Appendix 4

Transversity amplitudes

We briefly introduce the concept of transversity amplitudes and mention some of their key properties.

A4.1 Definition of transversity amplitudes

It has been known for a long time that certain simplifications occur if the spin quantization axis for each particle in the reaction

$$A + B \to C + D$$

is taken along the normal to the reaction plane (Dalitz, 1966). The usefulness of *transversity states* and *transversity amplitudes* in a modern context was emphasized by Kotanski (1970).

The transversity amplitudes $T_{cd;ab}(\theta)$ are defined by

$$\begin{aligned} T_{cd;ab}(\theta) = \sum_{\text{all } \lambda} & \mathscr{D}^{(s_C)^*}_{c\lambda_C} \mathscr{D}^{(s_D)^*}_{d\lambda_D} e^{i\pi(\lambda_D - \lambda_B)} \\ & \times H_{\lambda_C\lambda_D;\lambda_A\lambda_B}(\theta) \mathscr{D}^{(s_A)}_{\lambda_A a} \mathscr{D}^{(s_B)}_{\lambda_B b} \end{aligned} \tag{A4.1}$$

where the argument of each $\mathscr{D}$-function is

$$r_x(-\pi/2) = r(\pi/2, \pi/2, -\pi/2)$$

so that

$$\mathscr{D}^{(s)}_{\lambda\mu}\left(r_x(-\pi/2)\right) = \exp\left[i\pi(\mu - \lambda)/2\right] d^s_{\lambda\mu}(\pi/2). \tag{A4.2}$$

The transversity amplitudes measure the probability amplitudes for transitions amongst states of the type, $|\mathbf{p}_A; a\rangle_T$, which corresponds to particle A having spin component $s_z = a$ in the transversity rest frame S_A^T of A. S_A^T is obtained from the helicity rest frame S_A of A by a rotation

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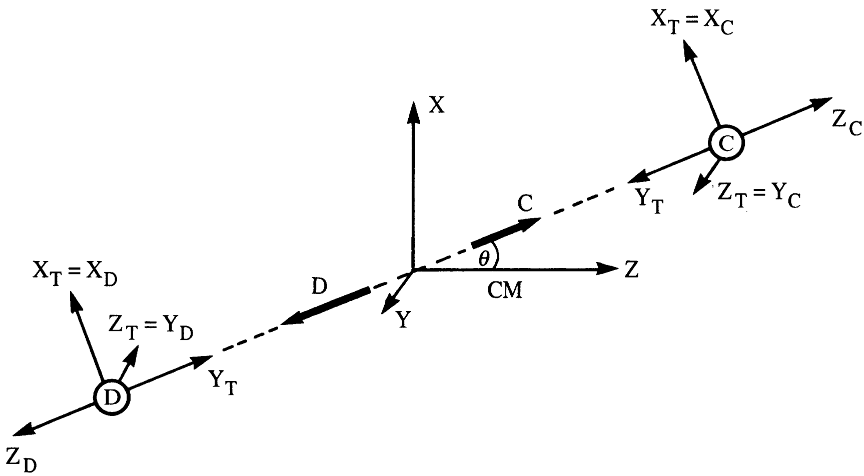

Fig. A4.1. Transversity rest frames for final particles in $A + B \to C + D$.

through $-\pi/2$ about the X axis of S_A. This is illustrated in Fig. A4.1[1] for particles C and D.

A4.2 Symmetry of transversity amplitudes

The symmetry properties of helicity amplitudes give rise to analogous properties for the transversity amplitudes as follows.

(*a*) *Parity.* With the intrinsic parities η_i one finds

$$T_{cd;ab}(\theta) = \frac{\eta_C \eta_D}{\eta_A \eta_B}(-1)^{a+b+c+d} T_{cd;ab}(\theta). \tag{A4.3}$$

Thus invariance under space inversion makes

$$T_{cd;ab}(\theta) = 0 \qquad \text{if} \qquad \frac{\eta_C \eta_D}{\eta_A \eta_B}(-1)^{a+b+c+d} = -1. \tag{A4.4}$$

This simplifies the appearance of the density matrix in the transversity basis giving it a 'chequer board' pattern, as discussed in subsection 5.4.1.

(*b*) *Time reversal.* In general

$$T_{cd;ab}(AB \to CD) = (-1)^{b-a+c-d} T_{ab;cd}(CD \to AB). \tag{A4.5}$$

and for elastic reactions $A + B \to A + B$

$$T_{a'b';ab} = (-1)^{b-a+a'-b'} T_{ab;a'b'}. \tag{A4.6}$$

[1] Note that some authors use a different convention. We have followed the original paper of Kotanski cited above.

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(*c*) *Identical particles.* For the correctly symmetrized amplitudes one finds the following.

For $A + B \to C + C$,

$$T^{\mathscr{S}}_{cc';ab}(\theta) = (-1)^{s_B - s_A + a + b + c + c'} T^{\mathscr{S}}_{-c'-c;-a-b}(\pi - \theta). \tag{A4.7}$$

For $A + A \to C + D$,

$$T^{\mathscr{S}}_{cd;aa'}(\theta) = (-1)^{s_D - s_C + a + a' + c + d} T^{\mathscr{S}}_{-c-d;-a'-a}(\pi - \theta). \tag{A4.8}$$

For $A + A \to C + C$, both the above, as well as

$$T^{\mathscr{S}}_{cc';aa'}(\theta) = T^{\mathscr{S}}_{c'c;a'a}(\theta). \tag{A4.9}$$

For states of definite isospin the right-hand side of (A4.7) and (A4.8) should contain an extra factor $(-1)^{I+1}$.

A4.3 Some analytic properties of transversity amplitudes

As remarked in Section 4.3 the analytic properties of the transversity amplitudes are only simple at thresholds and pseudothresholds. Their behaviour at $\theta = 0, \pi$ is just given by using (4.3.1) in (A4.1) and does not simplify.

In high energy models based on *t-channel* amplitudes the behaviour at the thresholds and pseudothresholds is important (Kotanski, 1970):

$$T^{(t)}_{cd;ab} \sim \varphi_{ab}^{\epsilon(a+b)} \varphi_{cd}^{\epsilon(c+d)} \psi_{ab}^{\epsilon\epsilon_{AB}(a-b)} \varphi_{cd}^{\epsilon\epsilon_{CD}(c-d)} \tag{A4.10}$$

where

$$\left.\begin{aligned} \varphi_{ij} &= [t - (m_i + m_j)^2]^{1/2} \\ \psi_{ij} &= [t - (m_i - m_j)^2]^{1/2} \\ \epsilon &= \ \text{sign}\left\{t(s-u) + (m_A^2 - m_B^2)(m_C^2 - m_D^2)\right\} \\ \epsilon_{ij} &= \ \text{sign}\left\{m_i - m_j\right\}. \end{aligned}\right\} \tag{A4.11}$$

If any of these thresholds or pseudothresholds is close to the physical region then the correct behaviour (A4.10) must be built into the models of $T^{(t)}_{cd;ab}$.

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