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ARTICLE

Heterogeneous biased expectations of young and old individuals: macroeconomic effects and policy implications

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Abstract
This paper examines the effects of heterogeneous biased expectations between the young and old on business cycles and explores its policy implications. Empirical findings reveal that individuals, particularly the young, can have more optimistic or pessimistic views about the future state of the economy compared to the data-generating measure. This study relates these results to the learning-from-experience literature, which suggests that individuals, particularly the young, place greater weight on recent observations when forming their expectations. Incorporating household weighting schemes into a life-cycle learning model, I show that household sensitivity to recent observations amplifies the effects of economic shocks. However, the amplification effects become less extensive as the population ages due to the lower sensitivity of the old. My simulation results indicate that a 10 percentage point increase in the old population ratio leads to a 16 percent decrease in output volatility. Regarding policy implications, this paper suggests that the government spending multiplier declines by approximately 10 percent when the old population ratio rises by 10 percentage points due to weak amplification effects. Moreover, the weakened output effects deteriorate the welfare of the population, particularly that of the young.

Keywords: heterogeneous biased expectation; population aging; adaptive learning; business cycle; fiscal policy

JEL classifications: D83; D84; E32; E62; J11

1. Introduction

Accelerated population aging has had significant economic consequences in the past decades. For instance, the aging population has been identified as a main factor for low economic growth or secular stagnation, a concept first described by Alvin Hansen in 1938. Another structural change that has captured the attention of scholars is the Great Moderation, which refers to the reduction in macroeconomic volatility since the 1980s. The literature identifies several factors that may have contributed to this trend, including monetary and fiscal policies, regulations on financial markets, and population aging. While this paper also acknowledges the role of the aging population in reducing macroeconomic volatility, it focuses on a novel channel through which heterogeneous biased expectations between young and older individuals play a crucial role.

Figure 1 supports the claim of this paper that population aging, characterized by a higher proportion of older individuals with lower sensitivity to economic conditions, is a key driver of the observed decline in macroeconomic fluctuations. Panel (a) shows the 10-year rolling window volatility of GDP growth rates excluding GFC and COVID-19 periods has decreased fast recently, but the old-age dependency ratio has surged during the same period. These facts imply that the

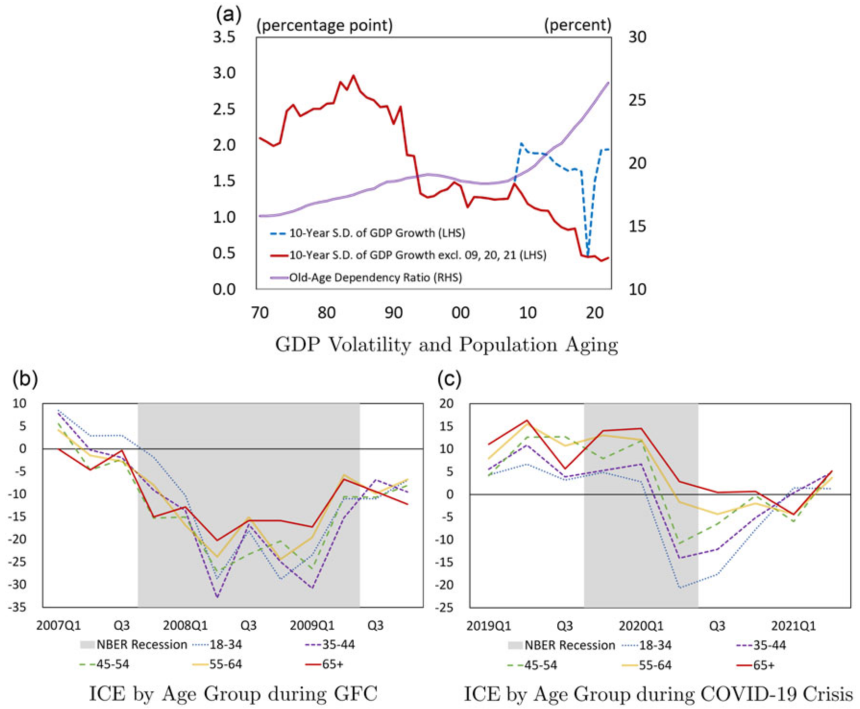

Figure 1. GDP volatility and population aging, and index of consumer expectations by age group during crises.
Notes: 10-Year S.D. of GDP Growth indicates the 10-year rolling window standard deviation of the quarterly GDP growth rates. The Old-Age Dependency Ratio is the ratio of people older than 64 to those ages 15–64. The y-axis in lower panels denotes the difference between ICE in a current period and the average ICE from 1978q1 to 2020q4. *Source*: FRED and Surveys of Consumers

Great Moderation can be linked to population aging. Panel (b) and (c) also plot the index of consumer expectations (ICE) during two economic crises and suggest that older individuals exhibit less sensitivity to recent observations in their expectations of the future economy.[1]

Thus, I aim to answer the following questions: 1) "How do heterogeneous biased expectations between young and old individuals affect dynamics of macro-variables?", and the related question is "How much does population aging contribute to the reduction in the volatility of business cycles in recent decades?", and 2) "What are the policy implications of heterogeneous biased expectations between the young and old for an aging society?"

To answer these questions, this paper first investigates the properties of household expectations, with particular emphasis on the heterogeneity between the expectations of young and old individuals. In particular, a belief wedge, defined as the disparity between the mean unemployment forecast one year ahead from the Michigan survey and the vector autoregression (VAR) or survey of professional forecasters (SPF) unemployment forecast, is introduced as a measure of the deviation of household expectations from the data-generating measure. The belief wedge analyses show household expectations can diverge significantly from the VAR or SPF forecast, which means they can be overly optimistic or pessimistic than current overall economic conditions. Especially, younger individuals have a stronger tendency towards biased expectations compared to their older counterparts. As a result, the younger individuals' biased expectations have a greater impact on our economy.

Although the literature offers various theories to explain this deviation, including sticky or noisy information and rational inattention, this paper adopts an adaptive learning mechanism from the learning-from-experience literature since it provides empirical evidence for heterogeneous biased expectations between young and older individuals. Malmendier and Nagel (2016) and Malmendier and Shen (2024) show that both young and older individuals place more weight on recent experiences when forming their expectations, with young individuals placing relatively more weight than older individuals. In particular, the constant gain learning algorithm with different gain parameters for each age group can effectively capture these expectation formation rules.

In this study, a life-cycle (LC) learning model is constructed by combining the real business cycle (RBC) learning and overlapping generations (OLG) framework based on the works of Eusepi and Preston (2011) and Gertler (1999). The model assumes bounded rationality, where households have an incomplete understanding of the economy and update their beliefs about the future market-clearing prices, a real wage and rental rate of capital, each period using a constant gain learning algorithm. Here, households act as if their beliefs will remain unchanged forever, as in Cogley and Sargent (2008)'s anticipated utility.[2] In particular, young households have larger gain parameters than old households, reflecting the young's relatively heavier reliance on recent data. The gain parameters for young and old households are derived from Malmendier and Nagel (2016).

The LC learning model incorporating heterogeneous household weighting schemes on past data presents the novel dynamics of macroeconomic variables and implications for business cycles and government policies.

At first, this study compares the dynamics of the macroeconomic variables in response to a technology shock in two different models: the LC learning model and the LC rational expectation (RE) model. By comparing these models, I shed light on how heterogeneous biased expectations arising from the learning mechanism differently affect economic fluctuations than rational expectations. Then, the LC learning model shows that household sensitivity to recent observations amplifies the effects of the technology shock. However, as the old population share increases, these amplification effects become less extensive, as older households have relatively lower sensitivity to recent shocks.

I also compare the LC learning model with the representative agent (RA) learning model and show the life-cycle assumption better explains the data in the real world. In other words, the RA learning model produces far greater amplification than the LC learning model, which does not match the data. This is because the RA does not consider the probability of death and losing a job in the future, and thus, they have a larger time discount factor. Therefore, the RA gives more importance to biased expectations about the future economy, which creates excessive amplification effects.

In addition, this paper examines the impact of heterogeneous biased expectations on the volatility of business cycles and finds household learning behavior creates fluctuations in economic activities that closely resemble those observed in the data. Furthermore, the lower sensitivity of old households to recent experiences leads to a decrease in business cycle volatility as the population ages. A 10 percentage point increase in the old population share (= the number of people aged 65 and over / 15 and over $\times$ 100) induces a 16 percent decrease in output volatility.[3] Notably, the United States has experienced a decline in GDP volatility of approximately 74 percent, coinciding with about a 6 percent point increase in the old population ratio from the 1980s to the 2010s, indicating the meaningful roles of heterogeneous biased expectations and population aging in reducing GDP volatility.

Lastly, the LC learning model provides important policy implications. First, household learning behavior amplifies the response of output to government spending shocks, although the magnitude of this response diminishes with an increase in the old household share due to their lower sensitivity. The government spending multiplier declines by approximately 10 percent when the old population ratio rises by 10 percentage points. I also conduct welfare analyses of government spending. The amplification of government spending effects under learning leads to an improvement in the welfare of the population compared to under rational expectations. However, as the old household share increases, the welfare of both young and old households deteriorates due to weakened output effects. Especially, the welfare of young households declines more drastically with an aging population due to their overreaction to the relatively poor performance of the economy in an aging society following a stimulus policy.

Related literature

This paper bridges the gap in three strands of literature: 1) biased expectations, 2) adaptive learning, and 3) the effects of population aging on business cycles and economic policies.

First, individuals often exhibit deviation from rational expectations, and I assume these biased expectations originate from heavy reliance on recent experiences. The learning-from-experience literature, including Malmendier and Nagel (2011), Malmendier and Nagel (2016), and Malmendier and Shen (2024), supports this claim and further shows that young individuals are more sensitive to recent observations than old individuals. Blanchard (2010) also suggests that the "deep scars" of the great financial crisis have a lasting impact on the economy. Moreover, there is growing literature on expectation-driven business cycles, as evidenced by Lorenzoni (2009), Angeletos and La'O (2013), Benhabib et al. (2015), Benhabib et al. (2016), etc. Similar to these papers, I study the effects of biased expectations on business cycles but the biased expectations are heterogeneous between the young and old, which has not been sufficiently dealt with in the literature.

This paper especially adopts adaptive learning to reflect the deviation from rational expectations. Adam et al. (2021) state that learning-from-experience could contribute to time-varying subjective expectation errors. Malmendier and Nagel (2016) show the constant gain adaptive learning rule captures average survey expectations well. Eusepi and Preston (2011) introduce imperfect information and learning behavior in the RBC framework. In particular, households in their model employ the minimum state variable (MSV) constant gain learning algorithm like Mitra et al. (2013), and Evans and Honkapohja (2001). I also adopt the MSV constant gain learning rule but add the heterogeneity between the young and old. Specifically, different gain parameters represent the heterogeneous learning rule between the young and old. Branch and McGough (2009) and Honkapohja and Mitra (2005) also employ the different gain parameters to model heterogeneity in household expectations.

This study also contributes to the literature on factors driving the stylized facts of an aging society, i.e., low business cycle volatility and less effective fiscal policy. Improved monetary policy (Stock and Watson, 2003) and the low volatility of old individuals' employment and hours worked (Jaimovich and Siu, 2009) are identified as the main drivers that lead to the reduction in the volatility of business cycles in recent decades, known as the Great Moderation. Basso and Rachedi (2021) and Honda and Miyamoto (2020) show that the government spending multiplier declines as the population ages due to weak responses of private consumption and employment. However, I offer a novel perspective by examining the role of heterogeneous biased expectations between the young and old in driving economic changes in an aging society, which has not been thoroughly investigated in the literature.

The structure of this paper is as follows. Section 2 presents the empirical evidence for heterogeneous biased expectations and estimates their impacts on the economy. Then, I discuss mechanisms for heterogeneous biased expectations based on previous papers. Next, Section 3 builds the life-cycle learning model in which young and old households use a different adaptive learning rule when they form expectations. After that, Section 4 explores the macroeconomic effects of heterogeneous biased expectations, and Section 5 provides their fiscal policy implications for an aging society. Finally, Section 6 concludes.

2. Empirical evidence for heterogeneous biased expectations

This section analyzes biased expectations of individuals by computing belief wedges and investigates the differences between the young and older generations. Then, I examine the impacts of biased expectations by each age group on the economy and present the mechanisms through which heterogeneous biased expectations arise.

2.1 Biased expectations

The belief wedges, the measures of biased expectations, suggest that individuals can perceive the future economic situation more optimistically or pessimistically than current overall economic conditions.

Definition

The belief wedge is defined as the difference between the mean unemployment rate forecast one year ahead from the Michigan survey and the VAR unemployment rate forecast—henceforth, VAR Wedge. Here, the VAR forecasts are considered the data-generating measure. I also exploit the unemployment forecast from a SPF conducted by the Federal Reserve Bank of Philadelphia as an alternative to the VAR forecast—henceforth, SPF Wedge.[4]

$$Belief\ Wedge = Expected\ Unemployment\ of\ Michigan\ Survey\ - VAR\ (or\ SPF)\ Forecast$$

Methodology

The belief wedges are calculated based on Bhandari et al. (2024) and Mankiw et al. (2003). Specifically, the VAR forecasts are generated from a standard quarterly forecasting VAR model containing nine variables: CPI inflation, real GDP, unemployment rate, the relative price of investment goods, capital utilization rate, hours worked, consumption rate (=Consumption/GDP), investment rate (=Investment/GDP), and federal funds rate. The time lag for the VAR model is two, and the data for VAR estimation and SPF forecasts are obtained from FRED and FRB of Philadelphia. The sample period for SPF Wedge ranges from 1960Q1 to 2023Q3, but for VAR Wedge, it is only from 1960Q1 to 2019Q4 since the COVID-19 shock causes a structural break in the VAR model.

Results

Figure 2 illustrates the changes in two belief wedges over time, which are similar to the findings of Bhandari et al. (2024). Both VAR and SPF Wedges exhibit a noticeable trend of increasing sharply during economic recessions, suggesting over-pessimistic expectations as a result of negative economic shocks, and gradually decreasing post-recession. In particular, during the COVID-19 crisis, these biased expectations are even stronger than in any previous crisis. Overall, these patterns suggest that individuals hold time-varying biased expectations.

2.2 Heterogeneous biased expectations

Belief wedges also suggest that biased expectations vary by age. Figure 3 shows the difference in the belief wedge between the age group "under 65" and "over 65," especially using the SPF Wedge.

$$Difference\ in\ Belief\ Wedge = Belief\ Wedge\ Under\ 65 - Belief\ Wedge\ Over\ 65.$$

Noticeably, the difference increases sharply during recessions and gradually decreases in the post-recession periods. Hence, it can be inferred that young individuals react more sensitively to recent macroeconomic shocks than older individuals, which causes the young to hold more

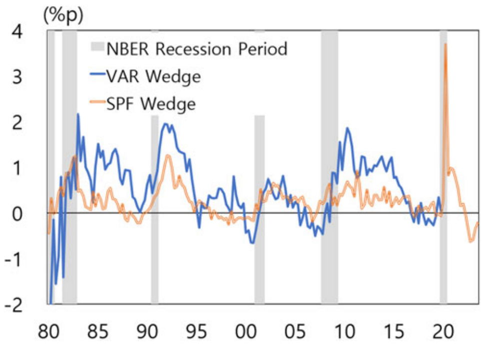

Figure 2. Belief wedges.
Notes: The belief wedges are computed based on Bhandari et al. (2024) and Mankiw et al. (2003). This paper does not report VAR Wedge after 2020 as the COVID-19 shock causes a structural break in the VAR model.

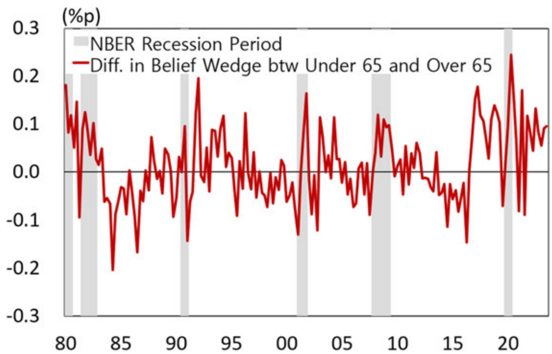

Figure 3. Difference in belief wedge between "Under 65" and "Over 65."

biased pessimistic expectations than the older during and right after economic downturns. The detailed mechanism for the difference in biased expectations by age is discussed in Section 2.3.

Now, this paper estimates Equation (1) to verify how biased expectations of each age group, i.e., "Over 65" or "Under 65," differently affect the economy, especially consumption

$$PCE_t^{detrend} = \alpha + \sum_{h=0}^{1} \beta_h BW_{t-h}^{over65} + \sum_{h=0}^{1} \gamma_h BW_{t-h}^{under65} + e_t \tag{1}$$

where $PCE_t^{detrend}$ is the detrended personal consumption expenditure using Equation (2)

$$PCE_t^{detrend} = \frac{PCE_t - PCE_t^{trend}}{PCE_t^{trend}} \times 100\,(\%) \tag{2}$$

.

Here, PCE_t^{trend} is the trend of personal consumption expenditure from HP-filtering. Also, BW^{over65} and $BW^{under65}$ indicate the belief wedges of the age group "Over 65" and "Under 65," respectively.

For data, personal consumption expenditure is from Bureau of Economic Analysis, and the sample period is from 1980Q1 to 2023Q3 for estimation using SPF Wedge and from 1980Q1 to 2019Q4 for estimation using VAR Wedge.

Table 1. Effects of biased expectations by age group on consumption

	(1) $PCE_t^{detrend}$	(2) $PCE_t^{detrend}$
$SPFWedge_t^{over65}$	0.1951	–
	(0.9195)	–
$SPFWedge_{t-1}^{over65}$	0.6175	–
	(0.9068)	–
$SPFWedge_t^{under65}$	−2.3140**	–
	(1.0397)	–
$SPFWedge_{t-1}^{under65}$	−0.5332	–
	(0.9361)	–
$VARWedge_t^{over65}$	–	0.9786
	–	(0.7919)
$VARWedge_{t-1}^{over65}$	–	0.4489
	–	(0.7951)
$VARWedge_t^{under65}$	–	−1.6682**
	–	(0.7992)
$VARWedge_{t-1}^{under65}$	–	−0.4800
	–	(0.8298)
Constant	0.5716***	0.4134***
	(0.1006)	(0.7890)
$R-squared$	0.4998	0.2725

Notes: Standard errors in parentheses, and ***p<0.01, **p<0.05, *p<0.1.

Table 1 provides estimation results and shows only the young individuals' belief wedge is inversely related to private consumption. Specifically, the coefficient on $BW^{under65}$ has a statistically significant negative value, which means excessively negative expectations about future unemployment result in a reduction in consumption below the trend. However, the coefficients on BW^{over65} are not statistically significant and have positive signs, making it difficult to derive economic meanings.

Through these empirical findings, this paper argues that older individuals are less overly affected by recent experiences, resulting in less biased expectations and subsequently weaker impacts on economic activities. From this perspective, the impact of biased expectations is expected to decrease as the proportion of older individuals increases.

2.3 Mechanisms for heterogeneous biased expectations

This paper explains heterogeneous biased expectations with a mechanism devised in the learning-from-experience literature. Although previous papers suggest that the deviation from rational expectations may be attributed to sticky or noisy information, rational inattention, etc, these assumptions cannot fully support the empirical findings that the expectations of older individuals are less biased than those of younger individuals. However, the learning-from-experience literature provides clear evidence for that. In particular, the literature highlights that the biased expectation formation rule can be well-captured by the constant gain learning algorithm with a different gain parameter by age.

The following are more details about the learning-from-experience literature. According to Malmendier and Nagel (2011), individuals tend to assign more weight to recent stock returns when forming their expectations, leading to more optimistic beliefs about future stock returns after experiencing higher returns. Similarly, Malmendier and Nagel (2016) find that people overweight inflation realized during their lifetimes, especially more recent data when predicting future inflation. Especially, these beliefs are heterogeneous between the young and old since young individuals place more weight on recent data than older individuals. Malmendier and Shen (2024) also show that personal experiences of unemployment, particularly recent ones, have long-lasting effects on consumption decisions. However, this beliefs-based channel is weaker in an old cohort than in a younger cohort.

3. Model

The model, based on the RBC framework, includes two market-clearing prices: real wages for the labor market and rental rates of capital for the capital market. While competitive firms take these factor prices as given, households hold biased expectations about them. Specifically, following Eusepi and Preston (2011), households possess an incomplete understanding of the economy and update their beliefs regarding market-clearing prices by relying on historical patterns in observed data.[5] They especially tend to assign greater weight to recent observations when forming expectations, using an adaptive learning algorithm. I also incorporate a life-cycle assumption, i.e., young households or workers and old households or retirees, into the RBC learning model following Gertler (1999), Blanchard (1985), and Gali (2021). As discussed in the previous sections, young individuals weigh relatively more on the recent data than old individuals in the learning model. Including heterogeneous agents with distinct weighting schemes leads to unique macroeconomic dynamics and implications for business cycles and government policies.

The adaptive learning rules relax the assumptions under rational expectations that agents optimally forecast future variables and solve dynamic optimization problems. In contrast, households in the learning model lack information about other agents' behavior and, as a result, cannot directly infer the aggregate laws of motion as they would in the RE model. Consequently, they rely on econometric time-series models to form their forecasts. Evans (2019) and Evans et al. (2009) explain how the adaptive learning model operates: at time t, agents make decisions based on the current state, realizations of exogenous shocks, and expectations of relevant variables. The aggregation of heterogeneous agents' behavior, combined with market clearing, determines the temporary equilibrium outcomes for endogenous variables in the economy. At time $t+1$, expectations are updated using the new data point provided by the temporary equilibrium, and the process repeats. Over time, this sequence of temporary equilibrium may yield parameter estimates for the forecasting models that converge to a fixed point, corresponding to a rational expectations equilibrium for the economy. For more detailed explanations see Evans and Honkapohja (2001).

3.1 Firms

There are identical competitive firms of mass one. Each firm produces goods using capital K_t and labor H_t. The production function is

$$Y_t = (K_t)^{\alpha}(X_t H_t)^{1-\alpha} \tag{3}$$

where $0 < \alpha < 1$. Here, X_t denotes the aggregate labor-augmenting technical progress which evolves via

$$\ln\left(\frac{X_{t+1}}{X_t}\right) = \ln(\chi_{t+1}) = \ln(\overline{\chi}) + u_{\chi,t+1} \tag{4}$$

where $u_{\chi,t}$ indicates an *i.i.d.* random variable with zero mean and standard deviation σ_{u_χ}. The stochastic process for the evolution of the technological shock is assumed known to the agents.

Firms maximize their profits with factor prices, the real wage W_t and returns to capital R_t^k, as given. The optimality conditions are

$$W_t = (1-\alpha)K_t^{\alpha}X_t^{1-\alpha}H_t^{-\alpha} = (1-\alpha)\frac{Y_t}{H_t} \tag{5}$$

$$R_t^k = \alpha K_t^{\alpha-1}(X_tH_t)^{1-\alpha} = \alpha\frac{Y_t}{K_t} \tag{6}$$

.

From now on, lowercase letters denote variables normalized with the technology X_t. Then, the normalized production function and optimality conditions are

$$y_t = (k_t)^{\alpha}(\chi_t)^{-\alpha}(H_t)^{1-\alpha} \tag{7}$$

$$w_t = (1-\alpha)(k_t)^{\alpha}(\chi_t)^{-\alpha}H_t^{-\alpha} = (1-\alpha)\frac{y_t}{H_t} \tag{8}$$

$$R_t^k = \alpha k_t^{\alpha-1}(\chi_t)^{-\alpha}(H_t)^{1-\alpha} = \alpha\frac{y_t}{k_t} \tag{9}$$

.

3.2 Belief updating

Households are assumed to use an economic model to forecast future market-clearing prices. The model relates the wage and returns to capital to the aggregate capital and the distribution of the wealth between young and old households, which are two minimum state variables (MSV) in the model

$$\hat{R}_t^k = \mu_r + \mu_{rk}\hat{k}_t + \mu_{r\lambda}\lambda_t + e_t^r \tag{10}$$

$$\hat{w}_t = \mu_w + \mu_{wk}\hat{k}_t + \mu_{w\lambda}\lambda_t + e_t^w \tag{11}$$

$$\hat{k}_{t+1} = \mu_k + \mu_{kk}\hat{k}_t + \mu_{k\lambda}\lambda_t + e_t^k \tag{12}$$

$$\lambda_{t+1} = \mu_\lambda + \mu_{\lambda k}\hat{k}_t + \mu_{\lambda\lambda}\lambda_t + e_t^\lambda \tag{13}$$

where $\lambda_t = \frac{\hat{a}_t^o}{\hat{a}_t}$ represents the distribution of the wealth between young and old households.[6] Here, a_t^y, a_t^o, and a_t are normalized A_t^y, A_t^o, and A_t, which are the assets of aggregate young, old, and total households, respectively, and $\hat{x}$ indicates the log-linearization of the variable x around a balanced growth path (BGP). e_t denotes a regression error or households consider e_t an idiosyncratic disturbance (i.e., a perceived white noise unobserved shock). As in Eusepi and Preston (2011), this paper excludes the technology shock from the household forecasting system since the technology shock is the only disturbance in the model, and thus, households learn quickly, which means they correctly expect market prices right away, if it is included.[7]

Equation (10)-(13) can be rewritten in a matrix form as Equation (14)

$$z_t' = \begin{pmatrix} \hat{R}_t^k \\ \hat{w}_t \\ \hat{k}_{t+1} \\ \lambda_{t+1} \end{pmatrix}, \; x_{t-1} = \begin{pmatrix} 1 \\ \hat{k}_t \\ \lambda_t \end{pmatrix}, \; \zeta_t = \begin{pmatrix} \mu_{r,t} & \mu_{w,t} & \mu_{k,t} & \mu_{\lambda,t} \\ \mu_{rk,t} & \mu_{wk,t} & \mu_{kk,t} & \mu_{\lambda k,t} \\ \mu_{r\lambda,t} & \mu_{w\lambda,t} & \mu_{k\lambda,t} & \mu_{\lambda\lambda,t} \end{pmatrix}, \; e_t' = \begin{pmatrix} e_t^r \\ e_t^w \\ e_t^k \\ e_t^\lambda \end{pmatrix}$$

$$z_t = x_{t-1}'\zeta_t + e_t. \tag{14}$$

Under rational expectations, $\mu_{r,t} = \mu_{w,t} = \mu_{k,t} = \mu_{\lambda,t} = 0$, and the coefficients of the model are time-invariant (i.e., $\mu_{ij,t} = \overline{\mu}_{ij}$ where $i \in \{r, w, k, \lambda\}$ and $j \in \{k, \lambda\}$). Also, $e_t^r = \overline{\mu}_{r\chi}\, \hat{\chi}_t$, $e_t^w = \overline{\mu}_{w\chi}\, \hat{\chi}_t$, $e_t^k = \overline{\mu}_{k\chi}\, \hat{\chi}_t$, and $e_t^\lambda = \overline{\mu}_{\lambda\chi}\, \hat{\chi}_t$. In other words, agents in the RE model have a complete understanding of the model and know the individual preferences and technologies of other market participants. Thus, they are aware of the exact relationship among the state variables, technology shocks, and factor prices. Therefore, the technology shocks are included in the forecasting system and the coefficients in Equation (10)–(13) are true and time-invariant. However, under learning, households update the coefficients every period as they observe new data. I employ constant gain recursive least squares estimates as an updating algorithm

$$\zeta_t^i = \zeta_{t-1}^i + g^i (Q_t^i)^{-1} x_{t-1} \underbrace{(z_t - x'_{t-1}\zeta_{t-1}^i)}_{\text{Forecast Error}} \tag{15}$$

$$Q_t^i = Q_{t-1}^i + g^i (x_t x_t' - Q_{t-1}^i) \tag{16}$$

where $i \in \{y, o\}$ and Q_t^i is the estimate of the second-moment matrix of regressors.

Equations (15) and (16) imply that the coefficients estimated in the previous period are updated according to the forecast errors produced for the current period. So, households use the previous period's coefficient estimates when forming expectations, and then, update their coefficients at the end of each period.

The gain parameter g allows the deviation from rational expectations. That is, the larger g, the more weight is given to recent data.[8] Here, young and old households have different gain parameters, i.e., g^y and g^o, based on Malmendier and Nagel (2016)'s empirical findings.[9,10] In particular, young individuals are assumed to put more weight on recent data than old individuals, resulting in a larger gain parameter for the former, i.e., g^y is greater than g^o. Collin-Dufresne et al. (2017) also assume that the gain parameter of the young agents is five times larger than that of the old agents.[11] Since young and old households have different weighting schemes on the past data, their adaptive expectations about the factor prices can be distinct from each other. These differences between age groups finally produce the novel dynamics of the macro-variables rather than assuming a representative agent.

There is one more assumption worth mentioning. In my model, agents do not know what other agents expect, and thus, their expectations are not affected by others. If young households are aware of old households' expectations or vice versa, they can internalize the impacts of updating their beliefs on the economy. So, households quickly learn and find the true evolution of wages and interest rates since the household learning behavior does not create forecast errors.

3.3 Households

This paper assumes an economy with overlapping generations following Gertler (1999), Gali (2021), and Blanchard (1985). As explained in Section 3.2, households have heterogeneous expectations depending on their age due to the different rules of forming expectations. Therefore, I index households according to their age, i.e., young (workers) or old (retirees) households. The size of the population is constant and normalized to one. Each individual has the constant probability γ of surviving into the following period, independently of their age and economic status. Moreover, each worker faces the constant probability $1 - \nu$ of becoming old and retired permanently. This probability is also independent of their age and economic status. Consequently, the size of young individuals or workers at any time is the constant $\phi = \frac{1-\gamma}{1-\nu\gamma} \in (0, 1]$ and that of old individuals or retirees is $1 - \phi = \frac{\gamma(1-\nu)}{1-\nu\gamma}$.

This study adopts the perfect annuity market assumption introduced by Gertler (1999) and Gali (2021), where agents are insured against the risk of death. More specifically, households have

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an annuity contract with a perfectly competitive insurance company that issues payments proportional to the household's financial wealth. Upon death, the household's wealth is transferred to the insurance company. The surviving households receive all returns in this market.[12] Furthermore, an insurance market is introduced to mitigate the risk of income loss during retirement. The complete market assumption makes the model more tractable, especially in terms of aggregating household consumption.

3.3.1 Old households (retirees)

The old household or retiree of cohort 'a' is the agent who retired 'a' quarters ago.[13] Each agent maximizes the following Bellman equation[14]

$$V^o(A^o_{a,t}, A^o_t, A_t, K_t) = \text{Max}\{\ln C^o_{a,t} + \gamma\beta\tilde{E}^o_t V^o(A^o_{a+1,t+1}, A^o_{t+1}, A_{t+1}, K_{t+1})\} \tag{17}$$

and the budget constraint for old households is

$$C^o_{a,t} + \gamma A^o_{a+1,t+1} = R_t A^o_{a,t} + S_t \tag{18}$$

where β is a time discount factor, $\tilde{E}_t$ indicates *subjective* expectations for the future, and the superscript o denotes old or retired households. Also, $C^o_{a,t}$ is consumption, and $A^o_{a,t}$ is the asset that old households hold at the beginning of time t. As mentioned above, only survivors receive all the returns, and households who die are paid nothing. The real interest rate R_t satisfies $R_t = R^k_t + 1 - \delta$ where δ is the depreciation rate of capital due to the absence of arbitrage between loans and capital. S_t is the social security benefit that old households receive. The state variables $\{A^o_t, A_t, K_t\}$—aggregate old household asset, total asset, and total capital—are used when households expect the future wages and returns to capital, which is discussed in Section 3.2 in detail. Households know only their own objectives, constraints, and beliefs as in Eusepi and Preston (2011).

Then, Euler equation is

$$(C^o_{a,t})^{-1} = \beta\tilde{E}^o_t[(C^o_{a+1,t+1})^{-1} R_{t+1}] \tag{19}$$

and the Euler equation and budget constraint are normalized with technology as follows

$$(c^o_{a,t})^{-1} = \beta\tilde{E}^o_t[(c^o_{a+1,t+1})^{-1} \chi^{-1}_{t+1} R_{t+1}] \tag{20}$$

$$c^o_{a,t} + \gamma a^o_{a+1,t+1} = R_t a^o_{a,t} \chi_t^{-1} + s_t \tag{21}$$

.

To get the aggregate consumption of old households, this paper derives the intertemporal budget constraint (IBC) from the one-period budget constraint, Equation (21). After that, the IBC is log-linearized around a BGP and then I aggregate each cohort's consumption using Euler equation, Equation (20). Finally, we obtain log-linearized aggregate consumption of old households

$$\begin{aligned} \hat{c}_t{}^o = {} & \eta_{ay}(1-\nu)\hat{a}^y_{t-1} + \eta_{ao}\hat{a}^o_t + \eta_{r\chi}(\hat{R}_t - \hat{\chi}_t) + \eta_s\hat{s}_t \\ & \underbrace{- \eta^e_r \tilde{E}^o_t \sum_{h=0}^{\infty} (\gamma\beta)^h \hat{R}_{t+1+h}}_{\text{EPV of Returns to Capital}} + \eta^e_s \tilde{E}^o_t \sum_{h=0}^{\infty} (\gamma\beta)^h \hat{s}_{t+1+h} \end{aligned} \tag{22}$$

where η's consist of primitive model parameters.[15] Note that Equation (22) shows how current and expected variables, such as real interest rates, affect the consumption decisions of old households.

3.3.2 Young households (workers)

The young household or worker of cohort 'b' is the agent who started to work (was born) 'b' quarters ago. Each agent maximizes the following Bellman equation

$$V^y(A^y_{b,t}, A^y_t, A_t, K_t) = \text{Max}\{\ln C^y_{b,t} + \theta \ln(1 - H_{b,t}) + \gamma\beta\tilde{E}^y_t[\nu V^y(A^y_{b+1,t+1}, A^y_{t+1}, A_{t+1}, K_{t+1}) + (1-\nu)V^o(A^{yo}_{b+1,t+1}, A^y_{t+1}, A_{t+1}, K_{t+1})]\} \tag{23}$$

and the budget constraint for young households is

$$C^y_{b,t} + \nu\gamma A^y_{b+1,t+1} + (1-\nu)\gamma A^{yo}_{b+1,t+1} = R_t A^y_{b,t} + W_t H_{b,t} - T_t \tag{24}$$

where the superscript y denotes young households or workers, $C^y_{b,t}$ is consumption, and since there are two possible future states, young households save $A^y_{b+1,t+1}$ for staying young and $A^{yo}_{b+1,t+1}$ for retiring next period. Complete asset markets insure young households against the risk of retirement. W_t is the real wage, $H_{b,t}$ is the labor supply, and T_t is the lump-sum tax. The state variables $\{A^y_t, A_t, K_t\}$—aggregate young household asset, total asset, and total capital—are used when young households expect the future wages and returns to capital, which is discussed in Section 3.2 in detail.

Young households solve the maximization problem, Equation (23), subject to the budget constraint, Equation (24). Then, the normalized Euler equations, labor supply condition, and budget constraint of young households are as follows

$$(c^y_{b,t})^{-1} = \beta\tilde{E}^y_t[(c^y_{b+1,t+1})^{-1}\chi^{-1}_{t+1}R_{t+1}] \tag{25}$$

$$(c^y_{b,t})^{-1} = \beta\tilde{E}^y_t[(c^o_{b+1,t+1})^{-1}\chi^{-1}_{t+1}R_{t+1}] \tag{26}$$

$$\frac{\theta c^y_{b,t}}{1 - H_{b,t}} = w_t \tag{27}$$

$$c^y_{b,t} + \nu\gamma a^y_{b+1,t+1} + (1-\nu)\gamma a^{yo}_{b+1,t+1} = R_t a^y_t \chi_t^{-1} + w_t H_{b,t} - \tau_t \tag{28}$$

.

Finally, we obtain log-linearized aggregate consumption of young households using the same way to get aggregate consumption of old households

$$\begin{aligned}\hat{c}^y_t = {} & \psi_{ay}\hat{a}^y_t + \psi_{r\chi}(\hat{R}_t - \hat{\chi}_t) + \psi_w\hat{w}_t - \psi_\tau\hat{\tau}_t - \psi^e_\tau\tilde{E}^y_t\sum_{h=0}^{\infty}(\nu\gamma\beta)^h\hat{\tau}_{t+1+h} \\ & + \psi^e_w \underbrace{\tilde{E}^y_t\sum_{h=0}^{\infty}(\nu\gamma\beta)^h\hat{w}_{t+1+h}}_{\text{EPV of Wages}} - \psi^e_r \underbrace{\tilde{E}^y_t\sum_{h=0}^{\infty}(\nu\gamma\beta)^h\hat{R}_{t+1+h}}_{\text{EPV of Returns to Capital}} \\ & - (1-\nu)\gamma\psi^e_{ro} \underbrace{\tilde{E}^y_t\sum_{h=1}^{\infty}\left(\frac{1-\nu^h}{1-\nu}\right)(\gamma\beta)^h\hat{R}_{t+1+h}}_{\text{EPV of Returns to Capital after Retiring}} \\ & + (1-\nu)\gamma\psi^e_s\tilde{E}^y_t\sum_{h=1}^{\infty}\left(\frac{1-\nu^h}{1-\nu}\right)(\gamma\beta)^h\hat{s}_{t+h}\end{aligned} \tag{29}$$

where ψ's consist of primitive model parameters.[16] Note that Equation (29) shows how current and expected variables such as the real interest rates and real wages affect the consumption decisions of young households. In particular, the last two terms in Equation (29) appear since young households or workers also consider they can lose their jobs and become retirees at any time in the future with the probability of $1 - \nu$.

3.4 Government

The government levies lump-sum taxes T_t on young households or workers and consumes G_t. Also, it pays retirees social security benefits S_t each period. Thus, the government satisfies the following budget constraint

$$\phi T_t = G_t + (1 - \phi)S_t \tag{30}$$

.

The government maintains its policy variables, T_t, G_t, and S_t, at steady-state levels, which is announced in advance. In particular, the policy announcement is credible, so agents believe there will be no policy changes in the future. These assumptions are based on Mitra et al. (2013).

3.5 Market clearing

Goods and asset market-clearing conditions are as follows:

$$Y_t = C_t + I_t + G_t \tag{31}$$

$$C_t = C_t^y + C_t^o \tag{32}$$

$$K_{t+1} = (1 - \delta)K_t + I_t \tag{33}$$

$$A_t = K_t \tag{34}$$

$$A_t = A_t^y + A_t^o \tag{35}$$

.

3.6 Model parameter calibration

Table 2 presents calibrated model parameter values. The RBC model parameters such as α, β, and δ are set as commonly used values in literature like King and Rebelo (2000), etc. The capital share α is 1/3, and the time discount factor $\beta = 0.995$ corresponds to the annual discount rate of 2 percent. The depreciation rate $\delta = 0.025$ matches to the annual depreciation rate of 10 percent. The proportion of young households $\phi = 0.8$ is set based on the relevant data. Specifically, the average ratio of the population aged 15–64 to the population aged 15 and older from 2014 to 2023 is 80.3 percent. The probability of surviving in the next period γ is 0.9959 following Gali (2021)'s calibration in which he uses lifetime expectancy.[17] Then, this paper assigns 0.9989 to the probability of staying workers in the following period ν to set the young households' share to 80 percent ($\phi = 0.8$). The weight on utility from leisure θ is 3.39 which satisfies the labor supply condition of young households in the steady state. Most importantly, the gain parameters for old and young households, $g^o = 0.010$ and $g^y = 0.018$, are derived from Malmendier and Nagel (2016) in which they estimate the gain parameter by age using inflation expectation data. For robustness checks, I also employ various gain parameter values, which is discussed in Section 4.1.

Table 2. Model parameter calibration

Parameter	Value	Description
α	1/3	Standard value for capital share
β	0.995	Corresponds to annual discount rate of 2 percent
δ	0.025	Corresponds to annual depreciation rate of 10 percent
ϕ	0.8	Corresponds to ratio of people aged 15-64 to 15 and over of 80 percent
γ	0.9959	Gali (2021) in which lifetime expectancy used
ν	0.9989	From $\gamma = 0.9959$ and $\phi = 0.8$
θ	3.39	Satisfies labor supply condition in SS
g^o	0.010	Malmendier and Nagel (2016)
g^y	0.018	
σ_{u_χ}	0.0078	Matches volatility of output in model to relevant data
$\bar{\chi}$	1.0053	Eusepi and Preston (2011)
$\bar{H}$	1/4	Standard value for labor in SS

The standard deviation of the technology shock σ_{u_χ} is calibrated to 0.0078 to match the volatility of output in the model to that in data as seen in Table 4. The growth of productivity in the steady state $\bar{\chi}$ is 1.0053 based on Eusepi and Preston (2011), and the labor supply in the steady state $\bar{H}$ is 1/4.

4. Effects of heterogeneous biased expectations on business cycles

This section compares the LC model under learning with the LC model under RE to investigate how heterogeneous biased expectations affect the dynamics of macroeconomic variables compared to RE. Then, I also compare the LC learning model with the representative agent (RA) learning model to show the life-cycle assumption better explains the data in the real world. Lastly, this paper studies the impacts of heterogeneous biased expectations on the volatility of business cycles and presents the implications for an aging society.

4.1 LC model under learning vs LC model under RE

The simulation process for obtaining the impulse responses of macro-variables is as follows. First of all, the 2000-period simulation under RE provides initial steady-state coefficients for the household forecasting model, Equation (10) – (13), and the simulation data are discarded. Then, the coefficients are updated every period in the learning model but keep their initial values in the RE model. Next, an N-period impulse response is obtained with a 1 percent permanent technology shock in the period 2001 and no more shocks after that. Then, this simulation is repeated 2,000 times, and I report the median response of model variables to the technology shock for IRF. The simulation methods are based on the work of Eusepi and Preston (2011) and Mitra et al. (2013).

4.1.1 Effects on aggregate variables

Figure 4 illustrates the responses of output, consumption, investment, and hours to a 1 percent permanent technology shock. Then, the upper left panel reveals that the household sensitivity to recent observations in the learning model generates amplification effects on output but not in the RE model due to no biased expectations under RE. We can interpret this amplification as households become overly optimistic about the future economy after the positive economic shock. Particularly, the output response exhibits a hump-shaped profile as found in Eusepi and Preston

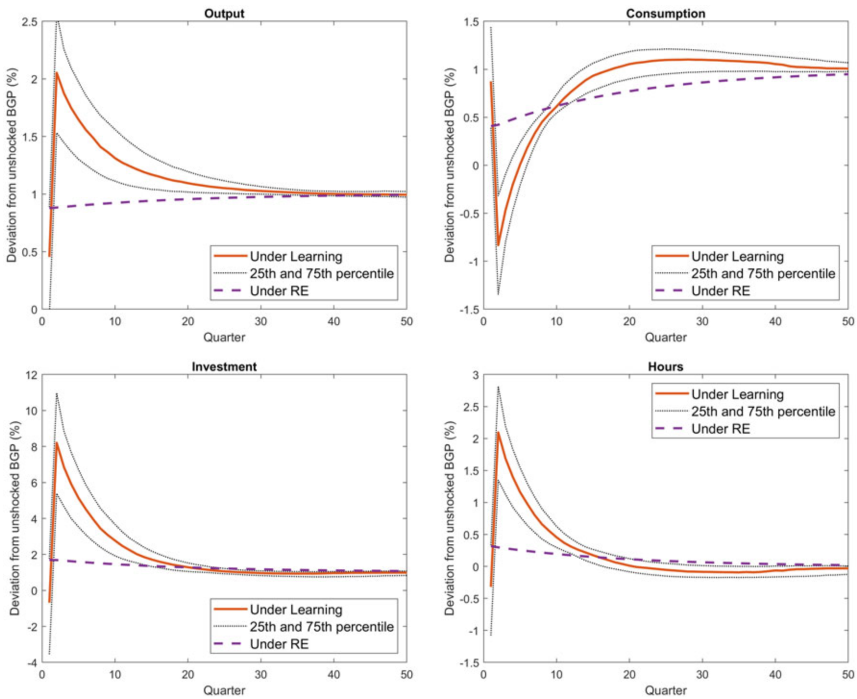

Figure 4. Impulse responses of macrovariables to 1 percent permanent technology shock.
Notes: The solid and dotted lines denote the median, 25th, and 75th percentile impulse response under learning, respectively. The dashed line indicates the impulse responses under RE. Unshocked BGP signifies the initial balanced growth path prior to the permanent technology shock.

(2011) and Cogley and Nason (1995).[18] The upper right panel shows the consumption response in the learning model is less than that in the RE model immediately after the shock, as households have overly optimistic expectations about the future returns to capital, and thus, increase investment sharply, as depicted in the lower left panel. However, in the long run, households in the learning model consume more than those in the RE model by utilizing their over-savings. The lower right panel suggests an overshooting in labor supply under learning.

Mechanism. To explicate the mechanisms of the heterogeneous biased expectations in the learning model, I provide a graphical representation in Figure 5. This figure illustrates the young and old households' expected present values (EPV) of returns to capital and labor after a positive technology shock under learning and RE. For example, the EPV of returns to capital and labor for young households under learning and RE are

$$\text{Under Learning:} \quad \tilde{E}_t^y \sum_{h=0}^{\infty} (\nu\gamma\beta)^h \hat{R}_{t+1+h} \quad \text{and} \quad \tilde{E}_t^y \sum_{h=0}^{\infty} (\nu\gamma\beta)^h \hat{w}_{t+1+h}$$

$$\text{Under RE:} \quad E_t \sum_{h=0}^{\infty} (\nu\gamma\beta)^h \hat{R}_{t+1+h} \quad \text{and} \quad E_t \sum_{h=0}^{\infty} (\nu\gamma\beta)^h \hat{w}_{t+1+h}$$

where each term appears in the young households' aggregate consumption decision rule. Here, the only difference between learning and RE models is the way of forming expectations, i.e., $\tilde{E}$ and E.

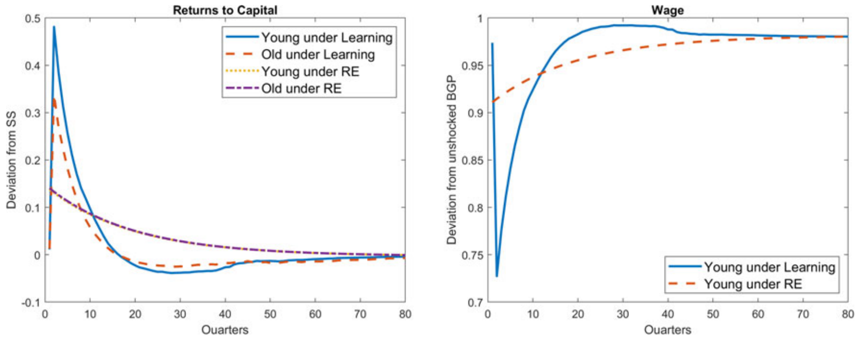

Figure 5. Expected present value of returns to capital and labor.

Then, the EPV of market prices in the learning model is significantly different from those in the RE model: i.e., the expected returns to capital are higher, while wages are lower under learning than under RE. This is because households in the learning model have biased expectations due to their overweight on recent observations, which causes forecast errors. In particular, young households have more optimistic expectations about the future returns to capital than old households, primarily because of their relatively higher sensitivity to recent shocks. However, in the RE framework, both young and old households have almost the same expectations since no different weighting schemes and heterogeneous biased expectations exist between the young and old.

Consumption, investment, and hours. The mechanism above gives further explanations about the dynamics of the macro-variables illustrated in Figure 4. The households' overly positive expectations regarding future returns to capital result in lower consumption and higher investment right after the shock, followed by an increase in consumption in the future through savings. Thus, households smooth their consumption through investment. Also, young households seek to work more today to offset their overly pessimistic wage expectations and to boost their investment and future consumption.

Oscillations in expectations. As seen in Figure 5, the household learning behavior that places more weight on recent observations results in oscillations in expectations, which can lead to fluctuations in economic activities. The paper argues that these oscillations in expectations can create further volatility in business cycles, a topic that is discussed in Section 4.3 with more details.

Persistence. An important distinction between the learning and RE models is whether persistent effects occur. As shown in Figure 4, the responses in the learning model are more persistent than those in the RE model. This persistence arises from the household learning behavior illustrated in Figure 5. Since households assign more significance to recent data, it takes a considerable amount of time for expectations under learning to approach those under RE. In other words, the persistence in the learning model is caused by adjustments in beliefs. However, in the absence of additional technology shocks, household expectations will converge to rational expectations as forecasting errors gradually diminish.

4.1.2 Effects on variables by age group

Based on the impulse responses depicted in Figure 6, it can be inferred that the biased expectations of young households play a crucial role in the amplified response of macroeconomic variables. The upper panels display the response of consumption by age group in the learning and RE model. As shown in the left panel, young households in the learning model exhibit a greater reaction to the

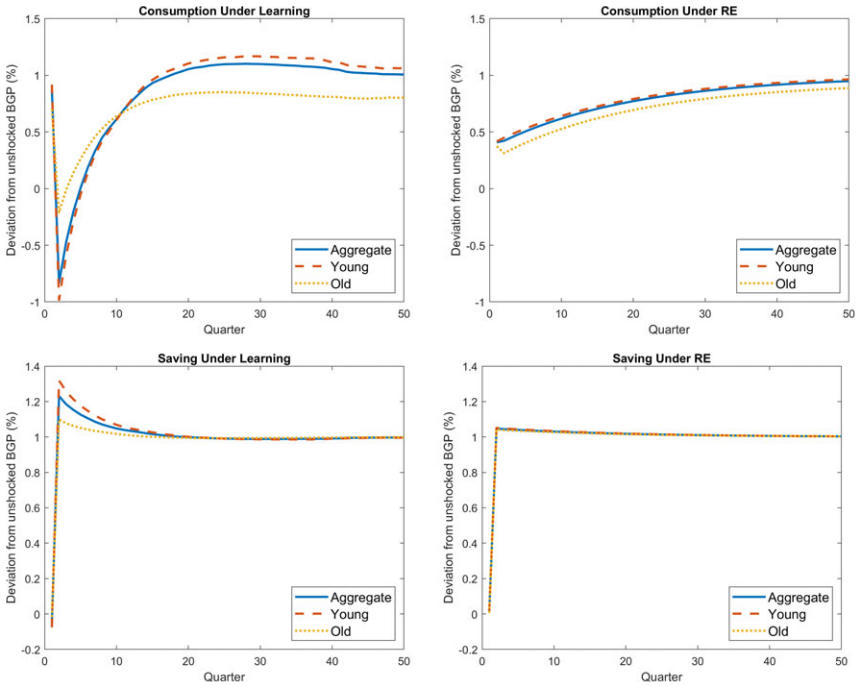

Figure 6. Impulse responses of consumption and saving to 1 percent permanent technology shock by age group.

technology shock than old households, owing to their relatively higher sensitivity to the shock. Conversely, the right panel indicates no discernible difference between young and old households' consumption in the RE model since rational expectations do not generate forecast errors or biased expectations. The lower panels also show that heterogeneous biased expectations cause young and old households to respond differently in terms of saving, particularly with young households engaging in more over-saving.

Figure 7 denotes the response of output to a 1 percent permanent technology shock in the learning model by the proportion of young households in the economy ϕ: 0.7 (=70 percent), 0.8 (=80 percent), and 0.9 (=90 percent). Consistent with previous results, a decrease in the proportion of young households, which corresponds to an increase in the share of the old population, results in a smaller response of output. Therefore, it can be concluded that the amplification effects of biased expectations become weaker in an aging society since population aging leads to a higher proportion of old households who exhibit lower sensitivity to recent shocks.

Gain parameters. To ensure the robustness of the results, this paper explores how the response of output to the positive technology shock varies as the gain parameters change, which is shown in Figure 8. When the gain parameters of young and old households, i.e., g^y and g^o, are zero, then the response under learning is identical to the one under RE. The zero gain parameters imply no biased expectations in the economy. Thus, the household expectations under learning match rational expectations. The left panel also illustrates that the response of output increases as the gain parameters rise. Larger gain parameters signify households react more sensitively to recent shocks, leading to more significant amplification effects. The right panel suggests an increase in the gain parameter of young households and a decrease in that of old households result in a larger response than the opposite case. This is because young households account for a larger share of the population and have biased expectations not only about future returns to capital but also future wages.

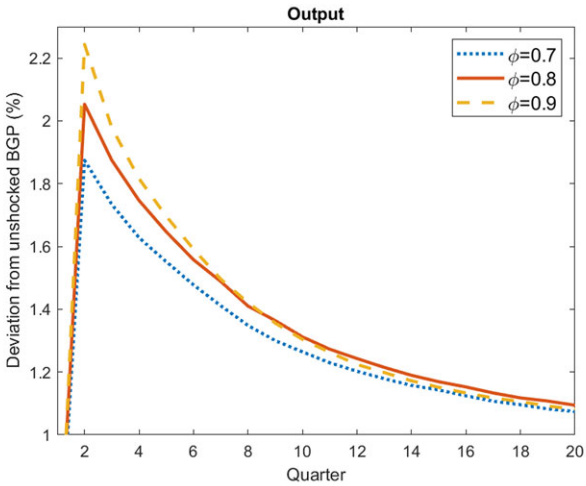

Figure 7. Impulse response of output by proportion of young households (ϕ).

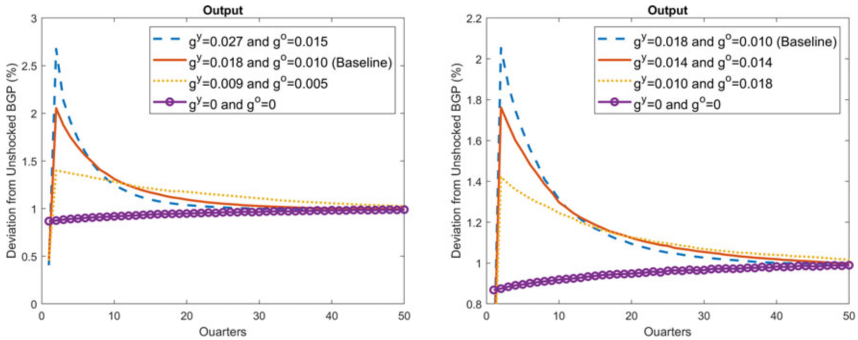

Figure 8. Impulse response of output under learning by gain parameter.

4.2 LC model under learning vs RA model under learning

I compare the LC learning model featuring heterogeneous beliefs and adaptive expectations with the RA learning model incorporating adaptive learning and homogeneous beliefs to offer a more comprehensive perspective. In particular, by comparing the RA and LC learning models, I show the life-cycle assumption better explains the data in the real world.

4.2.1 Representative agent learning model

I modify some parts of the LC learning model to obtain the RA learning model.

Firms. The firm sector in the RA learning model is assumed to be the same as in the LC learning model.

Households. An infinitely-lived representative agent maximizes the following Bellman equation

$$V(A_t^{RA}, K_t^{RA}) = \text{Max}\{\ln C_t^{RA} + \theta \ln(1 - H_t^{RA}) + \beta \tilde{E}_t^{RA}[V(A_{t+1}^{RA}, K_{t+1}^{RA})]\} \tag{36}$$

and their budget constraint is

$$C_t^{RA} + A_{t+1}^{RA} = R_t^{RA} A_t^{RA} + W_t^{RA} H_t^{RA} - T_t^{RA} \tag{37}$$

where $\tilde{E}_t^{RA}$ is the *subjective* expectation or belief by the representative agent, C_t^{RA} is consumption, A_t^{RA} is the asset, W_t^{RA} is the real wage, H_t^{RA} is the labor supply, and T_t^{RA} is the lump-sum tax.

Then, this paper obtains log-linearized aggregate consumption of the representative agent using the intertemporal budget constraint and Euler equation

$$\begin{aligned}\bar{c}\hat{c}_t^{RA} = & \frac{(1-\beta)\bar{a}}{(1+\theta)\beta}\hat{a}_t^{RA} + \left[\bar{c} - \frac{\bar{w}}{(1+\theta)}\right](\hat{R}_t^{RA} - \hat{\chi}_t^{RA}) \\ & + \frac{(1-\beta)\bar{w}}{(1+\theta)}\hat{w}_t^{RA} - \frac{(1-\beta)\bar{\tau}}{(1+\theta)}\hat{\tau}_t^{RA} \\ & - \frac{\beta(\bar{w}^{-}\bar{\tau})}{(1+\theta)}\tilde{E}_t^{RA}\sum_{h=0}^{\infty}\beta^h[\hat{R}_{t+1+h}^{RA} - \hat{\chi}_{t+1+h}^{RA}] \\ & + \frac{(1-\beta)\beta\bar{w}}{(1+\theta)}\tilde{E}_t^{RA}\sum_{h=0}^{\infty}\beta^h\hat{w}_{t+1+h}^{RA} \\ & - \frac{(1-\beta)\beta\bar{\tau}}{(1+\theta)}\tilde{E}_t^{RA}\sum_{h=0}^{\infty}\beta^h\hat{\tau}_{t+1+h}^{RA}\end{aligned} \tag{38}$$

.

For the comparison, log-linearized aggregate consumption in the LC learning model, which is the summation of Equation (22) and (29), is as follows

$$\begin{aligned}\bar{c}\hat{c}_t = & \left[\frac{(1-\gamma\beta)(1-\nu)}{\beta} + \psi^{'}\nu\right]\bar{a}^y\hat{a}_t^y + \frac{(1-\gamma\beta)}{\beta}\bar{a}^o\hat{a}_t^o \\ & + \left[\bar{c} - \frac{\psi^{'}\phi\beta\bar{w}}{(1-\nu\gamma\beta)}\right](\hat{R}_t - \hat{\chi}_t) + \psi^{'}\phi\beta\bar{w}\hat{w}_t - \psi^{'}\phi\beta\bar{\tau}\hat{\tau}_t \\ & - \left[\frac{\psi^{'}\phi\nu\gamma\beta^2(\bar{w}-\bar{\tau})}{(1-\nu\gamma\beta)}\right]\tilde{E}_t^y\sum_{h=0}^{\infty}(\nu\gamma\beta)^h[\hat{R}_{t+1+h} - \hat{\chi}_{t+1+h}] \\ & + \psi^{'}\phi\nu\gamma\beta^2\bar{w}\tilde{E}_t^y\sum_{h=0}^{\infty}(\nu\gamma\beta)^h\hat{w}_{t+1+h} \\ & - \psi^{'}\phi\nu\gamma\beta^2\bar{\tau}\tilde{E}_t^y\sum_{h=0}^{\infty}(\nu\gamma\beta)^h\hat{\tau}_{t+1+h}\end{aligned} \tag{39}$$

where

$$\psi^{'} = \frac{(1-\nu\gamma\beta)(1-\gamma\beta)}{(1+\theta)\beta(1-\gamma\beta)+(1-\nu)\gamma\beta^2}$$

.

Belief updating. The representative agent is assumed to use an economic model to forecast future wages and returns to capital. The model relates the wage and returns to capital to the aggregate capital which is the unique state variable in the RA model

$$\hat{R}_t^{k,RA} = \mu_r^{RA} + \mu_{rk}^{RA}\hat{k}_t^{RA} + e_t^{r,RA} \tag{40}$$

$$\hat{w}_t^{RA} = \mu_w^{RA} + \mu_{wk}^{RA}\hat{k}_t^{RA} + e_t^{w,RA} \tag{41}$$

$$\hat{k}_{t+1}^{RA} = \mu_k^{RA} + \mu_{kk}^{RA}\hat{k}_t^{RA} + e_t^{k,RA} \tag{42}$$

where e_t denotes a regression error.

Table 3. Coefficients and time discount factors on expected terms in aggregate consumption

(A) $\hat{w}_{t+1+h}$	
Representative Agent Model	Life-cycle Model
$\frac{(1-\beta)\beta\bar{w}}{(1+\theta)}\tilde{E}_t^{RA}\sum_{h=0}^{\infty}\beta^h\hat{w}_{t+1+h}^{RA}$	$\phi\frac{(1-v\gamma\beta)v\gamma\beta\bar{w}}{(1+\theta)+(1-v)\frac{\gamma\beta}{(1-\gamma\beta)}}\tilde{E}_t^y\sum_{h=0}^{\infty}(v\gamma\beta)^h\hat{w}_{t+1+h}$
(B) $\hat{R}_{t+1+h}$	
Representative Agent Model	Life-cycle Model
$\frac{\beta(\bar{w}-\bar{\tau})}{(1+\theta)}\tilde{E}_t^{RA}\sum_{h=0}^{\infty}\beta^h\hat{R}_{t+1+h}^{RA}$	$\phi\frac{v\gamma\beta(\bar{w}-\bar{\tau})}{(1+\theta)+(1-v)\frac{\gamma\beta}{(1-\gamma\beta)}}\tilde{E}_t^y\sum_{h=0}^{\infty}(v\gamma\beta)^h\hat{R}_{t+1+h}$

Gain parameter. The gain parameter for the representative agent is calibrated as $g^{RA} = 0.0164$ $[=g^o \times (1-\phi) + g^y \times \phi = 0.010 \times 0.2 + 0.018 \times 0.8]$ based on the Malmendier and Nagel (2016)'s estimation.

4.2.2 Main differences between LC and RA learning model

Differences in model. The LC and RA learning models have similar aggregate consumption equations, Equation (38) and (39), but with some notable differences. First, the LC learning model has one more state variable: other than aggregate capital ($\hat{k}_t$), the distribution of wealth between young and old households ($\hat{a}_t^y$ and $\hat{a}_t^o$) is also necessary to determine aggregate consumption. Thus, there is one more explanatory variable in the household forecasting system in the LC model. Second, Table 3 compares the coefficients and time discount factors on the expected terms from both LC and RA learning models and suggests that the differences in consumption behavior between the two models mostly stem from whether households consider the probability of surviving (γ) and staying workers (v). In the LC learning model, households know they may not survive into the next period or retire in the following period, which leads to greater discounting of the future and so smaller coefficients and time discount factors on the expected terms. On the other hand, the representative household in the RA learning model places relatively greater weight on the expectations about market prices as they do not consider the possibility of death or retirement in the future.

Results. Figure 9 displays the impulse responses in the LC and RA learning models. Despite their overall similarities, the responses in the RA learning model are found to be 1.5 to 2 times larger than those in the LC learning model. This is because the representative household places much more weight on their overly optimistic expectations after a positive shock, in comparison to households in the LC model. However, this poses a challenge as the excessive over-reactions of the representative household do not align with the actual data observed in the real world. Section 4.3 presents quantitative results about this issue.

4.3 Effects on volatility of business cycles

This paper utilizes the LC learning model to examine the impact of population aging on the recent reduction in business cycle fluctuations.

Simulation method. The 2000-period simulation under RE presents initial steady-state coefficients for the household forecasting system, Equation (10)–(13), and then the simulated data are discarded. After that, the 300-period simulation is conducted to match the sample size for the U.S. data from 1947Q1 to 2019Q4. Here, I do not include the COVID-19 period data due

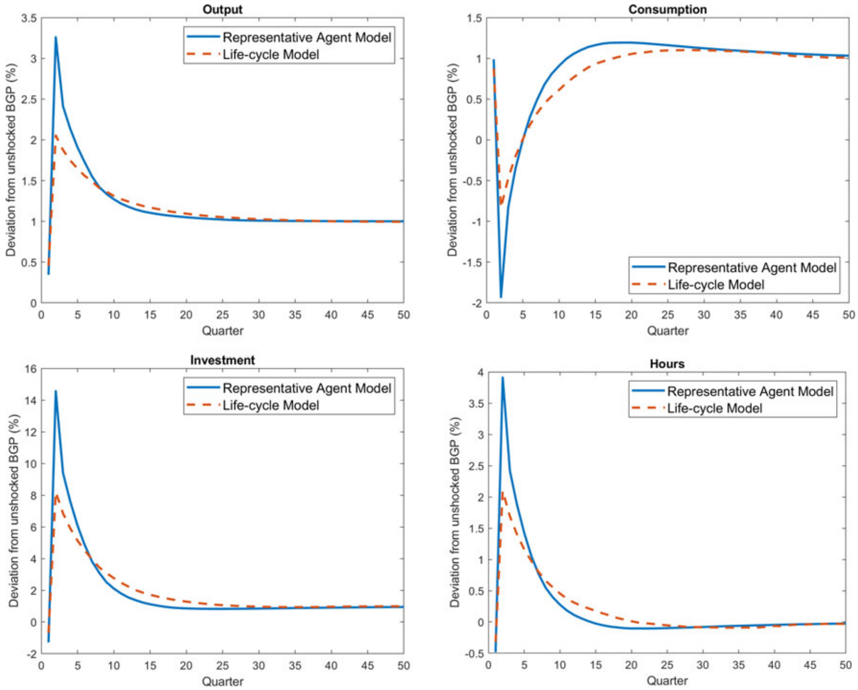

Figure 9. Impulse responses of output, consumption, investment, and hours to 1 percent permanent technology shock in LC and RA learning model.

to unprecedentedly high volatility. Finally, I calculate the volatility of business cycles using this 300-period simulated data.

Results. There are several findings from the relative standard deviations of the macroeconomic variables provided by the data and model simulations in Table 4. First, the amplification effects in the LC learning model produce macroeconomic variables' variations that are almost identical to the data when the proportion of young households ϕ is at its baseline value of 0.8 (=80 percent).[19] However, the RA learning model generates more substantial fluctuations due to the extensive over-response of the representative household as seen in Section 4.2.[20] This is because the representative household does not consider the probability of death and losing a job, and so they discount over-expectation less than households in the LC model, which triggers extensive over-responses. Second, the LC learning model shows that the fluctuations in business cycles decrease as the population ages. For instance, when the proportion of young households ϕ decreases from 0.9 (=90 percent) to 0.7 (=70 percent), the relative standard deviation of output decreases from 1.77 to 1.49.[21] Lastly, the RE models, regardless of the RA or LC assumption, can only explain half of the volatility of the macro-variables seen in the data.

Implications. The findings above imply a meaningful role of heterogeneous biased expectations and population aging in the recent moderation of business cycle fluctuations. The stylized facts suggest population aging is associated with a reduction in the fluctuations of the macro-variables. Table 5 indicates that the volatility of macroeconomic variables has decreased sharply in recent years. Specifically, the standard deviation of GDP, Consumption, and Investment has declined 74.2 percent, 59.6 percent, and 57.9 percent respectively from the 1980s to 2010s. Moreover, during the same period, the population has aged fast. The ratio of the population aged 65 and over to the

Table 4. Relative standard deviations of macro-variables from model simulations

	(1) σ_Y/σ_{Pr}	(2) σ_C/σ_Y	(3) σ_I/σ_Y	(4) σ_H/σ_Y
A. Data	1.64	0.82	4.33	1.20
B. Learning				
(i) Life-Cycle				
$\phi = 0.7$	1.49	0.75	4.12	1.66
$\phi = 0.8$ (Baseline)	**1.64**	**0.74**	**4.32**	**1.88**
$\phi = 0.9$	1.77	0.85	4.61	2.11
(ii) Representative	2.59	0.75	4.63	3.26
C. Rational Expectation				
(i) Life-Cycle				
$\phi = 0.8$	0.88	0.47	1.96	0.32
(ii) Representative	0.90	0.49	1.93	0.36

Notes: All macro-variables are logged and detrended with the HP filter. *Pr* stands for productivity.

Table 5. Standard deviations of annual growth rates of macro-variables from U.S. Data

(1) Standard Deviations			
Period	A. GDP	B. Consumption	C. Investment
1960Q1-2019Q4	2.23	1.88	9.46
(2) Standard Deviations, Relative to 1960Q1-2019Q4			
Period	A. GDP	B. Consumption	C. Investment
(i) 1960Q1-1969Q4	0.92	0.90	0.85
(ii) 1970Q1-1979Q4	1.22	1.20	1.20
(iii) 1980Q1-1989Q4	1.20	1.09	1.33
(iv) 1990Q1-1999Q4	0.68	0.80	0.69
(v) 2000Q1-2009Q4	0.97	1.01	0.99
	(0.73)	(0.69)	(0.80)
(vi) 2010Q1-2019Q4	0.31	0.44	0.56

Notes: This table is the extended version of the table in Stock and Watson (2003). The parenthesis ( ) indicates the volatility excluding the data for the Great Recession from 2007Q4 to 2009Q2. I exclude the COVID-19 period data due to unprecedentedly high volatility.

population aged 15 and over in the U.S. has risen from 14.6 percent in 1999 to 20.2 percent in 2019 according to OECD.stat. In line with these facts, the LC learning model simulations also show a 10 percentage point increase in the old population ratio leads to about a 16 percent decrease in output volatility.[22]

It is a novel channel that the increase in the proportion of old individuals who are relatively less sensitive to recent observations contributes to the lower volatility of business cycles. Literature lists the improved monetary policy, regulatory changes, financial market innovation, etc. as the causes of the decline in macroeconomic volatility since the 1990s. Jaimovich and Siu (2009) also claim that demographic changes account for 1/5 to 1/3 of the moderation of the US economy. However, they mainly point out the channel that the low volatility of old individuals' employment and hours worked leads to the Great Moderation.

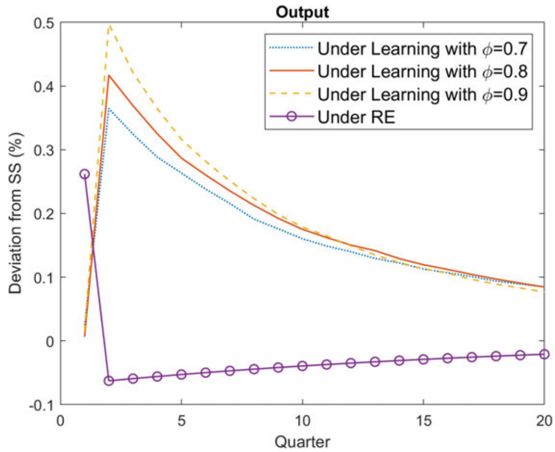

Figure 10. Impulse response of output to government spending shock by proportion of young households (ϕ).

5. Policy implications of heterogeneous biased expectations

This section investigates how heterogeneous biased expectations affect the output effects and multipliers of government spending in an aging population. Then, I also examine the welfare effects of government spending under population aging.

5.1 Population aging and effects of government spending

Figure 10 illustrates the impact of a government spending shock on output, where the government spending is 5 percent of the steady-state output level and is financed by an equal amount of lump-sum tax increase.[23] Here, the government spending shock is a *one-time surprise* shock. Specifically, normalized government spending g_t evolves via

$$g_t = \begin{cases} \bar{g} + 0.05\bar{y}, & \text{for } t = 1 \\ \bar{g}, & \text{for } t = 2, 3, 4, \cdots \end{cases}$$

where $\bar{y}$ and $\bar{g}$ are the steady-state values for output and government spending.

Then, the key takeaway is that the responsiveness of output to the government spending shock is inversely related to the proportion of old households in the economy. Since old households have relatively lower sensitivity to the recent government spending shock, the amplification effects become weak in an aging society, which is consistent with the findings from the technology shock analysis shown in Figure 7.

I also note that after government spending, the learning model generates an output response that is the opposite of the response from the RE model. The government spending shock decreases consumption and investment and increases hours worked, causing a rise in returns to capital and a decline in wages. In the learning model, households overreact to increased capital returns and raise investment dramatically. However, in the RE model, households know returns to capital will decrease immediately, so they reduce investment. Therefore, the output responses show opposite reactions.

Figure 11 provides the cumulative government spending multipliers in the first, second, and third year after the shock using the impulse responses in Figure 10. The cumulative government spending multiplier (M) is defined as

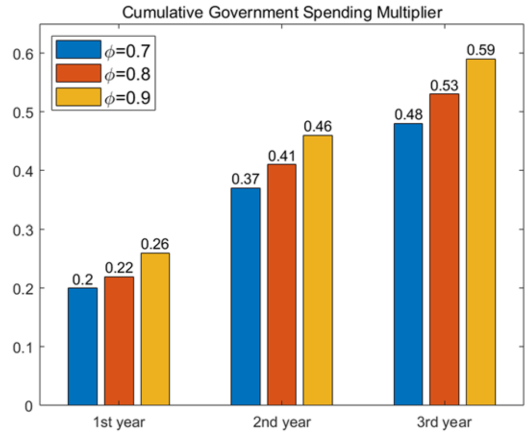

Figure 11. Cumulative government spending multiplier by proportion of young households (ϕ).

$$M_T = \frac{\sum_{i=0}^{T} (\beta\gamma)^i (y_{t+i} - \bar{y})}{g_t - \bar{g}} \tag{43}$$

where T is the cumulation period. Overall, the government spending multiplier declines about 10 percent when the old population ratio rises 10 percentage points. This result is consistent with previous studies, which empirically and theoretically show that the government spending multiplier is low in an aging society [e.g., Basso and Rachedi (2021) and Honda and Miyamoto (2020)]. However, my model offers a novel factor for the lower government spending multiplier: i.e., heterogeneous biased expectations between the young and old.

5.2 Welfare analyses of government spending

This paper investigates the welfare effects of government spending in the LC learning and LC RE model by conducting welfare experiments and finds the implications for an aging society.

5.2.1 Welfare experiments

This paper conducts welfare experiments under the following scenarios to study the welfare effects of government spending. Specifically, during 500 periods, a random technology shock with mean zero and standard deviation σ^u takes place every period in the LC learning and LC RE model.[24] Then, I assume two economies in each LC learning and LC RE model. In the first economy, the government counteracts negative technology shocks by increasing its spending.[25] However, in the second economy, the government does not respond to the technology shocks. By comparing the welfare of the population in these two economies, we can estimate the welfare effects of the government spending policy.

In this setting, government spending shocks can be beneficial to households by reducing the adverse effects of negative technology shocks. In particular, it can be mitigated more in the learning model due to the amplification effects of biased expectations.

Equivalent variations. Following literature such as Hunt (2021), this paper estimates the welfare effects of government spending using equivalent variations (EV) which are calculated by:

Table 6. Equivalent variations under learning by proportion of young households (ϕ)

Cohort	EV from	Welfare Effects of Gov Spending ($\Delta^{GOV}_{Learning}$%) A. $\phi = 0.7$	**B. $\phi = 0.8$**	C. $\phi = 0.9$	A-C (p.p.)
Old	Consumption	0.0009	**0.0014**	0.0064	−0.0055
Young	Labor Supply	−3.0553	−**2.6538**	−2.3468	−0.7085
	Consumption	−0.9306	−**0.7154**	−0.5764	−0.3542
	Sum	−3.9859	−**3.3692**	−2.9232	−1.0627
Total Population		−2.7898	−**2.6951**	−2.6303	−0.1596

$$\text{RE:} \quad \sum_{t=1}^{T} (\beta\gamma)^{t-1} U(C^{NonGov}_{RE,t}(1 + \Delta^{Gov}_{RE})) = \sum_{t=1}^{T} (\beta\gamma)^{t-1} U(C^{Gov}_{RE,t}) \tag{44}$$

$$\text{Learning:} \quad \sum_{t=1}^{T} (\beta\gamma)^{t-1} U(C^{NonGov}_{Learning,t}(1 + \Delta^{Gov}_{Learning})) = \sum_{t=1}^{T} (\beta\gamma)^{t-1} U(C^{Gov}_{Learning,t}) \tag{45}$$

where C^{NonGov}_t and C^{Gov}_t indicate consumption in the economy without and with the government spending policy, respectively. Also, the welfare experiment length T is 500. The interpretation of Equation (44) and (45) is that the EV equates the utility of household in the first economy with the government spending policy to the utility of household in the second economy without government spending plus a fraction Δ. In other words, for example, the household in the second economy without government spending needs to consume Δ^{Gov}% more every period to have the same welfare as the household has in the first economy with government spending. As a result, Δ^{Gov}_{RE}% and $\Delta^{Gov}_{Learning}$% represent the welfare effects of government spending in the LC RE and LC learning model.

5.2.2 Welfare effects of government spending

Table 6 provides details of the welfare effects of government spending in the LC learning model, denoted by ($\Delta^{Gov}_{Learning}$%), across varying proportions of young households. As the proportion of old households increases, both young and old households experience a decline in welfare due to the weakened amplification effects. However, young households experience a more significant decline in their welfare compared to that of old households. With an increase in the proportion of old households, fewer individuals have a high degree of optimism triggered by government spending. Consequently, the economy exhibits relatively poor performance after the government spending shock. Then, young households respond more strongly to this weaker economic performance by reducing consumption and increasing labor supply, while old households are less responsive. Thus, the welfare of young households deteriorates more significantly as the population ages.

Table 7 presents the welfare effects of government spending in the LC learning and LC RE models ($\Delta^{Gov}_{Learning}$% and Δ^{Gov}_{RE}%, respectively). The welfare effects are more significant in the LC learning model compared to the LC RE model since the biased optimism induced by government spending amplifies the output effects, resulting in an overall improvement in households' welfare. However, this learning behavior only benefits young households, while it reduces the welfare of the old households. Young households or workers in the LC learning model increase labor supply and investment, leading to higher future consumption. This excessive consumption, funded by savings, causes an improvement in young households' welfare, despite the disutility caused by the

Table 7. Equivalent variations under learning and rational expectation (RE)

Cohort	EV from	Welfare effects of Gov Spending: A. RE (Δ_{RE}^{Gov}%)	Welfare effects of Gov Spending: B. Learning ($\Delta_{Learning}^{Gov}$%)	Learning vs RE: **B-A (p.p.)**
Old	Consumption	0.0168	0.0014	**−0.0154**
Young	Labor Supply	−2.5715	−2.6538	**−0.0824**
	Consumption	−2.1436	−0.7154	**1.4281**
	Sum	−4.7150	−3.3692	**1.3458**
Total Population		−3.7687	−2.6951	**1.0736**

Notes: In this experiment, ϕ is 0.8. The last column in bold shows the welfare effects of the household expectations under learning relative to under RE.

increased labor supply. However, the young's over-consumption crowds out the consumption of old households, causing it to fall below the RE level, resulting in a reduction in the old's welfare.

6. Conclusion

Economists have long considered the expectations of economic agents one of the key drivers of economic fluctuations. To that end, numerous studies have been conducted to deal with this topic. I contribute to this body of literature by examining the implications of the heterogeneous biased expectations between young and old individuals on business cycles and economic policies. The life-cycle learning model incorporating heterogeneous biased expectations reveals that the fluctuations of business cycles decline as the population ages due to the relatively lower sensitivity of older households to recent observations. Additionally, in an aging society, the output effects of government spending, also known as government spending multipliers, decrease, and these reduced effects cause a welfare loss for households, particularly the young.

Based on these findings, several policy recommendations can be made. First, economic policies should be devised and implemented from a long-term perspective in an aging society since population aging reduces the volatility of business cycles and hinders the short-term effects of government policies. Moreover, the government needs to consider how its policies affect young and old individuals differently, given their heterogeneous biased expectations. Finally, a faster and more extensive fiscal stimulus policy is necessary during recessions in an aging society to support a swift recovery, which also necessitates the government to raise fiscal space in advance for the aging population.

Notes

1 The ICE focuses on three areas: how consumers view prospects for their own financial situation, how they view prospects for the general economy over the near term, and their view of prospects for the economy over the long term.

2 Cogley and Sargent (2008) state "Agents are eager to learn at the beginning of each period, but their decisions reflect a pretense that this is the last time they will update beliefs, a pretense that is falsified at the beginning of every subsequent period."

3 In this paper, the old population share or ratio indicates $\frac{\text{Number of people aged 65 and over}}{\text{Number of people aged 15 and over}} \times 100(\%)$.

4 Keane and Runkle (1990), using a SPF, find that professional forecasters have rational expectations.

5 The anticipated returns to labor and capital are crucial elements influencing household consumption decisions, as noted by Milton Friedman's Permanent Income Hypothesis, which posits that consumption expenditures are contingent on anticipated future income, rather than current income.

6 See Section 4.2 for the minimum state variables in the LC learning model.

7 I find that the results of the learning model are almost the same as those of the RE model if the technology shock is included in the household forecasting system.

8 Equation (15) and (16) are derived from the following constant gain least squares estimator. Derivations are in Carceles-Poveda and Giannitsarou (2007).

$$\zeta_{k,t} = \left[\sum_{i=1}^{t} (1-g)^{i-1} x_{t-i} x'_{t-i} \right]^{-1} \left[\sum_{i=1}^{t} (1-g)^{i-1} x_{t-i} k_{t-i+1} \right]$$

Also, the model nests rational expectations since the model converges to the model under RE as g goes to zero.
9 According to Honkapohja and Mitra (2006), the agents' different degree of responsiveness in the updating function, i.e., different gain parameters, can be a source of heterogeneity in the learning model.
10 Finance literature such as Nakov and Nuño (2015) and Malmendier et al. (2020) adopts the different gain parameter by age. However, individuals in each cohort have the same gain parameter in this paper to make the model more tractable in the period-by-period aging framework.
11 2.5 percent for the young and 0.5 percent for the old.
12 In this setting, financial intermediaries solve a linear profit maximization problem and end up with zero profit, which means their decisions are not affected by the degree of uncertainty. So, they are risk-neutral.
13 I borrow notations from Baksa and Munkacsi (2019).
14 Log utility for consumption is necessary for steady-state labor supply along a balanced growth path [see King et al. (1988)].
15 The derivation of Equation (22) is explained in Appendix A.1.
16 The derivation of Equation (29) is explained in Appendix A.2.
17 In Gali (2021), "To calibrate γ, I use the expected lifetime at age 16, which is 63.2 years in the United States, and thus set $\gamma = 1 - (1/(4 \times 63.2)) \simeq 0.996$."
18 At time t when the productivity shock occurs, there are no biased expectations in the learning model since households have the same expectations as in the RE model. However, when households observe at time $t+1$ that the productivity shock in period t has occurred, they will have biased expectations due to forecast errors. These biased expectations then greatly impact the macroeconomic variables and the impulse responses become hump-shaped.
19 Eusepi and Preston (2011) also find that a learning model fits the data better.
20 Eusepi and Preston (2011) use the small gain parameter 0.002 (0.2 percent) that is below the normal range for it, 0.007 (0.7 percent) – 0.05 (5.0 percent), which is given by the literature. This small gain parameter helps reduce the extensive over-response of the representative agent in their learning model and match the data. See Section 4.2.
21 Instead of the "Perpetual Youth" assumption in this paper, we can also assume that young individuals have a lower probability of dying than old individuals. Then, young individuals give more weight to their biased expectations due to a higher time discount factor, which increases business cycle volatility more significantly. Thus, an aging population with fewer young individuals would reduce business cycle volatility more than when households have the same probability of death.
22 I calculate the output volatility from the model simulations using the growth rate of output for the comparison with Table 5.
23 Contrary to the assumptions of this paper, government spending could be financed by a reduction in social security payments. This would have different implications for the consumption and asset choices of young and old individuals. However, the impact of government spending on aggregate variables is not expected to change significantly.
24 Chari, Kehoe, and McGrattan (2007) show that input financing frictions causing inefficiency in the usage of input factors can be observationally equivalent to negative productivity shocks. So, Mitra et al. (2019) consider negative innovations to productivity as a convenient shortcut for modeling distortions related to the financial crisis.
25 For instance, the government increases its spending by 5 percent of the steady-state output level in response to a negative 1 percent technology shock.
26 More details are in Mitra et al. (2019).

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Appendix A. Log-linearized Aggregate Consumption of Young and Old Households in Life-Cycle Learning Model

This section derives log-linearized aggregate consumption of young and old households in the life-cycle learning model.

A.1 Old Households (Retirees)

This paper normalizes the one-period budget constraint of old households

$$c^o_{a,t} + \gamma a^o_{a+1,t+1} = R_t a^o_{a,t} \chi_t^{-1} + s_t \tag{46}$$

and iterate forward Equation (46). Then, the intertemporal budget constraint of old households is

$$\underbrace{\tilde{E}^o_t \sum_{n=0}^{\infty} \prod_{h=0}^{n} \gamma^n \frac{\chi_{t+h}}{R_{t+h}} c^o_{a+n,t+n}}_{=(i)} = \underbrace{a^o_{a,t}}_{=(ii)} + \underbrace{\tilde{E}^o_t \sum_{n=0}^{\infty} \prod_{h=0}^{n} \gamma^n \frac{\chi_{t+h}}{R_{t+h}} s_{t+n}}_{=(iii)} \tag{47}$$

.

After that, I log-linearize the intertemporal budget constraint using Euler equation

$$(i) = \frac{\beta \bar{c}^o_a}{1-\gamma\beta} \hat{c}^o_{a,t} + \frac{\beta \bar{c}^o_a}{1-\gamma\beta} (\hat{\chi}_t - \hat{R}_t)$$

$$(ii) = \bar{a}^o_a \hat{a}^o_{a,t}$$

$$\begin{aligned}(iii) = & \frac{\beta \bar{s}}{1-\gamma\beta}(\hat{\chi}_t - \hat{R}_t) + \frac{\gamma\beta^2 \bar{s}}{1-\gamma\beta} \tilde{E}^o_t \sum_{h=0}^{\infty} (\gamma\beta)^h (\hat{\chi}_{t+1+h} - \hat{R}_{t+1+h}) \\ & + \beta \bar{s} \hat{s}_t + \gamma \beta^2 \bar{s} \tilde{E}^o_t \sum_{h=0}^{\infty} (\gamma\beta)^h \hat{s}_{t+1+h}\end{aligned}$$

and plug (i), (ii), and (iii) into Equation (47)

$$\begin{aligned}\bar{c}^o_a \hat{c}^o_{a,t} = & \frac{(1-\gamma\beta)}{\beta} \bar{a}^o_a \hat{a}^o_{a,t} + (\bar{c}^o_a - \bar{s})(\hat{R}_t - \hat{\chi}_t) + (1-\gamma\beta)\bar{s}\hat{s}_t \\ & + \gamma\beta\bar{s}\tilde{E}^o_t \sum_{h=0}^{\infty} (\gamma\beta)^h (\hat{\chi}_{t+1+h} - \hat{R}_{t+1+h}) \\ & + \frac{(1-\gamma\beta)}{\gamma\beta} \bar{s} \tilde{E}^o_t \sum_{h=0}^{\infty} (\gamma\beta)^h \hat{s}_{t+1+h}\end{aligned} \tag{48}$$

.

To aggregate the consumption of old households, this paper multiplies both sides with $\sum_{a=0}^{\infty} N^o_{a,t}$ where $N^o_{a,t}$ is the number of old households who belong to cohort 'a'. Then, I get

$$\begin{aligned}\bar{c}^o\hat{c}_t^o = &\frac{(1-\gamma\beta)}{\beta}[(1-\nu)\bar{a}^y\hat{a}_{t-1}^y + \bar{a}^o\hat{a}_t^o] + [\bar{c}^o - (1-\phi)\bar{s}](\hat{R}_t - \hat{\chi}_t) \\ &+ (1-\phi)(1-\gamma\beta)\bar{s}\hat{s}_t \\ &+ (1-\phi)\gamma\beta\bar{s}\tilde{E}_t^o\sum_{h=0}^{\infty}(\gamma\beta)^h(\hat{\chi}_{t+1+h} - \hat{R}_{t+1+h}) \\ &+ (1-\phi)(1-\gamma\beta)\gamma\beta\bar{s}\tilde{E}_t^o\sum_{h=0}^{\infty}(\gamma\beta)^h\hat{s}_{t+1+h}\end{aligned} \tag{49}$$

.

In particular, obtaining Equation (49) is based on Baksa and Munkacsi (2019)'s following assumption

$$\sum_{a=o}^{\infty} N_{a,t}^o a_{a,t}^o = N_{0,t}^o a_{0,t}^o + \sum_{a=1}^{\infty} N_{a,t}^o a_{a,t}^o = (1-\nu)a_{t-1}^y + a_t^o \tag{50}$$

$$\left(\because\ N_{0,t}^o a_{0,t}^o \approx (1-\nu)N_{t-1}^y \frac{a_{t-1}^y}{N_{t-1}^y} = (1-\nu)a_{t-1}^y \quad \text{and} \quad \sum_{a=1}^{\infty} N_{a,t}^o a_{a,t}^o = \sum_{a=1}^{\infty} \gamma N_{a-1,t-1}^o a_{a,t}^o = a_t^o\right)$$

and therefore

$$\sum_{a=o}^{\infty} N_{a,t}^o \bar{a}_a^o \hat{a}_{a,t}^o = (1-\nu)\bar{a}^y\hat{a}_{t-1}^y + \bar{a}^o\hat{a}_t^o \tag{51}$$

.

Next, I rearrange Equation (49), and finally, the consumption decision rule of old households in the LC learning model is

$$\begin{aligned}\hat{c}_t^o = &\frac{(1-\gamma\beta)(1-\nu)}{\beta}\frac{\bar{a}^y}{\bar{c}^o}\hat{a}_{t-1}^y + \frac{(1-\gamma\beta)}{\beta}\frac{\bar{a}^o}{\bar{c}^o}\hat{a}_t^o \\ &+ \left[1 - (1-\phi)\frac{\bar{s}}{\bar{c}^o}\right](\hat{R}_t - \hat{\chi}_t) + (1-\phi)(1-\gamma\beta)\frac{\bar{s}}{\bar{c}^o}\hat{s}_t \\ &+ (1-\phi)\gamma\beta\frac{\bar{s}}{\bar{c}^o}\tilde{E}_t^o\sum_{h=0}^{\infty}(\gamma\beta)^h(\hat{\chi}_{t+1+h} - \hat{R}_{t+1+h}) \\ &+ (1-\phi)(1-\gamma\beta)(\gamma\beta)\frac{\bar{s}}{\bar{c}^o}\tilde{E}_t^o\sum_{h=0}^{\infty}(\gamma\beta)^h\hat{s}_{t+1+h}\end{aligned} \tag{52}$$

.

A.2 Young Households (Workers)

I rearrange the budget constraint of young households using the labor supply condition

$$(1+\theta)c_{b,t}^y + \nu\gamma a_{b+1,t+1}^y + (1-\nu)\gamma a_{b+1,t+1}^{yo} = R_t\chi_t^{-1}a_{b,t}^y + w_t - \tau_t \tag{53}$$

and iterate forward Equation (53). Then, the intertemporal budget constraint of young households is

$$\underbrace{(1+\theta)\tilde{E}_t^y\sum_{n=0}^{\infty}\prod_{h=0}^{n}(\nu\gamma)^n\frac{\chi_{t+h}}{R_{t+h}}c^y_{b+n,t+n}}_{=(i)}=\underbrace{a^y_{b,t}}_{=(ii)}+\underbrace{\tilde{E}_t^y\sum_{n=0}^{\infty}\prod_{h=0}^{n}(\nu\gamma)^n\frac{\chi_{t+h}}{R_{t+h}}w_{t+n}}_{=(iii)}$$
$$-\underbrace{\tilde{E}_t^y\sum_{n=0}^{\infty}\prod_{h=0}^{n}(\nu\gamma)^n\frac{\chi_{t+h}}{R_{t+h}}\tau_{t+n}}_{=(iv)}$$
$$-\underbrace{(1-\nu)\gamma\tilde{E}_t^y\sum_{n=0}^{\infty}\prod_{h=0}^{n}(\nu\gamma)^n\frac{\chi_{t+h}}{R_{t+h}}a^{yo}_{b+1+n,t+1+n}}_{=(v)} \tag{54}$$

.

After that, this paper log-linearizes the intertemporal budget constraint of young households using Euler equations

$$(i)=\frac{(1+\theta)\beta\bar{c}^y_b}{(1-\nu\gamma\beta)}(\hat{\chi}_t-\hat{R}_t+\hat{c}^y_{b,t})$$

$$(ii)=\bar{a}^y_b\hat{a}^y_{b,t}$$

$$(iii)=\frac{\beta\bar{w}}{1-\nu\gamma\beta}(\hat{\chi}_t-\hat{R}_t)+\beta\bar{w}\hat{w}_t+\frac{\nu\gamma\beta^2\bar{w}}{1-\nu\gamma\beta}\tilde{E}_t^y\sum_{h=0}^{\infty}(\nu\gamma\beta)^h(\hat{\chi}_{t+1+h}-\hat{R}_{t+1+h})$$
$$+\nu\gamma\beta^2\bar{w}\tilde{E}_t^y\sum_{h=0}^{\infty}(\nu\gamma\beta)^h\hat{w}_{t+1+h}$$

$$(iv)=\frac{\beta\bar{\tau}}{1-\nu\gamma\beta}(\hat{\chi}_t-\hat{R}_t)+\beta\bar{\tau}\hat{\tau}_t+\frac{\nu\gamma\beta^2\bar{\tau}}{1-\nu\gamma\beta}\tilde{E}_t^y\sum_{h=0}^{\infty}(\nu\gamma\beta)^h(\hat{\chi}_{t+1+h}-\hat{R}_{t+1+h})$$
$$+\nu\gamma\beta^2\bar{\tau}\tilde{E}_t^y\sum_{h=0}^{\infty}(\nu\gamma\beta)^h\hat{\tau}_{t+1+h}$$

$$(v)=(1-\nu)\gamma\Bigg[\frac{\beta^2\bar{c}^y_b}{(1-\gamma\beta)(1-\nu\gamma\beta)}(\hat{\chi}_t-\hat{R}_t+\hat{c}^y_{b,t})-\frac{\beta^2\bar{s}}{1-\gamma\beta}(\hat{\chi}_t-\hat{R}_t)$$
$$-\frac{\beta^2\bar{s}}{1-\gamma\beta}\tilde{E}_t^y\sum_{h=0}^{\infty}(\nu\gamma\beta)^h(\hat{\chi}_{t+1+h}-\hat{R}_{t+1+h})$$
$$-\frac{\beta^2\bar{s}}{1-\gamma\beta}\tilde{E}_t^y\sum_{h=1}^{\infty}(\frac{1-\nu^h}{1-\nu})(\gamma\beta)^h(\hat{\chi}_{t+1+h}-\hat{R}_{t+1+h})$$
$$-\frac{\beta^2\bar{s}}{\gamma\beta}\tilde{E}_t^y\sum_{h=1}^{\infty}(\frac{1-\nu^h}{1-\nu})(\gamma\beta)^h\hat{s}_{t+1+h}\Bigg]$$

.

Next, I plug (i), (ii), (iii), (iv), and (v) into Equation (54) and rearranges it, and then, multiply both sides with $\sum_{b=0}^{\infty}N^y_{b,t}$ where $N^y_{b,t}$ is the number of young households who belong to cohort 'b'.

Then, I get

$$\bar{c}^y\hat{c}_t^y = \left[\frac{(1-\gamma\beta)(1-\nu\gamma\beta)}{(1+\theta)\beta(1-\gamma\beta)+(1-\nu)\gamma\beta^2}\right]\Bigg[\nu\bar{a}^y\hat{a}_t^y \tag{55}$$
$$+\left(\left(\frac{(1+\theta)\beta}{1-\nu\gamma\beta}+\frac{(1-\nu)\gamma\beta^2}{(1-\gamma\beta)(1-\nu\gamma\beta)}\right)\bar{c}^y-\phi\left(\frac{\beta(\bar{w}-\bar{\tau})}{1-\nu\gamma\beta}+\frac{(1-\nu)\gamma\beta^2\bar{s}}{1-\gamma\beta}\right)\right)(\hat{R}_t-\hat{\chi}_t)$$
$$+\phi\beta\bar{w}\hat{w}_t-\phi\beta\bar{\tau}\hat{\tau}_t+\phi\bar{w}\nu\gamma\beta^2\tilde{E}_t^y\sum_{h=0}^{\infty}(\nu\gamma\beta)^h\hat{w}_{t+1+h}-\phi\bar{\tau}\nu\gamma\beta^2\tilde{E}_t^y\sum_{h=0}^{\infty}(\nu\gamma\beta)^h\hat{\tau}_{t+1+h}$$
$$+\phi\left(\frac{(\bar{w}-\bar{\tau})\nu\gamma\beta^2}{1-\nu\gamma\beta}+\frac{(1-\nu)\gamma\beta^2\bar{s}}{1-\gamma\beta}\right)\tilde{E}_t^y\sum_{h=0}^{\infty}(\nu\gamma\beta)^h(\hat{\chi}_{t+1+h}-\hat{R}_{t+1+h})$$
$$+\phi(1-\nu)\frac{\gamma\beta^2\bar{s}}{1-\gamma\beta}\tilde{E}_t^y\sum_{h=1}^{\infty}\left(\frac{1-\nu^h}{1-\nu}\right)(\gamma\beta)^h(\hat{\chi}_{t+1+h}-\hat{R}_{t+1+h})$$
$$+\phi(1-\nu)\frac{\gamma\beta^2\bar{s}}{\gamma\beta}\tilde{E}_t^y\sum_{h=1}^{\infty}\left(\frac{1-\nu^h}{1-\nu}\right)(\gamma\beta)^h\hat{s}_{t+h}\Bigg]$$

and rearrange Equation (55). Finally, the consumption decision rule of young households in the LC learning model is

$$\hat{c}_t^y = \psi\nu\bar{a}^y\hat{a}_t^y+\psi\phi\beta\bar{w}\hat{w}_t-\psi\phi\beta\bar{\tau}\hat{\tau}_t \tag{56}$$
$$+\left[1-\psi\phi\left(\frac{\beta(\bar{w}-\bar{\tau})}{1-\nu\gamma\beta}+\frac{(1-\nu)\gamma\beta^2\bar{s}}{1-\gamma\beta}\right)\right](\hat{R}_t-\hat{\chi}_t)$$
$$+\psi\phi\bar{w}\nu\gamma\beta^2\tilde{E}_t^y\sum_{h=0}^{\infty}(\nu\gamma\beta)^h\hat{w}_{t+1+h}-\psi\phi\bar{\tau}\nu\gamma\beta^2\tilde{E}_t^y\sum_{h=0}^{\infty}(\nu\gamma\beta)^h\hat{\tau}_{t+1+h}$$
$$+\psi\phi\left[\frac{(\bar{w}-\bar{\tau})\nu\gamma\beta^2}{1-\nu\gamma\beta}+\frac{(1-\nu)\gamma\beta^2\bar{s}}{1-\gamma\beta}\right]\tilde{E}_t^y\sum_{h=0}^{\infty}(\nu\gamma\beta)^h(\hat{\chi}_{t+1+h}-\hat{R}_{t+1+h})$$
$$+(1-\nu)\gamma\psi\phi\frac{\beta^2\bar{s}}{1-\gamma\beta}\tilde{E}_t^y\sum_{h=1}^{\infty}\left(\frac{1-\nu^h}{1-\nu}\right)(\gamma\beta)^h(\hat{\chi}_{t+1+h}-\hat{R}_{t+1+h})$$
$$+(1-\nu)\gamma\psi\phi\frac{\beta^2\bar{s}}{\gamma\beta}\tilde{E}_t^y\sum_{h=1}^{\infty}\left(\frac{1-\nu^h}{1-\nu}\right)(\gamma\beta)^h\hat{s}_{t+h}$$

where

$$\psi = \frac{(1-\gamma\beta)(1-\nu\gamma\beta)}{\bar{c}^y[(1+\theta)\beta(1-\gamma\beta)+(1-\nu)\gamma\beta^2]}$$

.

B. Solutions to Life-Cycle Learning Model

This section briefly shows the solutions to the LC learning model.[26] As in Equation (52) and (56), we need to calculate the expected present value of returns to capital and labor to have the solutions for the consumption of young and old households. Variables other than the market prices in the consumption decision rules are predetermined or perfectly foresighted.

From the household forecasting model, Equation (10) to (13), I calculate the expected evolution of the aggregate capital and wealth distribution between young and old households as follows

$$x_t' = \begin{pmatrix} 1 \\ \hat{k}_t \\ \lambda_t \end{pmatrix}, \; \tilde{B} = \begin{pmatrix} 1 & 0 & 0 \\ \mu_{k,t-1} & \mu_{kk,t-1} & \mu_{k\lambda,t-1} \\ \mu_{\lambda,t-1} & \mu_{\lambda k,t-1} & \mu_{\lambda\lambda,t-1} \end{pmatrix},$$

$$x_{t+1} = \tilde{B} x_t \tag{57}$$

and iterating Equation (57) gives

$$x_{t+h} = \tilde{B}^h x_t$$

.

Then, the expected present value of the market prices by young households are

$$\begin{aligned} \tilde{E}_t^y \sum_{h=0}^{\infty} (\nu\gamma\beta)^h \hat{w}_{t+1+h} &= \frac{1}{\nu\gamma\beta} \tilde{E}_t^y \sum_{h=1}^{\infty} (\nu\gamma\beta)^h \hat{w}_{t+h} \\ &= \frac{1}{\nu\gamma\beta} \tilde{E}_t^y \sum_{h=1}^{\infty} (\nu\gamma\beta)^h (\mu_{w,t-1} \; \mu_{wk,t-1} \; \mu_{w\lambda,t-1}) \tilde{B}^h x_t \\ &= \frac{1}{\nu\gamma\beta} (\mu_{w,t-1} \; \mu_{wk,t-1} \; \mu_{w\lambda,t-1}) (\nu\gamma\beta) \tilde{B} (I - \nu\gamma\beta\tilde{B})^{-1} x_t \\ &= (\mu_{w,t-1} \; \mu_{wk,t-1} \; \mu_{w\lambda,t-1}) \tilde{B} (I - \nu\gamma\beta\tilde{B})^{-1} x_t \end{aligned} \tag{58}$$

and

$$\tilde{E}_t^y \sum_{h=0}^{\infty} (\nu\gamma\beta)^h \hat{R}_{t+1+h} = (\mu_{r,t-1} \; \mu_{rk,t-1} \; \mu_{r\lambda,t-1}) \tilde{B} (I - \nu\gamma\beta\tilde{B})^{-1} x_t \tag{59}$$

.

Finally, I can have the solution to young households' consumption, and the solution to old households' consumption also can be obtained in the same way above.

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