Second Meeting, December 11th, 1885.

Dr Ferguson, F.R.S.E., President, in the Chair.

On certain Integrals.

By Professor Tait.

This paper was based mainly on the results of an investigation which will appear in full in the *Transactions* of the Royal Society of Edinburgh. Incidentally, however, it led to a discussion of the question :—*Find the law of density of a planet's atmosphere, supposing Boyle's law to be true for all pressures, and the temperature to be uniform throughout.*

Boyle's law gives $p = k\rho$, where ρ is the density at distance r from the planet's centre.

The Hydrostatic condition is $\frac{dp}{dr} = -\rho\mathrm{R}$, where R is the attraction on unit of mass.

Hence $k\frac{d\rho}{dr} = -\rho\frac{\mathrm{M} + \int_{r_0}^{r} 4\pi r^2 \rho\, dr}{r^2}$, where r_0 is the radius, and M the mass of the planet.

Write this as

$$\frac{kr^2}{\rho}\frac{d\rho}{dr} = -\mathrm{M} - \int_{r_0}^{r} 4\pi r^2 \rho\, dr$$

and differentiate ; and we obtain the curious equation

$$\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{d\rho}{dr}\right) = -\frac{4\pi}{k}r^2\rho \quad \ldots \quad \ldots \quad (1)$$

A *special* value of ρ (corresponding to the absence of a nucleus) is

$$\rho = +\frac{k}{2\pi r^2},$$

but this cannot be generalised.

The finding of the integral of (1) in a form convergent for all values of r greater than r_0 presents novel and grave difficulties ; but it is clear from the physical question on which the whole is based that such a solution exists.

If we change the independent variable to s, where $rs=1$, (1) becomes

$$\frac{d^2\log\rho}{ds^2} = -\frac{4\pi}{k}\frac{\rho}{s^4};$$

or, if $\log\rho = u$, $\frac{4\pi}{k} = e$,

$$\frac{d^2u}{ds^2} = -\frac{e}{s^4}\epsilon^u.$$

This seems to be the simplest form into which the equation can be transformed.

To transform a rectangle into a square.

By W. Peddie.

Let l be the length of the rectangle and b its breadth, while s is the side of the square equal to it in area; s is found, of course, by taking $l : s = s : b$. A number of different cases arise.

I. $2s < l + b$. Fig. 18 represents the method of cutting the rectangle in this case. AC is the rectangle and AF the equal square. Make HM = BC, QF = MC, NB = QM. The proof is evident.

II. $2s = l + b$. In this case EB = QF, BC = DQ, ∴ EC is similar and equal to GQ.

III. $2s > l + b$. Three cases arise.

- α. $2s > l$. This includes method I., and the method shown in Fig. 19.
- β. $2s = l$. Then AQ = EC.
- γ. $2s < l$.

Let $l = (k+\theta)s$, where k is an integer and θ a proper fraction. If we cut off portions of the rectangle of length s, the above method applies directly if $2\theta s \nless s$ *i.e.*, if $\theta \nless \frac{1}{2}$. Again if $\theta < \frac{1}{2} = \frac{1}{p}$ where p is an integer, the solution is easy. If p is an improper fraction consistent with the condition $\theta < \frac{1}{2}$, the simplest method is to cut up the rectangle into an equal number of parts, so as to form a rectangle of suitable breadth. In some of these cases the transformation is produced by simple sliding of the various parts parallel to themselves. In others the parts have to be rotated through a right angle.

Another solution of the problem is given in Dr Charles Hutton's *Recreations in Mathematics and Natural Philosophy*.