Formulae connected with the Radii of the Incircle and the Excircles of a Triangle.

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The following formulae may be added to the list given in the *Proceedings of the Edinburgh Mathematical Society*, Vol. XII., pp. 86–102 (1894).

$$\left.\begin{aligned} a &= \frac{r_1(r_2+r_3)}{s} = \frac{r(r_2+r_3)}{s_1} \\ &= \frac{s^2(r+r_3)+r_3{}^2(r_1-r_2)}{2sr_3} = \frac{s_1{}^2(r_1-r_2)+r_2{}^2(r+r_3)}{2s_1r_2} \end{aligned}\right\} \qquad (81)$$

and so on.

$$b+c = \frac{s(r_1+r)}{r_1} = \frac{s_1(r_1+r)}{r}, \;\ldots\ldots \qquad (82)$$

$$b-c = \frac{r_1(r_2-r_3)}{s} = \frac{r(r_2-r_3)}{s_1}, \;\ldots\ldots \qquad (83)$$

$$\left.\begin{aligned} (b+c)(c+a)(a+b) : abc &= h_1+h_2+h_3-r : r \\ (b+c)(a-c)(a-b) : abc &= h_1-h_2-h_3+r_1 : r_1 \end{aligned}\right\} \qquad (84)$$

$$\left.\begin{aligned} &4r^2(r_1{}^2+r_2{}^2+r_3{}^2)+4(r_2{}^2r_3{}^2+r_3{}^2r_1{}^2+r_1{}^2r_2{}^2) \\ &= 4\,\Delta^4\left(\frac{1}{r_3{}^2r_3{}^2}+\frac{1}{r_3{}^2r_1{}^2}+\frac{1}{r_1{}^2r_2{}^2}+\frac{1}{r^2r_1{}^2}+\frac{1}{r^2r_2{}^2}+\frac{1}{r^2r_3{}^2}\right) \\ &= (a^2+b^2+c^2)^2-8\Delta^2 \end{aligned}\right\} \qquad (85)$$

$$\Delta^4\left(\frac{1}{r^2}+\frac{1}{r_1{}^2}+\frac{1}{r_2{}^2}+\frac{1}{r_3{}^2}\right)^2 = (a^2+b^2+c^2)^2 \qquad (86)$$

$$2\Delta^4\left(\frac{1}{r^4}+\frac{1}{r_1}{}^4+\frac{1}{r_2{}^4}+\frac{1}{r_3{}^4}\right) = (a^2+b^2+c^2)^2+8\Delta^2 \qquad (87)$$

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$$
\left.\begin{aligned}
r : r_1 &= h_1 - 2r : h_1 = h_1 : h_1 + 2r_1 \\
r : r_2 &= h_2 - 2r : h_2 = h_2 : h_2 + 2r_2 \\
r : r_3 &= h_3 - 2r : h_3 = h_3 : h_3 + 2r_3
\end{aligned}\right\} \quad (88)
$$

$$
\left.\begin{aligned}
r^2 : r_1^2 &= h_1 - 2r : h_1 + 2r_1 \\
r^2 : r_2^2 &= h_2 - 2r : h_2 + 2r_2 \\
r^2 : r_3^2 &= h_3 - 2r : h_3 + 2r_3
\end{aligned}\right\} \quad (89)
$$

$$
\frac{(h_1 - 2r)(h_2 - 2r)(h_3 - 2r)}{(h_1 + 2r_1)(h_2 + 2r_2)(h_3 + 2r_3)} = \frac{r^4}{s^4} \quad (90)
$$

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