

## SWELLING PRESSURE CALCULATED FROM MINERALOGICAL PROPERTIES OF A JURASSIC OPALINUM SHALE, SWITZERLAND

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**Abstract**—Nineteen drill core samples of lower Dogger opalinum shale from wells drilled in connection with a tunnel project near Brugg, northern Switzerland, were investigated. The shale is a well known swelling rock that has caused problems in underground construction work. Swelling pressures determined under constant volume conditions to obtain maximum values were 0.7 to 2.2 N/mm<sup>2</sup>. The samples contained 37–59% clay-size material and about 35% quartz, 7–18% carbonate minerals, and about 5% feldspar, pyrite, and organic matter. In addition to kaolinite, illite, and chlorite, the clay-size fraction also contained mixed-layer illite/smectite with about 30% swelling layers. The specific surface area of the clay fraction was 135 m<sup>2</sup>/g. The specific surface charge of the clay ( $6.7 \times 10^4$  esu/cm<sup>2</sup>), the ion concentration in the pore fluid of the specimen after the swelling test ( $10^{-2}$  mmole/cm<sup>3</sup>), the valence of the ions in the double layer of the clay particles (+1), and the half distance between the clay plates in the specimen (8–15 Å) allowed the calculation of the swelling pressure for each sample according to the Gouy double layer theory.

The mean value of the calculated swelling pressures was found to be of the same order of magnitude as the measured values, indicating that the technique can be used where cylindrical or rectangular specimens are not available for direct measurement.

**Key Words**—Gouy double layer theory, Mineralogical composition, Shale, Soil mechanics, Surface area, Surface charge, Swelling pressure.

### INTRODUCTION

In recent years many Swiss railway tunnels have had to be repaired due to damage caused by swelling rocks. In new highway tunnels swelling rocks have also caused great difficulties (Grob, 1976; Einstein, 1979). The rock movement in the tunnels is mostly in the form of heave of the floor, followed by a convergence of the abutments. From laboratory tests and in-situ observations it is known that the prevention of swelling strains induces considerable compressive stresses, the so-called swelling pressure. Swiss rocks particularly susceptible to swelling are Tertiary marlstones (molasse) which contain ~30% montmorillonite in the clay fraction, the Triassic Keuper shale which contains corrensite and anhydrite, and a lower Dogger opalinum shale (named after the ammonite *Leioceras opalinum*) containing mixed-layer minerals illite/montmorillonite.

In connection with a new tunnel project near Brugg in northern Switzerland, the swelling pressure of 19 samples of opalinum shale was investigated. The measurements were made on drill cores from between 15- and 155-m depth. The variation of the results could not be explained on the basis of sample depths, and it was assumed that differences in the mineralogical composition could be the reason for the different swelling pressures. Therefore, the mineralogical composition of the specimens was investigated to allow the calculation

of the swelling pressure from mineralogical parameters according to the Gouy double layer theory.

### SWELLING PRESSURE THEORY

In clay minerals, the electric double layer consists of a negative surface charge and a compensating counterion charge, which is accumulated in the liquid near the surface of the clay particle. The positive counter-ions are electrostatically attracted by the negative charged surface. The concentration of the positive ions near the surface is high, and it decreases with increasing distance from the surface. Simultaneously, a deficiency of negative ions exists near the surface, inasmuch as these ions are electrostatically repelled by the clay particle. The distribution of the ions as a function of the distance from the clay surface can be calculated according to the electrostatic and diffusion theory using the Poisson-Boltzmann equation (Gouy, 1910, 1917; Chapman, 1913).

According to the double layer theory (Verwey and Overbeek, 1948; Bolt, 1955, 1956; van Olphen, 1977), the interacting force between two double layers can be derived from the ion concentration midway between the clay surfaces, which is determined by the value of the electric potential. The repulsive pressure (Langmuir, 1938) is given by the osmotic pressure midway between the surfaces with respect to that of the equi-

librium solution. Because these osmotic pressures are determined by the ion concentration, the pressure can be calculated directly from the excess concentration midway between the clay surfaces. This excess concentration is equal to

$$n(\exp(u) - 1) + n(\exp(-u) - 1) = 2n(\cosh u - 1)$$

and the pressure is

$$p = 2nkT(\cosh u - 1), \quad (1)$$

where  $p$  = repulsive pressure,  $n$  = ion concentration far from the clay surface,  $u$  = electric potential midway between the clay surfaces,  $k$  = Boltzmann constant, and  $T$  = absolute temperature.

The ion concentration  $n$ , which here is equal to the ion concentration in the pore fluid of the swelling specimen, and the temperature  $T$  can be easily determined. The electric potential  $u$  cannot be measured directly, but must be calculated according to the double layer theory as follows: For equilibrium conditions in the double layer, the average local concentration of positive and negative ions at a distance  $x$  from the clay surface can be expressed as a function of the average electric potential at that distance according to Boltzmann's theorem:

$$\begin{aligned} n_+ &= n_+^* \exp(-\nu_+ e\psi/kT) \text{ and} \\ n_- &= n_-^* \exp(\nu_- e\psi/kT), \end{aligned} \quad (2)$$

in which  $n_+$  and  $n_-$  = local concentrations of positive and negative ions,  $n_+^*$  and  $n_-^*$  = concentrations of positive and negative ions far away from the surface (potential = 0),  $\nu_+$  and  $\nu_-$  = valences of the ions,  $e$  = elementary charge, and  $\psi$  = electric potential. The local density of charge  $\rho$  in a distance  $x$  is

$$\rho = \nu_+ en_+ - \nu_- en_-.$$

Because the valence of the ions of the same sign as the surface charge is not very important,  $\nu_+$  is assumed to equal  $\nu_-$ ; thus,  $n_+^* = n_-^* = n$ . Then,

$$\rho = \nu e(n_+ - n_-). \quad (3)$$

Combining Eqs. (2) and (3) gives

$$\rho = (-2\nu e) \sinh(\nu e\psi/kT), \quad (4)$$

which describes the local density of charge as a hyperbolic function of the potential. The local density of charge and the local electric potential are also related by the Poisson equation

$$\frac{d^2\psi}{dx^2} = -\frac{4\pi\rho}{\epsilon}, \quad (5)$$

which describes the variation of the field strength as a function of the local density of charge.  $\epsilon$  is the dielectric constant of the medium (water). Combining Eqs. (4) and (5), the fundamental equation for the double layer is obtained,

$$\frac{d^2\psi}{dx^2} = \frac{8\pi ne\nu}{\epsilon} \sinh(\nu e\psi/kT), \quad (6)$$

which describes the variation of the field strength as a hyperbolic function of the potential. Substituting  $y = \nu e\psi/kT$ ,  $z = \nu e\psi_0/kT$ ,  $\xi = \chi x$ , and  $\chi^2 = \frac{8\pi ne^2\nu^2}{\epsilon kT}$  gives

$$\frac{d^2y}{d\xi^2} = \sinh y, \quad (6a)$$

where  $\psi_0$  = surface potential. In the midway plane between two clay surfaces, at a distance  $d$  from either surface, the field strength is zero. The potential midway between the surfaces is  $\psi_d$ , and  $u = \nu e\psi_d/kT$ .

Integrating Eq. (6a) once with the boundary condition that for  $x = d$ ,  $y = u$  and  $\frac{dy}{d\xi} = 0$  gives

$$\frac{dy}{d\xi} = -(2 \cosh y - 2 \cosh u)^{0.5}. \quad (7)$$

The second integration between the limits  $z$  and  $u$  for  $y$  or 0 and  $d$  for  $x$  gives

$$\int_z^u (2 \cosh y - 2 \cosh u)^{-0.5} dy = -\int_0^d d\xi = -\chi d.$$

Evaluating this elliptic integral using tables (Jahnke and Emde, 1933) gives

$$\begin{aligned} \chi d &= 2 \exp\left(-\frac{u}{2}\right) \\ &\cdot \left\{ F_1 \left[ \exp(-u), \frac{\pi}{2} \right] \right. \\ &\quad \left. - F_2 \left[ \exp(-u), \arcsin \exp\left(-\left(\frac{z-u}{2}\right)\right) \right] \right\} \quad (8) \end{aligned}$$

where the functions  $F_1$  and  $F_2$  are obtained using the tables.

The charge of the clay particle is a result of cation substitutions in the crystal. Therefore, the surface charge of the particle is a constant. Thus, the potential midway between the clay surfaces  $u$  is a function of the surface potential  $z$  and the half distance  $d$  and  $z$  is a function of the surface charge  $D$  and  $d$ .

The surface charge  $D$  is given by

$$D = -\int_0^d \rho \, dx$$

and according to Eq. (5)

$$D = -\left(\frac{\epsilon}{4\pi}\right) \left(\frac{d\psi}{dx}\right)_0. \quad (9)$$

As  $y = \nu e\psi/kT$  and  $\xi = \chi x$ , Eq. (9) is transformed to

$$D = -\left(\frac{\epsilon nkT}{2\pi}\right)^{0.5} \left(\frac{dy}{d\xi}\right)_0. \quad (9a)$$

Combining Eqs. (9a) and (7) gives as  $y = z$  for  $\xi = 0$

$$D = \left( \frac{\epsilon n k T}{2\pi} \right)^{0.5} (2 \cosh z - 2 \cosh u)^{0.5}.$$

Therefore, for a constant charge,

$$(2 \cosh z - 2 \cosh u)^{0.5} (n)^{0.5} = \text{constant}$$

independent of the distance between the clay surfaces. To calculate the change of the midway potential  $u$  with varying distance  $2d$  at a given electrolyte concentration  $n$ ,  $\chi$  sets of values for  $z$  and  $u$  are required for which

$$(2 \cosh z - 2 \cosh u)^{0.5} = D \left( \frac{2\pi}{\epsilon n k T} \right)^{0.5} = \text{constant}.$$

Knowing the values of surface charge  $D$ , ion concentration  $n$ , valence  $\nu$ , temperature  $T$ , and dielectric constant  $\epsilon$ , it is possible to calculate the midway potential  $u$  as a function of the particle distance, and also to calculate the repulsive pressure  $p$  as a function of the particle distance.

The steps of the calculation are:

- (1)  $\chi$ , according to  $\chi = \left( \frac{8\pi n e^2 \nu^2}{\epsilon k T} \right)^{0.5}$ ;
- (2)  $\left( \frac{dy}{d\xi} \right)_0$ , according to  $\left( \frac{dy}{d\xi} \right)_0 = \frac{4\pi \nu D}{\epsilon k T \chi}$  which is obtained from Eq. (9a) with  $n = \frac{\chi^2 \epsilon k T}{8\pi e^2 \nu^2}$ ;
- (3)  $z$  for chosen values of  $u$ , according to  $\left( \frac{dy}{d\xi} \right)_0 = -(2 \cosh z - 2 \cosh u)^{0.5}$ ;
- (4)  $d$  with values of  $\chi$ ,  $z$ , and  $u$ , according to Eq. (8);
- (5)  $p(d)$  for the values of  $u(d)$ , according to Eq. (1).

The half distance  $d$  was calculated as follows: If, as assumed, all water in the saturated sample was associated with the clay particles, the half distance  $d$  was the quotient of the specific water volume of the sample after the measurement of the swelling pressure and the specific surface area of the sample, i.e.,

$$d = \frac{\text{specific water volume}}{\text{specific surface area}}.$$

## EXPERIMENTAL

### Apparatus

The swelling pressure apparatus as shown in Figure 1 consisted essentially of a loading device capable of continuous adjustment to maintain the specimen at constant height as the swelling process develops. The adjustment was made by tightening the upper nuts, whereas the force required to resist the axial displacement was measured by the pressure cell. The displacement, i.e., the swelling strain, was measured by the two

micrometer dial gauges. The specimen was embedded tightly in a stainless steel ring for rigid radial restraint. Porous plates at the top and bottom of the specimen allowed the water access. A stainless steel plate, 1.5-cm thick, at the top of the upper porous plate, ensured the rigid application of the force. The specimen assembly was contained in a 15-cm diameter cell, capable of being filled with water to a level above the top of the specimen.

### Mineralogical investigations

From the theoretical considerations, the following parameters of the specimen must be known in order to calculate its swelling pressure: valence of the counter-ions in the double layer ( $\nu$ ), ion concentration far away from the clay surface, i.e., pore water of the specimen ( $n$ ), surface charge of the clay ( $D$ ), and the half distance between the clay surfaces ( $d$ ). After measuring the swelling pressure, which took about 14 days for each sample, the valence  $\nu$  and the ion concentration  $n$  were determined by analyzing the water from the cell of the swelling pressure apparatus (Figure 1). The amount of  $\text{Na}^+$  and  $\text{K}^+$  was measured with a flame photometer;  $\text{Ca}^{2+}$  and  $\text{Mg}^{2+}$  were obtained by the complexometric method. One half of the swelling specimen was used to determine the water content by drying it for 24 hr at  $105^\circ\text{C}$ . The other half of the swelling specimen was dried at room temperature and used for the remainder of the determinations according to Figure 2.

The external specific surface area was measured by the one-point BET method with  $\text{N}_2$ . The specimen was degassed for 18 hr at  $150^\circ\text{C}$  and  $10^{-6}$  torr before the adsorption was measured. To ensure a representative composition of the specimens being investigated, a mechanical sample splitter was used to divide the bulk specimen. The external specific surface area was used in the calculation of the total specific surface area (external plus internal area). The carbonate content was determined by means of an evacuated system using the method of Hutchinson and MacLennan as reported in Piper (1944). The carbonate, mainly  $\text{CaCO}_3$ , was treated with  $\text{HCl}$ ;  $\text{CO}_2$  was passed into a  $\text{NaOH}$  solution, and after precipitation with  $\text{BaCl}_2$ , it was determined by titration. The carbonate content was used to recalculate the original sample composition after determining the size of clay fraction from the decarbonated sample.

Before the amount of the clay fraction ( $<0.002$  mm) was obtained by the pipet-method, it was necessary to eliminate the carbonates. For this purpose  $\sim 5$  g of sample was treated for 10 min in an ultrasonic bath with a frequency of 28 kHz and an amplitude of 0.028 mm (Müller-Vonmoos, 1971), and then boiled in a pH 5 sodium acetate-acetic buffer solution. After removing the organic matter with  $\text{H}_2\text{O}_2$ , the noncrystalline

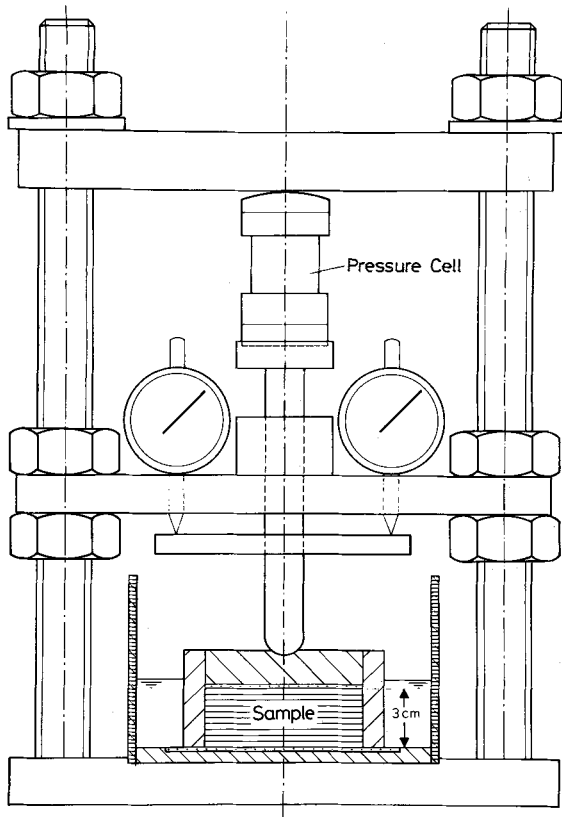


Figure 1. Apparatus for measuring the swelling pressure.

coatings on the clay particles were eliminated by extraction with dithionite-citrate (Mehra and Jackson, 1960). The clay fraction was then saturated with  $\text{Ca}^{2+}$  for the further investigations. The clay minerals were characterized by X-ray powder diffraction (XRD) ( $\text{CuK}\alpha$  radiation) using orientated specimens. The specimens were treated with glycerol and with hydrazine.

The internal specific surface area of the clay fraction was determined by glycerol sorption on a Mettler-thermobalance (Madsen, 1977) and recalculated using the above obtained amount of the clay fraction to the original sample composition. About 100 mg of the clay fraction was treated with glycerol and heated to  $300^\circ\text{C}$  at  $1^\circ\text{C}/\text{minute}$ . Between  $100^\circ$  and  $170^\circ\text{C}$  the loss of weight was about 0.5 mg/min. At  $170^\circ\text{C}$  the loss of weight slowed, and between about  $200^\circ$  and  $230^\circ\text{C}$  it was  $\sim 0.08$  mg/min. Above  $230^\circ\text{C}$  a loss of weight of  $\sim 0.5$  mg/min was again observed. It was assumed that at about  $200^\circ\text{C}$  all glycerol except a monolayer was lost. The specific surface area of the clay was calculated assuming a value of  $1967\text{ m}^2/\text{g}$  for a monolayer thickness of  $4.5\text{ \AA}$  for glycerol. The cation-exchange capacity of the clay fraction was determined by the method of Mackenzie (1951) using  $\text{NH}_4^+$ .

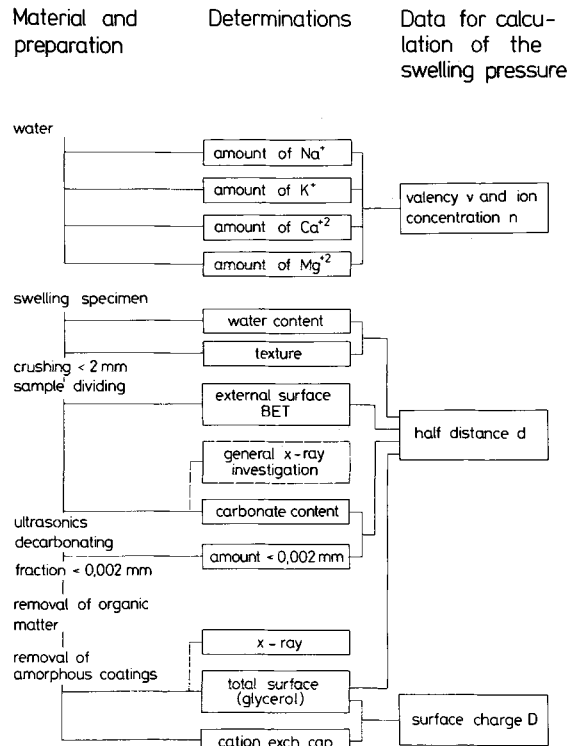


Figure 2. Procedure for the determination of the mineralogical data.

### Measurement of the swelling pressure

The swelling pressure was measured according to the recommendations of the International Society for Rock Mechanics (1979). The test measures the pressure necessary to constrain the rock specimen at constant volume when it is immersed in water. The measurements were made on 8-cm diameter drilling cores from between 15 m and 155 m depth. The cores were sawed into discs with a thickness of 3 cm. The disc was fitted into the stainless steel ring, and the apparatus was assembled. A small axial stress, about  $0.025\text{ N}/\text{mm}^2$ , was applied to the specimen, and the cell was flooded with water to cover the top of the porous plate. Because the chemistry of the water in the rock mass from which the samples were obtained was unknown, distilled water was used in all tests. The swelling heave was measured by the dial gauges, and after a heave of about 0.01 to 0.02 mm, the original height of the sample was re-established by increasing the axial stress. The value of the stress,  $\sigma^*$ , at which no further swelling strain was observed, was designated the swelling pressure.

## RESULTS

### Clay data

The determination of the amount of  $\text{Na}^+$ ,  $\text{K}^+$ ,  $\text{Ca}^{2+}$ , and  $\text{Mg}^{2+}$  from the water in the swelling pressure cell

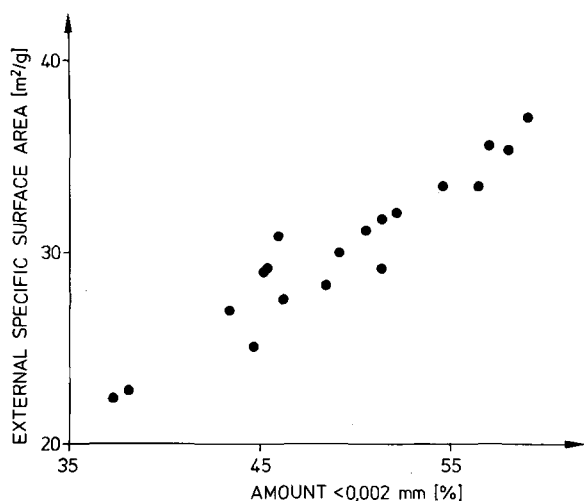


Figure 3. External specific surface area as a function of the size of the clay fraction.

proved that the ratio of mono- to bivalent ions was 80:20.  $\text{Al}^{3+}$  or  $\text{Fe}^{3+}$  were not present. About 75% of the ions were  $\text{Na}^+$ . The mean value of the ion concentration was  $10^{-2}$  mmole/cm<sup>3</sup>. The water content of the saturated swelling specimens varied between 6.3 and 8.3% of the dry sample weight. The external specific surface area varied between 22 and 37 m<sup>2</sup>/g. Because the composition of the clay fraction of the opalinum shale was quite homogeneous, the specific surface area depended only on the size of the clay fraction (<0.002 mm), as shown in Figure 3. The carbonate content varied between 7 and 18%, and the clay fraction, calculated for the original sample composition, was between 37 and 59% of the dry sample weight.

Besides clay minerals and carbonate, the opalinum shale consists mainly of quartz (~35%) with small amounts (<5%) of feldspar, pyrite, and organics. According to the nomenclature of Füchtbauer (1959), the opalinum shale is a sandy or carbonitic sandy claystone. Figure 4 shows the XRD pattern of the clay fraction. On the left is the air-dried specimen with mixed-layer peaks at 12 and 35 Å. In the middle is the same specimen after treatment with glycerol. According to Brown (1961) and Brindley (1966), the mixed-layer is an illite/montmorillonite with about 30% swelling layers. The righthand diagram shows a specimen after treatment with hydrazine. The 001 peak of kaolinite thus moved from 7 to about 10 Å. The clay fraction therefore consists of mixed-layer illite/montmorillonite, illite, kaolinite, and chlorite.

The internal specific surface area of the clay fraction was about 75 m<sup>2</sup>/g for all specimens. These data mean that the clay fraction contains about 35% mixed-layer illite/montmorillonite with 30% swelling layers. The total specific surface area of the clay fraction (external plus internal) for all samples was between 130 and 140

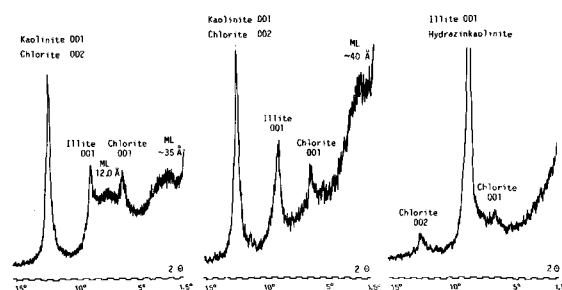


Figure 4. X-ray powder diffractograms of the clay fraction using  $\text{CuK}\alpha$  radiation. Left: air-dried specimen; middle: glycerol-treated specimen; right: hydrazine-treated specimen.

m<sup>2</sup>/g (mean value 135 m<sup>2</sup>/g). The cation-exchange capacity of the clay fraction varied from 28 to 34 meq/100 g with a mean value of 31 meq/100 g.

#### Swelling data

The swelling test took about 14 days for each sample. After this time, no further swelling strain was observed, and equilibrium was assumed to have been established between the pore water in the sample and the water in the cell of the apparatus. The incremental values of stress,  $\Delta\sigma_i$ , and observed strain,  $\Delta\epsilon_i$ , are represented as a summation curve for one sample in Figure 5. The value of stress  $\sigma^*$  at which no further swelling strain was observed was designated the swelling pressure. The swelling pressure of the 19 investigated cores was between 0.7 and 2.2 N/mm<sup>2</sup> (Table 1). (1 Newton/mm<sup>2</sup> = 142 lb/sq in.)

#### DOUBLE LAYER CALCULATIONS

The following data were chosen for the calculation of the swelling pressure function  $p(d)$ : valence,  $\nu = 1$ ; ion concentration,  $n = 10^{-2}$  mmole/cm<sup>3</sup> =  $6.02 \times 10^{18}$  ions/cm<sup>3</sup>; specific surface area of clay fraction, 135 m<sup>2</sup>/g; and cation-exchange capacity, 31 meq/100 g. The constants are:  $k = \text{Boltzmann constant} = 1.38 \times 10^{-16}$

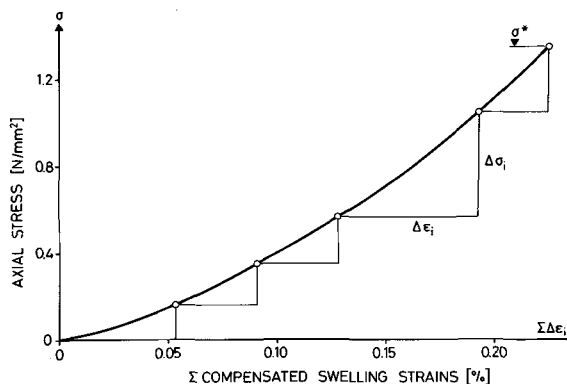


Figure 5. Determination of the swelling pressure  $\sigma^*$  at constant volume of sample.



erg/°K;  $\epsilon$  = dielectric constant of water = 80;  $e$  = elementary charge =  $4.80 \times 10^{-10}$  esu; and  $T$  = absolute temperature = 293°K (20°C).

The mean value of the specific surface charge,  $D$ , of the clay fraction was computed from the specific surface area and the cation-exchange capacity (CEC) as follows:

$$\text{area per ion} = \frac{\text{specific surface area}}{\text{CEC(Avogadro's number)}}$$

$$\begin{aligned} \text{area per ion} &= \frac{135 \times 10^4 \text{ cm}^2/\text{g}}{31 \times 10^{-5} \cdot 6.02 \times 10^{23} \text{ ions/g}} \\ &= 72 \times 10^{-16} \text{ cm}^2/\text{ion}, \end{aligned}$$

$$\begin{aligned} D &= \frac{\text{charge}}{\text{area}} \\ &= \frac{4.80 \times 10^{-10} \text{ esu}}{72 \times 10^{-16} \text{ cm}^2} \\ &= 6.7 \times 10^4 \text{ esu/cm}^2. \end{aligned}$$

The value  $u(d)$  was calculated from:

$$\chi = \left( \frac{8\pi n e^2 \nu^2}{\epsilon k T} \right)^{0.5} = 328 \times 10^4 \text{ cm}^{-1}$$

and

$$\left( \frac{dy}{d\xi} \right)_0 = \frac{4\pi e \nu D}{\epsilon k T \chi} = 38.1.$$

The value of  $z$  for chosen values of  $u$  was calculated from:

$$\left( \frac{dy}{d\xi} \right)_0 = -(2 \cosh z - 2 \cosh u)^{0.5}$$

and

$$\cosh z = 0.5 \left( \frac{dy}{d\xi} \right)_0^2 + \cosh u,$$

which yielded the following values:

$u$	$\cosh u$	$\cosh z$	$z$
2	3.7622	729.5672	7.2856
3	10.0677	735.8727	7.2942
3.5	16.5728	742.3768	7.3030
4	27.3082	753.1132	7.3174
4.5	45.0141	770.8191	7.3406
5	74.2099	800.0149	7.3778

Values of  $d$  for values of  $\chi$ ,  $z$ , and  $u$  were calculated as follows:

$$\begin{aligned} \chi d &= 2 \exp\left(-\frac{u}{2}\right) \\ &\cdot \left\{ F_1 \left[ \exp(-u), \frac{\pi}{2} \right] \right. \\ &\quad \left. - F_2 \left[ \exp(-u), \arcsin \exp\left(-\frac{z-u}{2}\right) \right] \right\}, \end{aligned}$$

$u$	$2 \exp\left(-\frac{u}{2}\right)$	$F_1$	$F_2$	$\chi d$	$d(\text{\AA})$
2	0.7358	1.5788	0.07122	1.1093	33.8
3	0.4463	1.5725	0.11707	0.6496	19.8
3.5	0.3475	1.5718	0.14988	0.4941	15.1
4	0.2707	1.5714	0.1915	0.3735	11.4
4.5	0.2108	1.5712	0.2440	0.2798	8.5
5	0.1642	1.5710	0.3095	0.2071	6.3

Values of  $p(d)$  for the values of  $u(d)$  were calculated according to:

$$p = 2nkT(\cosh u - 1)$$

$u$	$p \text{ (dyn/cm}^2\text{)}$	$d \text{ (\AA)}$
2	$0.134 \times 10^7$	33.8
3	$0.441 \times 10^7$	19.8
3.5	$0.758 \times 10^7$	15.1
4	$1.28 \times 10^7$	11.4
4.5	$2.14 \times 10^7$	8.5
5	$3.56 \times 10^7$	6.3

Normally, double layer calculations are made in the cm-g-sec-system, and the swelling pressure  $p$  is reported in dyn/cm<sup>2</sup>. According to the new international system of units (SI), a force is measured in Newtons where 1 N = 1 m·kg/sec<sup>2</sup>, and the swelling pressure therefore is given in N/mm<sup>2</sup>.

$$1 \text{ N/mm}^2 = 10^7 \text{ dyn/cm}^2 = 142 \text{ lb/sq in.}$$

The function  $p(d)$ , as calculated according to the double layer theory with the mineralogical parameters of the opalinum shale, is plotted in Figure 6 in the terms of N/mm<sup>2</sup>.

The swelling pressure  $p$  was not calculated for each sample, but obtained for values of half distance  $d$  from the plot of  $p(d)$ . The half distance  $d$  is computed for each sample according to

$$d = \frac{\text{specific water volume of the sample}}{\text{specific surface area of the sample}}$$

The specific water volume used was the water content of the sample after the swelling test, expressed in cm<sup>3</sup> water/g dry sample. It was assumed that 1 g water = 1 cm<sup>3</sup> water. In the calculation of  $d$ , no difference was assumed between the external and internal surfaces of the clay minerals. The specific surface area of the sample was calculated from the specific surface area of the clay fraction and the amount of clay fraction. The deviation in the determinations of the amount of clay was about  $\pm 1\%$  and in the specific surface area of the clay, about  $\pm 5 \text{ m}^2/\text{g}$ . These deviations are included in the values of the surface area of the sample (Table 1) and thus also in the half distance  $d$ . For a known range of  $d$ , the maximum and minimum calculated swelling pressure  $p$  was deduced from the function  $p(d)$  in Figure 6.

Table 1. Mineralogical data and swelling pressure for the lower Dogger opalinum shale from northern Switzerland.

Sample	H <sub>2</sub> O <sup>1</sup> (%)	Carbonate content (%)	Clay fraction (%)	External surface area <sup>2</sup> (m <sup>2</sup> /g)	Surface area of clay fraction <sup>3</sup> (m <sup>2</sup> /g)	Surface area of sample (m <sup>2</sup> /g)	Half distance (Å)	Swelling pressure calculated (N/mm <sup>2</sup> )	Swelling pressure measured (N/mm <sup>2</sup> )
1	8.3	13	45 ± 1	29	135 ± 5	64.4–57.2	12.9–14.5	1.0–0.8	0.7
2	8.3	12	52 ± 1	32	135 ± 5	74.2–66.3	11.2–12.5	1.3–1.1	1.3
3	7.5	12	51 ± 1	29	135 ± 5	72.8–65.0	10.3–11.5	1.6–1.3	1.6
4	8.3	12	46 ± 1	31	135 ± 5	65.8–58.5	12.6–14.2	1.1–0.8	0.8
5	7.6	12	45 ± 1	29	135 ± 5	64.4–57.2	11.8–13.3	1.2–1.0	1.2
6	7.0	11	45 ± 1	25	135 ± 5	64.4–57.2	10.9–12.2	1.4–1.1	1.4
7	7.9	7	49 ± 1	30	135 ± 5	70.0–62.4	11.3–12.7	1.3–1.1	1.3
8	6.6	10	57 ± 1	36	135 ± 5	81.2–72.8	8.1–9.1	2.3–1.9	1.3
9	7.2	9	56 ± 1	34	135 ± 5	79.8–71.5	9.0–10.1	2.0–1.6	2.0
10	6.8	11	59 ± 1	37	135 ± 5	84.0–75.4	8.1–9.0	2.3–1.9	1.7
11	7.1	9	58 ± 1	35	135 ± 5	82.6–74.1	8.6–9.6	2.1–1.7	1.0
12	7.8	17	43 ± 1	27	135 ± 5	61.6–54.6	12.7–14.3	1.1–0.8	0.8
13	6.3	18	38 ± 1	23	135 ± 5	54.6–48.1	11.5–13.1	1.3–1.0	1.2
14	6.7	10	46 ± 1	28	135 ± 5	65.8–58.5	10.2–11.5	1.6–1.3	1.5
15	6.9	11	51 ± 1	32	135 ± 5	72.8–65.0	9.5–10.6	1.8–1.5	1.7
16	6.7	7	55 ± 1	33	135 ± 5	78.4–70.2	8.5–9.5	2.1–1.7	2.0
17	8.2	8	48 ± 1	28	135 ± 5	68.6–61.1	12.0–13.4	1.2–0.9	0.8
18	7.1	9	51 ± 1	31	135 ± 5	72.8–65.0	9.8–10.9	1.7–1.4	2.2
19	7.2	16	37 ± 1	22	135 ± 5	53.2–46.8	13.5–15.4	1.0–0.7	0.7

<sup>1</sup> Water content after swelling test.

<sup>2</sup> External specific surface area of sample by BET.

<sup>3</sup> External plus internal specific surface area by glycerol.

For sample 1 (Table 1), the clay fraction was found to be 45% ± 1% of the sample; the specific surface area of the clay fraction was 135 ± 5 m<sup>2</sup>/g. These data give maximum and minimum specific surface areas 64.4 m<sup>2</sup>/g and 57.2 m<sup>2</sup>/g, respectively. The minimum half distance, *d*, is, with a water content after swelling test of 8.3%, equal to

$$d = \frac{0.083 \text{ cm}^3/\text{g}}{64.4 \times 10^4 \text{ cm}^2/\text{g}}$$

$$= 12.9 \times 10^{-8} \text{ cm}$$

$$= 12.9 \text{ Å}.$$

The maximum *d* was calculated to be 14.5 Å. These values give a range of the swelling pressure *p* (Figure 6) of 1.0 to 0.8 N/mm<sup>2</sup> (142 to 114 lb/sq in.). Figure 6 also summarizes the values of the measured swelling pressure  $\sigma^*$  as a function of the calculated half distance *d*. The values of *d* are drawn as small bars according to the above-mentioned range of uncertainty. The values are also listed in Table 1.

## DISCUSSION

The Gouy theory was used in the uncorrected form to calculate the swelling pressure function *p*(*d*). Bolt (1955) presented a corrected form of the theory by adding several secondary energy terms to the primary potential energy term for the ion in the electric field of the double layer. The uncorrected form, however, gives results almost identical with those of the corrected theory when applied to the problem of double layer in-

teraction. The greatest influence on the value of *p*(*d*) was the choice of the valence to be +1 despite the fact that only 80% of the ions were monovalent. The influence of the valence may be seen from the fact that a function *p*(*d*) calculated for divalent ions would give swelling pressures about four times smaller than for monovalent ions. The influence of the choice of the valence was partly corrected by the calculation of the ion concentration, *n*. Higher values of *n* result in lower values of swelling pressure. The correction was made by counting one divalent ion as two monovalent ions to get the higher ion concentration. On the whole, the function *p*(*d*) as shown in Figure 6 gave values which are slightly too high.

By calculating the half distance *d* all water in the saturated sample was assumed to be connected to the clay surfaces; however, the pore volume distribution of some specimens, determined by mercury intrusion porosimetry at 3000 atm, indicated that about 10 to 15% of the pores were of radius >100 Å. In the calculation of *d*, the water from these pores was also distributed on the clay surface and thus resulted in a calculated half distance that was too large. Larger *d* values means smaller swelling pressure *p* as read from *p*(*d*) in Figure 6. The fact that the function *p*(*d*) resulted in excessively large values was thus, up to a certain point, compensated by the excessively large values of *d*.

The effect of the similarity of the external and the internal surface of the clay on the calculation of *d* was difficult to estimate. For half distances of the ascertained order or maximum 15 Å and a clay with mainly

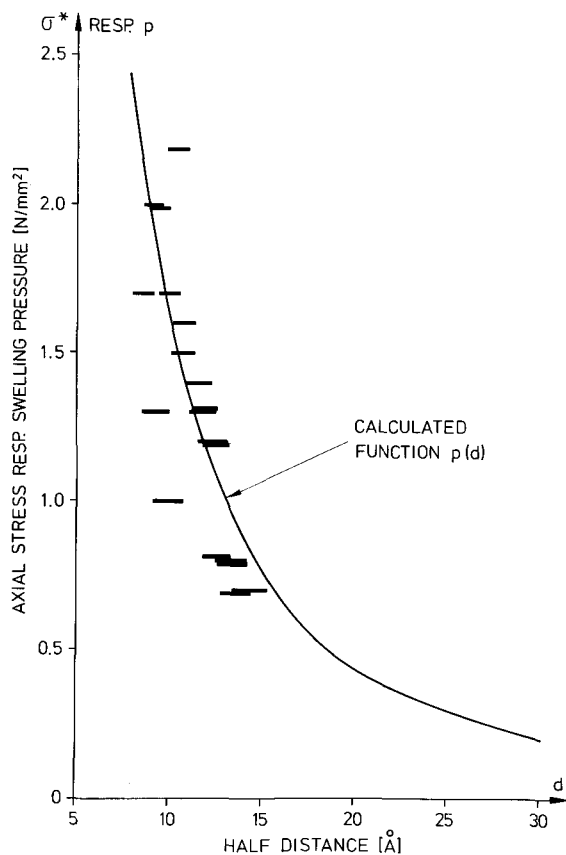


Figure 6. Calculated function  $p(d)$  with measured swelling pressure  $\sigma^*$  as a function of the half distance  $d$ .

$\text{Na}^+$  as counter-ions in the double layer, the influence was probably not very important. On the whole, the influence of the different assumptions seemed to compensate each other up to a certain value and thus gave an acceptable fit of the calculated with the measured swelling pressure.

### CONCLUSION

The investigation showed the possibility of calculating the swelling pressure of the opalinum shale from the mineralogical parameters using the Gouy theory. As the mean value of the calculated swelling pressures ( $1.39 \text{ N/mm}^2$ ) is of the same order of magnitude as the mean value of the measured swelling pressures ( $1.33 \text{ N/mm}^2$ ), the method was sufficiently accurate for tunnel-construction purposes. The advantage of the mineralogical method is that the swelling pressure can be obtained for samples of any shape, whereas the direct measurement of the swelling pressure requires samples in form of cylindrical or rectangular discs. The disadvantage of the mineralogical method is that it is rather complicated and in most cases takes more time than the direct measurement. As these investigations

normally are made in the preconstruction phase of the tunneling work, time is not a serious problem.

The real advantage of the mineralogical method is that once the  $p(d)$ -function for a swelling rock is known, it is very easy to determine the swelling pressure of a specimen by calculating the half distance  $d$  from the water content and the specific surface area of the sample. Because of the homogeneous composition of the clay fraction of the opalinum shale, the specific surface area of the sample depended only on the size of the clay fraction. The size of the clay fraction can, with a sufficient accuracy, be ascertained from Figure 3 by determining the external specific surface area. This routine method allows eight samples a day to be tested.

Therefore, for the opalinum shale, the swelling pressures which were determined from the drilling cores, can be checked in an easy way during tunnel construction by investigating undisturbed fresh samples. This will be an important control, inasmuch as drilling cores always are disturbed (partly swelled) to some extent, due to the use of water as drilling fluid, and therefore tend to give swelling pressures that are erroneously small.

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