The Bulletin of Symbolic Logic
Volume 27, Number 4, December 2021

IN MEMORIAM: J. MICHAEL DUNN, 1941–2021

The history of *relevance logic* cannot be written without mentioning J. Michael Dunn who played a prominent role in shaping this area of logic. In the late twentieth century, he was a doyen with a world-class reputation in the field of philosophical logic. Dunn's research also encompassed logics outside of relevance logic, from 2-valued first-order logic to quantum logic and substructural logics such as the Lambek calculi, intuitionistic logic, linear logic, etc. Each of the disciplines of philosophy, mathematics, and computer science has been impacted in intrinsic ways by some of the theorems proved by Dunn.

Jon Michael Dunn was born in Fort Wayne, Indiana, on June 19, 1941, and he passed away on April 5, 2021, in Bloomington, Indiana. After attending high schools in Fort Wayne and Lafayette, he obtained an AB degree in philosophy from Oberlin College, Ohio, in 1963. Dunn completed his Ph.D. Thesis entitled *The Algebra of Intensional Logics* at Pittsburgh University in 1966; his thesis supervisor was Nuel D. Belnap. In 1969, Dunn was appointed an associate professor in the Department of Philosophy at Indiana University in Bloomington, Indiana, and he stayed on the faculty at IU until 2007, when he retired as University Dean of the School of Informatics, Oscar R. Ewing Professor of Philosophy, Professor of Informatics, Professor of Computer Science, and Core Faculty in Cognitive Science. Dunn supervised 14 Ph.D. students in logic and 3 Ph.D. students in other areas of philosophy. He taught advanced graduate courses in logic, including courses on 2-valued logic (metalogic), modal logic, algebraic logics, and substructural logics. Dunn was the founding dean of the School of Informatics, and he served in other administrative positions such as Chair of the Department of Philosophy and Associate Dean of the College of Arts and Sciences in previous years. Dunn held multiple research grants and visiting positions at universities in the US, Europe, and Australia. Dunn, for his contributions to Indiana University, was honored by the *IU Provost Medallion* in 2007. The state of Indiana bestowed on Dunn the rank and title of *Sagamore of the Wabash* the same year. Dunn was elected a *Fellow of the American Academy of Arts and Sciences* in 2010.

The logic of relevant implication **R** combines Church's "weak implication" with lattice connectives and De Morgan negation. Alternatively, **R** results from Ackermann's system $\mathbf{\Pi'}$ by omitting a rule (the so-called γ rule) and adding permutation. Dunn started to investigate **R** from an algebraic point of view in his Ph.D. thesis [14]. This research continued the study of distributive lattices with De Morgan negation already underway in [1, 4, 5, 35, 43]. Dunn showed that $\boldsymbol{4}$, the four-element lattice with two incomparable elements on which negation has fixed points, plays a fundamental role among De Morgan lattices, and hence, for first-degree entailments **fde**; (the implication-free fragment of **R** and of the logic of entailment **E**). Dunn proved—using methods similar to those Stone utilized in his representation of Boolean algebras—that every De Morgan lattice is embeddable into a product of $\boldsymbol{4}$, that is, into $\prod_{i<\kappa} \boldsymbol{4}_i$, where $\kappa \leq \lambda^2$ and λ is the cardinality of the De Morgan lattice. Whenever possible, Dunn generalized theorems to complete lattices, complete homomorphisms, complete embeddings, and similar notions, which, strictly speaking, go beyond the purely algebraic approach. He also defined a new interpretation for **fde** that relies on pairs of sets of situations. Later, Dunn defined the four values (true, false, both, and neither) that emerge in the interpretation of **fde** as subsets of $\{T, F\}$, and Belnap provided motivations for these values by appeal to databases. Nowadays, this logic is often referred to as Dunn–Belnap (or Belnap–Dunn) logic (cf. [48]). The algebraization of **R** revolves around two concepts: *residuation* and the *intensional truth constant* (denoted by $\boldsymbol{t}$). Relevant

1079-8986/21/2704-0011
DOI:10.1017/bsl.2021.65

implication ($\to$) is not a residual of conjunction ($\wedge$)—unlike intuitionistic implication. Dunn introduced an intensional conjunction connective ($\circ$) and treated $\to$ as its residual. He defined De Morgan semi-groups and showed that the Lindenbaum algebra of **R** is not free in the class of De Morgan semi-groups. However, once $\boldsymbol{t}$ is added to **R** (conservatively), the Lindenbaum algebra of $\mathbf{R}^{\boldsymbol{t}}$ is free in the class of *De Morgan monoids*. This algebraization of propositional **R** parallels the algebraization of 2-valued propositional logic as a Boolean algebra.

Dunn and Belnap proved in [36] that 2-*valued first-order logic* (FOL) is incomplete with the substitutional interpretation of quantifiers. The latter approach to quantification is popular in some introductory logic courses, presumably, because it bypasses the function that assigns elements of the domain to variables. A simple form of this incompleteness is practically obvious to anybody who is familiar with standard definitions of the language and the interpretation of FOL; however, [36] proved incompleteness in a stronger sense using Gödel's incompleteness theorem. Dunn and Belnap also pointed out a notion of logical consequence that is not subject to this sort of incompleteness. To wit, logical consequence that is invariant under extensions of the language by new name constants circumvents the crucial step in the incompleteness argument; this notion is pivotal in Henkin-style completeness proofs for first-order logics.

Algebraic investigations of relevance logics by Dunn led to a proof of the *admissibility of* γ in joint work with Robert K. Meyer. The γ rule (as a rule of inference) was omitted from Ackermann's $\mathbf{\Pi'}$ calculus giving the logic **E**. [47] showed that if both A and $\sim A \vee B$ are theorems of **E**, then so is B; similarly, for **R** and **T** (the logic of ticket entailment). This resolved one of the most intriguing open problems concerning **E** (cf. [30] and [47]). $\sim A \vee B$ may be written as $A \supset B$ to resemble material implication, and then the admissibility of γ means that detachment for $\supset$ is OK on theorems. Another way to view the γ rule is as "disjunctive syllogism," which has a fascinating history (cf. [2, §25], [3, §82]). Urquhart [51] provides an account of how Dunn and Meyer came up with their proof.

Dunn proved that γ is admissible for **R**-mingle (**RM**), which adds the *mingle axiom* (M) $A \to (A \to A)$ to **R**. (Incidentally, **RM** was created by Dunn to parallel **EM**, which was suggested as a potentially interesting logic by Storrs McCall; see [2, p. 94], [13].) The addition of the axiom $A \to (B \to A)$ to **R** reduces it to 2-valued logic; however, **RM** is neither of those logics. Infinite Sugihara matrices, which are linearly ordered, are characteristic for **RM**, but for instance, $\vdash_{\mathbf{RM}} (A \to B) \vee (B \to A)$ and $\vdash_{\mathbf{RM}} (A \wedge \sim A) \to (B \vee \sim B)$ are not theorems of **R**. In [15], Dunn proved that **RM** is pretabular, and its extensions that have a finite characteristic Sugihara matrix without 0 admit γ. A similar pretabularity result was proved for **LC** (Dummett's logic) in [38] by Dunn and Meyer.

R-mingle turned out to be a very fruitful logic—despite its "not-quite-relevant" nature. The first relational (or Kripke-style) semantics for an intensional logic—beyond modal and intuitionistic logics—was defined for this logic. Dunn in [18, 19] gave semantics for propositional **R**-mingle and first-order **R**-mingle using model structures with a *binary accessibility relation*. The linear order of Sugihara matrices might hint at a reason why such a semantics works; however, another novelty in the semantics is equally important. Namely, Dunn defined a 3-*valued semantics*, which is unlike the common 2-valued semantics for modal logics and for intuitionistic logic (cf. [44–45]). The third value is the result of a formula getting *both* "usual" truth values (i.e., T and F); that is, the motivation for the third value here is different than in $Ł_3$ for $\frac{1}{2}$, or in Bochvar's and Kleene's 3-valued logics where truth-value gaps are introduced. Truth-value gluts have appeared, later on, in other logics, for instance, in **LP** (the logic of paradox).

The three-variable fragment of **R**-mingle ($\mathbf{RM}_3$) has a 3-valued linearly ordered characteristic matrix, which can be viewed as $\{-1, 0, +1\}$ with the usual $\leq$. 0 is its own negation; thus, the filter [0) contains an "inconsistent element." Dunn in [21] used this 3-valued logic to show that there is a three-element model of Peano arithmetic (**PA**) that does not make formulas that are not theorems of **PA** true (but it makes all theorems of **PA** at least true). A similar result follows for type theory. These theories illustrate Dunn's general theorem about 3-*valued structures*; the latter emerge, for instance, when the homomorphic image of a

https://doi.org/10.1017/bsl.2021.65 Published online by Cambridge University Press

structure results by a(n operational) homomorphism that is not a relational homomorphism. Arithmetic and type theory may be formulated in the language of a relevance logic such as **R**. If those theories contain the theorems of their 2-valued counterparts and admit the rule γ, then an elementary proof of the absolute consistency of the 2-valued theories results. The difficult steps in carrying out this plan are (i) finding a suitable formulation of the relevant version of a theory and (ii) showing that γ is admissible. An example provided by Dunn in [20] pertains to Robinson's arithmetic **Q**. $\mathbf{Q_R}$ (with 0 included) collapses into **Q**. Roughly speaking, the culprit is 0, or more precisely, $\lambda x.\, x \times 0$ being a constant function that (of course) yields 0 for any argument. Multiplication by 0 is not unlike the combinator K with reduction axiom $\mathsf{K}xy \rhd x$, which "disregards" y, or in other terms, $\mathsf{K}x$ is a constant function. Dunn proved that *relevant Robinson's arithmetic* of positive integers is not $\mathbf{Q}(1)$ (2-valued **Q** without 0). Then in [23], Dunn formulated **OM#**, arithmetic as an extension of orthomodular logic—owing to the craze in philosophy about the "One True Logic" (orthomodular logic) at the time. He demonstrated that **OM#**, *orthomodular arithmetic* proves the distributivity of $\wedge$ and $\vee$. Furthermore, orthomodular logic with a minimal amount of extensionality collapses into 2-valued logic. Dunn continued to scrutinize the interactions between extensionality and components of a logic such as the constants $\boldsymbol{T}$ and $\boldsymbol{F}$ as well as theorems such as truth table generalizations of excluded middle in [26]. He proved that such ingredients—independently of other features of a logic—suffice to reduce a higher-order logic to 2-valued higher-order logic. Dunn investigated applications of *quantified relevance logics with identity* to philosophical problems in a series of papers [25, 27–29, 34]. Often, distinctions are made between various kinds of properties such as intrinsic and essential properties, and there is no agreement in the philosophical literature as to existence being or not being a predicate. Relevance logics provide a more expressive framework than modal logics do (let alone FOL does) to analyze such problems.

Dunn also worked on the *proof-theoretic aspects* of relevance logics. He introduced LR^+, a *sequent calculus for* $\mathbf{R}_+$, the positive fragment of the logic **R** in [16]. Relevance logics avoid proving $A \to (B \to A)$ (the arrow version of the positive paradox) as a theorem, which means that the left weakening rule cannot be adopted as a rule in a sequent calculus. An innovation in LR^+ is that formulas can form two kinds of sequences that are indicated, respectively, by , and ; . Comma corresponds to $\wedge$, whereas semi-colon turns into $\circ$ on the left-hand side of the $\vdash$, and weakening is a rule only for sequences built with comma. Another novelty in LR^+ is the presence of $\boldsymbol{t}$, which is essential in applications of the cut rule where the left premise is a theorem (i.e., the $\vdash$ has nothing on the left). The insertion of $\boldsymbol{t}$ instead of the invisible empty sequence precludes the conflation of the two kinds of sequences of formulas (and thereby, it prevents bogus proofs of non-theorems to count as proofs). Dunn in [17] also showed that the most natural version of *analytic tableaux* in the style of Jeffrey formalizes **fde** (rather than 2-valued logic). Further, [22] gave a straightforward step-by-step method to find out whether a 2-valued theorem of the form $A \supset B$ (with no $\supset$ in A or B) is an **fde** theorem $A \to B$ (with $\neg$ rewritten as $\sim$).

The *cut rule*, in some form, is desirable in a sequent calculus, because it facilitates a proof of the replacement theorem as well as a proof of the equivalence of the sequent calculus formulation of the logic with its axiomatization. The cut rule, on the other hand, violates the subformula property; hence, its presence is sometimes undesirable. The admissibility of the cut rule, which delivers the benefits without the drawbacks, can be proved syntactically, but also semantically. Dunn and Meyer in [39] used insights from proofs of the admissibility of γ to prove the *admissibility of the cut* for K_1, Schütte's formulation of FOL. This was the first conceptually new proof of the cut theorem for FOL since the 1950s.

Curry noted a similarity between certain combinators and structural rules in the sequent calculi LK and LJ. The positive or the implicational fragments of the relevance logic **B** can supply the context to make this analogy precise. Dunn and Meyer in [40] introduced sequent calculi in which structural rules are fully supplanted by *combinatory rules*, hence the name "structurally free logics" for this group of logics. Indirectly, the ideas in structurally free logics together with the connections between implicational fragments of relevance logics and the simple principal type schemas of (proper) combinators led to a solution of an open problem by Dunn and Bimbó around 2010. [9] introduced a new sequent calculus for $\mathbf{R}_{\to}$ (denoted by $LT^{\textcircled{t}}_{\to}$), which extends a sequent calculus for $\mathbf{T}^{\boldsymbol{t}}_{\to}$ (from [6]). Then [10] showed how three sequent calculi, $LR^{\boldsymbol{t}}_{\to}$, $LT^{\textcircled{t}}_{\to}$, and $LT^{\boldsymbol{t}}_{\to}$, plus the decidability of $\mathbf{R}^{\boldsymbol{t}}_{\to}$, can be combined to prove the *decidability of* $\mathbf{T}_{\to}$, that is, the decidability of implicational ticket entailment.

The proof-search tree approach to decidability, which is based on sequent calculi and originated with Curry, was adapted to relevance logics (to $\mathbf{E}_{\to}$ and $\mathbf{R}_{\to}$) by Saul A. Kripke. Meyer showed that LR (lattice-**R**) and $LR^{\Box}$ (lattice-**R** with necessity) are decidable. LR differs from **R** by omitting the distributivity of conjunction and disjunction, and the $\Box$ in $LR^{\Box}$ is like the "exponential" ! , which makes $LR^{\Box}$ similar to LL (linear logic). [11] (cf. [12]) showed that LL (which has been believed to be undecidable for decades) is *decidable*, like a handful of other closely related logics.

Dunn claimed in his autobiography [7, p. xxix] that he had no grand research program. However, he invented a *grand framework*—gaggle theory—to deal with the semantics of substructural logics. The set-theoretical semantics that Dunn defined for **RM** did not seem to generalize to **R** or **E**. After the introduction of the Meyer–Routley semantics [49], together with the operational semantics of Urquhart [50] and the operational–relational semantics of Fine [42], Dunn started to develop a theory of set-theoretical semantics that would encompass a range of semantics somewhat similarly as the Jónsson–Tarski representation provides semantics for a range of logics the Lindenbaum algebra of which has a Boolean algebra reduct. Dunn gave the first "gaggle talk" in 1979, and presented the whole framework in a series of papers in the 1990s [30–33], and further, in two books in the 2000s [8, 37]. *Gaggles* (of various kinds) are algebras, many of which are reducts of the Lindenbaum algebra of intensional logics such as **R** or **T**. ("Gaggle" also serves as the pronunciation of the acronym "gGl" that stands for *generalized Galois logics*.) This uniform approach to the definition of the semantics of substructural logics yields a semantics once (abstractly) *residuated operations* are grouped together, and their *distribution types* (if there are lattice connectives in the logic) or their *tonicity types* are discerned. An interpretation for a logic emerges in two steps: first, the Lindenbaum algebra of the logic is seen as a gaggle (or more likely, several gaggles combined), and second, the set-theoretic representation of the algebra yields the semantics. Hopefully, this description outlines the big picture; however, the details are many and subtle. Kripke's semantics and the Meyer–Routley semantics dealt with non-classical logics of a certain kind, while Dunn's gaggle theory is applicable not only to those logics but to logics without $\wedge$ and $\vee$ distributing over each other, and even without one or both of the latter connectives. Furthermore, issues of canonicity, of correspondence between axioms and frame conditions, of topological characterizations of the image of the Lindenbaum algebra and of interactions between groups of operations are only some of the questions that have been investigated as part of generalized Galois logics. Gaggle theory combines an algebraic approach to logics—that was always preeminently present in Dunn's work—with the goal of producing informally palatable interpretations for logics—that is in harmony with Dunn's view of logics as tools for rational beings.

This brief overview has been unavoidably selective. Dunn published (sometimes with co-authors) on other topics too, which include information, its properties, and its role in logic and computing; pieces of history of relevance logics and informal interpretations of those logics; the logic of quantum computers; relational algebras; etc. The *complete list of publications* of Dunn (up to about 2015) is included in [7], and hopefully the reader is intrigued by now to find out more about Dunn's work and results in logic.

J. Michael Dunn was a highly respected logician whose legacy will thrive through his books and papers, through the knowledge of his students, as well as through his efforts to establish the Logic Program and to found the School of Informatics at Indiana University in Bloomington.

REFERENCES

[1] A. R. Anderson and N. D. Belnap, *First degree entailments*. ***Mathematische Annalen***, vol. 149 (1963), no. 4, pp. 302–319.

[2] ———, ***Entailment: The Logic of Relevance and Necessity***, ***vol.*** I, Princeton University Press, Princeton, 1975.

[3] A. R. Anderson, N. D. Belnap, and J. Michael Dunn, ***Entailment: The Logic of Relevance and Necessity***, ***vol.*** II, Princeton University Press, Princeton, 1992.

[4] N. D. Belnap and J. H. Spencer, *Intensionally complemented distributive lattices*. ***Portugalie Mathematica***, vol. 25 (1966), no. 2, pp. 99–104.

[5] A. Białynicki-Birula and H. Rasiowa, *On the representation of quasi-Boolean algebras*. ***Bulletin de l'Académie Polonaise des Sciences***, vol. 5 (1957), pp. 259–261.

[6] K. Bimbó, *Relevance logics*, ***Philosophy of Logic*** (D. Jacquette, editor), Handbook of the Philosophy of Science, vol. 5, Elsevier/North-Holland, Amsterdam, 2007, pp. 723–789.

[7] K. Bimbó (ed.), ***J. Michael Dunn on Information Based Logics***, Outstanding Contributions to Logic, vol. 8, Springer, Cham, 2016.

[8] K. Bimbó and J. Michael Dunn, ***Generalized Galois Logics: Relational Semantics of Nonclassical Logical Calculi***, CSLI Lecture Notes, vol. 188, CSLI Publications, Stanford, 2008.

[9] ———, *New consecution calculi for* $R^t_\to$. ***Notre Dame Journal of Formal Logic***, vol. 53 (2012), no. 4, pp. 491–509.

[10] ———, *On the decidability of implicational ticket entailment*. ***Journal of Symbolic Logic***, vol. 78 (2013), no. 1, 214–236.

[11] ———, *Modalities in lattice-R*, submitted, 2015, 39 pp.

[12] ———, *On the decidability of classical linear logic (abstract)*, this Journal, vol. 21 (2015), no. 3, p. 358.

[13] ———, *Entailment, mingle and binary accessibility*, ***Saul A. Kripke on Modal Logic*** (Y. Weiss and R. Padro, editors), Outstanding Contributions to Logic, Springer, Cham, forthcoming, 29 pp.

[14] J. M. Dunn, ***The Algebra of Intensional Logics***, Ph.D. thesis, University of Pittsburgh, Ann Arbor (UMI), 1966 (Published as Vol. 2 in the Logic PhDs series by College Publications, London, 2019).

[15] ———, *Algebraic completeness results for R-mingle and its extensions*. ***Journal of Symbolic Logic***, vol. 35 (1970), no. 1, pp. 1–13.

[16] ———, *A 'Gentzen system' for positive relevant implication (abstract)*. ***Journal of Symbolic Logic***, vol. 38 (1973), no. 2, pp. 356–357.

[17] ———, *Intuitive semantics for first-degree entailments and 'coupled trees'*. ***Philosophical Studies***, vol. 29 (1976), no. 3, pp. 149–168.

[18] ———, *A Kripke-style semantics for R-mingle using a binary accessibility relation*. ***Studia Logica***, vol. 35 (1976), no. 2, pp. 163–172.

[19] ———, *Quantification and RM*. ***Studia Logica***, vol. 35 (1976), no. 3, pp. 315–322.

[20] ———, *Relevant Robinson's arithmetic*. ***Studia Logica***, vol. 38 (1979), no. 4, pp. 407–418.

[21] ———, *A theorem in 3-valued model theory with connections to number theory, type theory, and relevant logic*. ***Studia Logica***, vol. 38 (1979), no. 2, pp. 149–169.

[22] ———, *A sieve for entailments*. ***Journal of Philosophical Logic***, vol. 9 (1980), no. 1, pp. 41–57.

[23] ———, *Quantum mathematics*, ***PSA 1980: Proceedings of the 1980 Biennial Meeting of the Philosophy of Science Association***, ***vol.*** 2 (P. D. Asquith and R. N. Giere, editors), Philosophy of Science Association, East Lansing, 1981, pp. 521–531.

[24] ———, *Relevance logic and entailment*, ***Handbook of Philosophical Logic***, ***vol.*** 3, first ed. (D. Gabbay and F. Guenthner, editors), D. Reidel, Dordrecht, 1986, pp. 117–224.

[25] ———, *Relevant predication* 1: *The formal theory*. ***Journal of Philosophical Logic***, vol. 16 (1987), no. 4, pp. 347–381.

[26] ———, *The impossibility of certain higher-order non-classical logics with extensionality*, ***Philosophical Analysis: A Defense by Example*** (D. F. Austin, editor), Philosophical Studies Series, vol. 39, Kluwer, Dordrecht, 1988, pp. 261–281.

[27] ———, *The frame problem and relevant predication*, ***Knowledge Representation and Defeasible Reasoning*** (H. E. Kyburg, R. P. Loui, and G. N. Carlson, editors), Studies in Cognitive Systems, vol. 5, Kluwer, Dordrecht, 1990, pp. 89–95.

[28] ———, *Relevant predication* 2: *Intrinsic properties and internal relations*. ***Philosophical Studies***, vol. 60 (1990), no. 3, pp. 177–206.

[29] ———, *Relevant predication* 3: *Essential properties*, ***Truth or Consequences: Essays in Honor of Nuel Belnap*** (J. M. Dunn and A. Gupta, editors), Kluwer, Amsterdam, 1990, pp. 77–95.

[30] ———, *Gaggle theory*: *An abstraction of Galois connections and residuation, with applications to negation, implication, and various logical operators*, ***Logics in AI: European Workshop JELIA'90*** (J. van Eijck, editor), Lecture Notes in Computer Science, vol. 478, Springer, Berlin, 1991, pp. 31–51.

[31] ———, *Partial gaggles applied to logics with restricted structural rules*, ***Substructural Logics*** (K. Došen and P. Schroeder-Heister, editors), Studies in Logic and Computation, vol. 2, Clarendon, Oxford, 1993, pp. 63–108.

[32] ———, *Gaggle theory applied to intuitionistic, modal and relevance logics*, ***Logik und Mathematik. Frege-Kolloquium Jena 1993*** (I. Max and W. Stelzner, editors), W. de Gruyter, Berlin, 1995, pp. 335–368.

[33] ———, *Generalized ortho negation*, ***Negation: A Notion in Focus*** (H. Wansing, editor), W. de Gruyter, New York, 1996, pp. 3–26.

[34] ———, *Is existence a* (*relevant*) *predicate*? ***Philosophical Topics***, vol. 24 (1996), no. 1, pp. 1–34.

[35] J. M. Dunn and N. D. Belnap Jr., *Homomorphisms of intensionally complemented distributive lattices*. ***Mathematische Annalen***, vol. 176 (1968), no. 1, pp. 28–38.

[36] ———, *The substitution interpretation of the quantifiers*. ***Noûs***, vol. 2 (1968), no. 2, pp. 177–185.

[37] J. M. Dunn and G. M. Hardegree, ***Algebraic Methods in Philosophical Logic***, Oxford Logic Guides, vol. 41, Oxford University Press, Oxford, 2001.

[38] J. M. Dunn and R. K. Meyer, *Algebraic completeness results for Dummett's LC and its extensions*. ***Zeitschrift für Mathematische Logik und Grundlagen der Mathematik***, vol. 17 (1971), no. 1, pp. 225–230.

[39] ———, *Gentzen's cut and Ackermann's gamma*, ***Directions in Relevant Logic*** (J. Norman and R. Sylvan, editors), Kluwer, Dordrecht, 1989, pp. 229–240.

[40] ———, *Combinators and structurally free logic*. ***Logic Journal of the IGPL***, vol. 5 (1997), no. 4, pp. 505–537.

[41] J. M. Dunn and G. Restall, *Relevance logic*, ***Handbook of Philosophical Logic, vol.*** 6, second ed. (D. Gabbay and F. Guenthner, editors), Kluwer, Amsterdam, 2002, pp. 1–128.

[42] K. Fine, *Models for entailment*. ***Journal of Philosophical Logic***, vol. 3 (1974), no. 4, pp. 347–372.

[43] J. A. Kalman, *Lattices with involution*. ***Transactions of the American Mathematical Society***, vol. 87 (1958), no. 2, pp. 485–491.

[44] S. A. Kripke, *A completeness theorem in modal logic*. ***Journal of Symbolic Logic***, vol. 24 (1959), no. 1, pp. 1–14.

[45] ———, *Semantical analysis of intuitionistic logic I*, ***Formal Systems and Recursive Functions. Proceedings of the Eighth Logic Colloquium*** (J. N. Crossley and M. A. E. Dummett, editors), North-Holland, Amsterdam, 1965, pp. 92–130.

[46] ———, *Semantical analysis of modal logic II. Non-normal propositional calculi*, ***The Theory of Models*** (J. W. Addison, L. Henkin, and A. Tarski, editors), North-Holland, Amsterdam, 1965, pp. 206–220.

[47] R. K. Meyer and J. Michael Dunn, *E, R and γ*. ***Journal of Symbolic Logic***, vol. 34 (1969), no. 3, pp. 460–474 (Reprinted in A. R. Anderson and N. D. Belnap, ***Entailment: The Logic of Relevance and Necessity, vol. 1***, Princeton University Press, Princeton, 1975, pp. 300–314, Sec. 25.2).

[48] H. Omori and H. Wansing (eds.), ***New Essays on Belnap–Dunn Logic***, Synthese Library, vol. 418, Springer, Cham, 2019.

[49] R. Routley and R. K. Meyer, *The semantics of entailment*, ***Truth, Syntax and Modality. Proceedings of the Temple University Conference on Alternative Semantics*** (H. Leblanc, editor), North-Holland, Amsterdam, 1973, pp. 199–243.

[50] A. Urquhart, *Semantics for relevant logic*. ***Journal of Symbolic Logic***, vol. 37 (1972), no. 1, pp. 159–169.

[51] ———, *The story of γ*, ***J. Michael Dunn on Information Based Logic*** (K. Bimbó, editor), Outstanding Contributions to Logic, vol. 8, Springer, Cham, 2016, pp. 93–105.

Katalin Bimbó