. unit weight  $\pm 0!0875$

##### Series B

29 IIaD plates with GG 495 filter: 1978.6 - 1980.9  
 630 stars used in adjustment  
 72 stars with  $15 < V < 16$ ;  $1.2 < B-V < 1.3$  used in constraints  
 s.e. unit weight  $\pm 0!0645$

Weights 1.0 or 0.5 were assigned according to image size as measured by GALAXY. If the errors are normally distributed, then the weighted sums of squares of residuals for individual stars, divided by the variance of unit weight should be distributed as  $\chi^2$ . The number of degrees of freedom is approximately  $2n-p$  where  $n$  is the number of plates and  $p$  the number of star constants determined for each star; this is not quite true since the plate constants have also been determined from the total subset of stars used in the adjustment, but we ignore this slight diminution in the number of degrees of freedom.

If some residuals are abnormally large, due for example to bad measurement of blended images, they will make the computed variance of unit weight also too large, and hence all the values of  $\chi^2$  systematically too small compared with the theoretical distribution. Accordingly it is necessary to reject stars with abnormally large residuals from the adjustment and then recompute  $\chi^2$  for each star until there is approximate consistency between the observed and theoretical distributions. This procedure was carried out for both series A and B, and the resulting observed distribution of  $\chi^2$  are compared with the theoretical distributions for 75 and 53 degrees of freedom respectively in Figs 1(a) and 1(b). Although the fits are not exact, the maximum frequency of  $\chi^2$  in each case is in reasonably good agreement with the theoretical values, and we regard these adjustments as satisfactory.

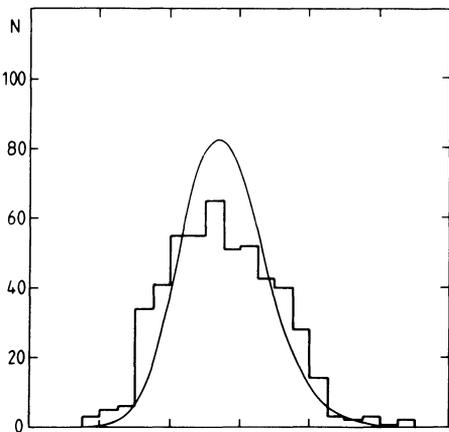


Fig 1(a)

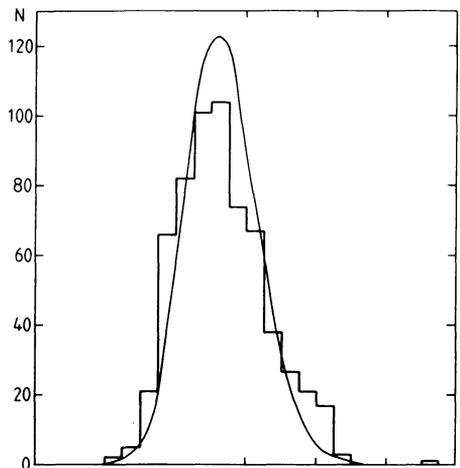


Fig 1(b)

The formal internal standard errors of a single component of position,  $\sigma$ , and annual proper motion,  $\sigma_{\mu}$ , and of trigonometric parallax,  $\sigma_{\pi}$ , for a star with full weight, in each series, are given in Table I.

Table I

	Series A	Series B
$\sigma$	$\pm \text{!}0142$	$\pm \text{!}0120$
$\sigma_{\mu}$	$\pm \text{!}0191$	$\pm \text{!}0172$
$\sigma_{\pi}$	$\pm \text{!}0156$	$\pm \text{!}0124$

The standard error of position refers to the weighted mean epoch of all plates.

#### 4.2 Combination of the two solutions for star constants

Because the samples of stars used in forming the linear constraints are different for series A and B, there may well be a systematic coordinate dependent linear transformation between the individual results derived from the two solutions. At the present stage in the work this possibility has been ignored and results from Series A have been transformed to those of Series B by means of zero point corrections only, to the proper motion components and parallaxes; these corrections were obtained from the mean values from the Series A solution, for the 72 stars which formed the constraints for the Series B solution. A reduction to absolute parallax of  $+\text{!}0032$  has been applied.

A first examination of the trigonometric parallaxes obtained in the two solutions showed a very marked dependence on magnitude of the differences between the two, in the sense that for bright stars the Series A solution (unfiltered IIA0 plates) gave parallaxes systematically more positive than the Series B solution (IIAD plates +GG 495 filter). There appeared to be little dependence on colour, over the rather small range, ( $1.2 < B-V < 1.8$ ) in colour in these samples. As yet no quantitative explanation for this effect has been found although it is reasonable to link it with the atmospheric dispersion problems associated with the unfiltered IIA0 plates. It was pointed out in Paper I that, because of the correlation between parallax factor and hour angle, the error of parallax will be about half the error in the refraction. Such an error could arise from the non-linearity of the photographic emulsion, by which the saturated images of bright stars will be shifted relative to those of unsaturated images of faint stars at the same colour (Bergstrand effect). We accordingly take the view that the true system of parallaxes is that defined by the Series B and have applied the empirical correction

$$\Delta\pi_A = -0\text{!}05 + 0\text{!}0009 (B-9.5)^2 \quad (1)$$

to the parallaxes derived from Series A before combining with Series B.

If  $\underline{p}_A$ ,  $\underline{p}_B$  denote the vectors of star constants for a particular star obtained from the two series and  $\underline{C}_A$ ,  $\underline{C}_B$  the corresponding covariance matrices, then the vector,  $\underline{p}$ , of star constants from the combination of the two series was computed from

$$(\underline{C}_A^{-1} + \underline{C}_B^{-1}) \underline{p} = \underline{C}_A^{-1} \underline{p}_A + \underline{C}_B^{-1} \underline{p}_B$$

In calculating the covariance matrices, the epoch of each separate solution was changed to that of the weighted mean epoch for each star, and the individual variances obtained from the residuals for each star, were used.

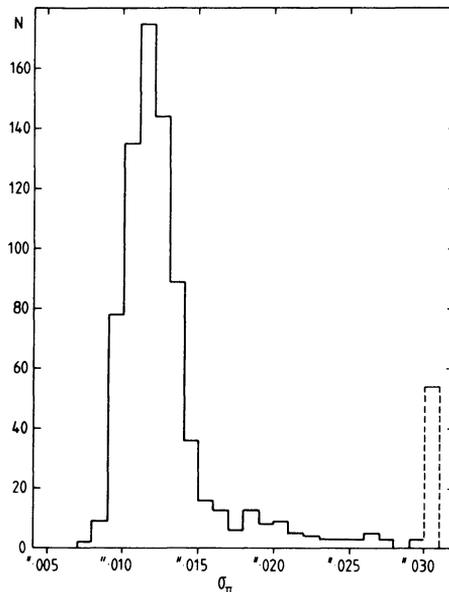


Fig 2

The distribution of the standard errors of the parallaxes of individual stars is shown in Fig. 2. There is a very satisfactorily small spread, about a mean value of  $\pm 0.012$ , with comparatively few in excess of  $\pm 0.02$ ; these however include some very large values for objects with poor or blended images.

## 5. ACCURACY OF THE TRIGONOMETRIC PARALLAXES

A full discussion of the parallaxes and proper motions obtained in this investigation will appear elsewhere. The present discussion is confined to an assessment of the real accuracy of the parallaxes. This can be examined in two ways, from internal evidence among the large number of parallaxes measured and from comparison between the trigonometric parallaxes and photometric parallaxes for any stars for which the latter can be obtained.

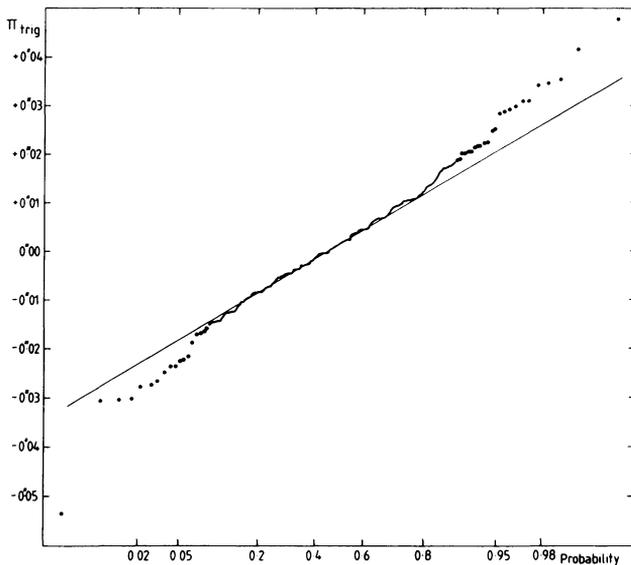


Fig 3

In Fig. 3 we show the probability plot of the parallaxes of 201 stars with formal standard errors less than  $0''.015$  and with  $1.2 < B-V < 1.3$ . Over most of the range of probability less than 0.8 the plot is satisfactorily linear, even if we include the highly discordant large negative parallax of  $-0''.053$ . Furthermore, the slope of the straight line corresponds to a standard deviation of  $\pm 0''.012$  which is exactly that predicted from the average internal standard error. The real random errors are thus consistent with the internal errors and the departure from linearity of the probability plot at probabilities larger than 0.8 shows statistical evidence for a distribution of real parallaxes, although it would obviously be dangerous to assign particular values to individual stars.

Spectroscopic or photometric data have been obtained, for a number of stars in our sample, by various authors. Jones et al. (1981) have published photometric parallaxes based on narrow band measurements for three dwarfs and Jones has kindly estimated parallaxes for four more dwarfs from unpublished material. Eggen's (1976a) photometric survey of large proper stars includes a further twelve of our stars which he classified as M dwarfs and for which he has given photometric parallaxes (Eggen 1976b). Recently Reid has carried out spectroscopic observations of seven of our stars, of which he finds two to be dwarfs; photometric parallaxes for these have been derived from unpublished V, I photometry by Reid and Gilmore. There are thus 21 dwarfs for which we can compare trigonometric and photometric parallaxes. In addition we have five giants identified by Reid and three by Alexander et al (1983) for which we assume the photometric parallax to be effectively zero.

We denote by  $\pi_A$ ,  $\pi_B$  the trigonometric parallaxes obtained from the Series A and B solutions respectively and  $\pi_{\text{phot}}$  the photometric parallax. In Fig. 4(a) we show  $\pi_A - \pi_{\text{phot}}$  plotted against B magnitude; the crosses denote giants and the filled circles dwarfs. The smooth curve represents the error  $-\Delta\pi_A$  (equation (1)) which was derived from all the available data  $\pi_A - \pi_B$ ; this shows reasonably good agreement with the plotted points, although it is based on many more stars. In Fig. 4(b) we show a similar plot for  $\pi_B - \pi_{\text{phot}}$ . In this case there seems to be no systematic dependence on magnitude; this supports the view that the source of the magnitude dependent error must be in the unfiltered IIa0 plates (Series A).

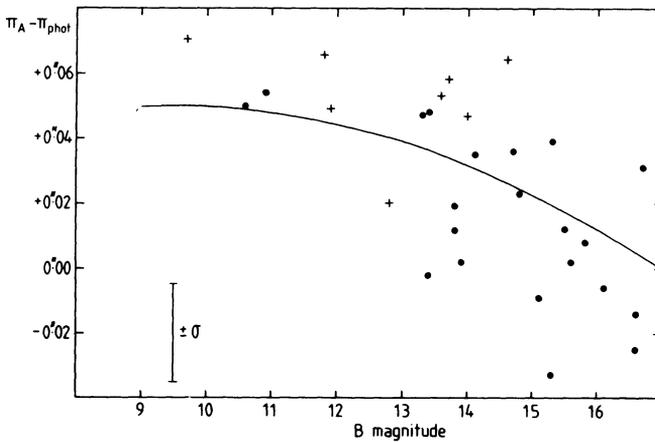


Fig 4(a)

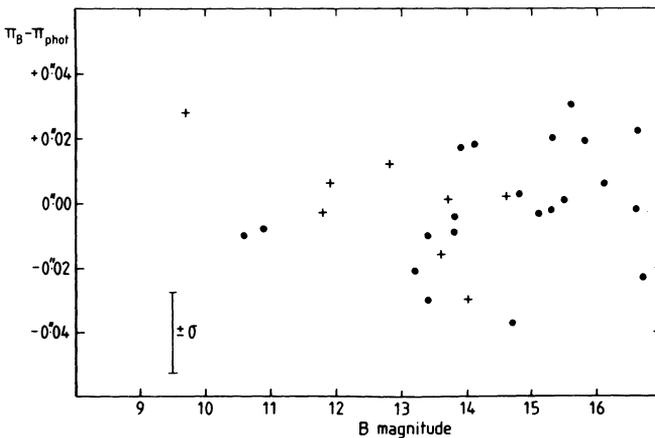
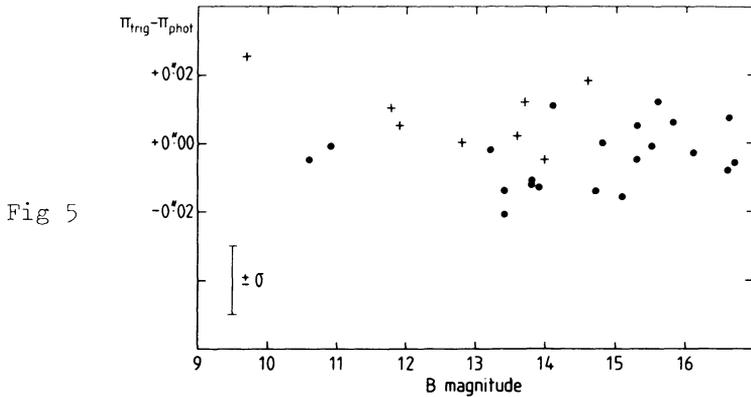


Fig 4(b)



The combined parallax  $\pi_{\text{trig}}$  after correcting  $\pi_A$  by  $\Delta\pi_A$  is compared with  $\pi_{\text{phot}}$  in Fig. 5. We now have excellent overall agreement between the trigonometric and photometric parallaxes, although there is some suggestion that the average trigonometric parallax of the giants may be systematically too large. We conclude that the formal error quoted for the parallaxes gives a realistic estimate of their uncertainty and that, though systematic errors depending on magnitude may still be present, the whole series of parallaxes provide a statistically homogeneous data set for further analysis of the space distribution of the stars.

## 6. ACKNOWLEDGEMENTS

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## References

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