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Correspondence

Steady-state water pressures in subglacial conduits: corrections to a model and recommendations for its use

INTRODUCTION

Recent efforts have been made to increase our understanding of the dynamics of ice-sheet hydrology. Notably, much work has focused on the southwest sector of the Greenland ice sheet (GrIS), with intense data collection on diurnal to interannual timescales (e.g. Bartholomew and others, 2012; Cowton and others, 2013; Doyle and others, 2013). Observations show a close correlation between surface meltwater production and the seasonal ice-sheet acceleration, and it is a well-accepted hypothesis that an increase in the former drives the latter via meltwater transfer through the subglacial drainage system (e.g. Zwally and others, 2002). However, due to the remote nature and complexity of the subglacial domain, a satisfactory description at the process level has remained elusive. Better understanding of the coupling of meltwater forcing on ice velocity through the subglacial component is therefore necessary to improve the physical integrity of ice-sheet models.

Meierbachtol and others (2013) retrieved subglacial water-pressure data from multiple boreholes over a time span of three summers. Using typical seasonal discharge series and a simple model for straight subglacial conduits, they model both the steady state and time-transient evolution of conduits and the resulting water pressures therein. They find, for both the steady-state and transient scenarios, that in the near-margin area (<30 km) it is possible to reconcile the data series with the modeled water pressures. That is, the modeled water pressures are lower than the measured, implying that the data probably are from an inefficient part of the subglacial system that is at high water pressures. However, further up-glacier (>34 km), their modeling suggests that conduit pressures are higher than the observed data. This in turn is interpreted as inconsistent, since a conduit in the form of a Röthlisberger (R-) channel should be at a relatively low pressure compared to the surrounding inefficient high-pressure system. Based on the results of the transient modeling, Meierbachtol and others (2013) conclude that, while velocity observations (Hoffman and others, 2011) and dye tracing (Chandler and others, 2013) suggest growth of an efficient drainage system in the interior reaches, it is unlikely that such a development is due to widespread growth of low-pressure, melt-dominated conduits. This leads to an important implication: in the majority of the area of the GrIS where we expect subglacial hydrology to play a significant role in driving ice-sheet acceleration, the conceptual step from alpine-type subglacial hydrology to ice-sheet hydrology may only be valid close to the margin.

Meierbachtol and others (2013) successfully show the implications of ice-sheet configuration on subglacial hydrology. Unfortunately their steady-state calculations include a conceptual error. By rectifying the error, we show that while the equation used in Meierbachtol and others (2013) to calculate the steady-state water pressure was oversimplified, the conclusions based on the observations and transient modeling remain unaffected.

Meierbachtol and others (2013) can be of interest to other research areas, such as paleo-ice-sheet reconstructions, so the transfer of their results to other scenarios is probable. Due to the intricacies of glacial hydrological theory, the limitations of the steady-state case could easily be overlooked. We therefore think, for the sake of completeness, there is a need to provide results from the correct steady-state formulation while at the same time de-emphasizing its importance compared to the transient case.

STEADY-STATE WATER PRESSURES

Following Clarke (1996), Meierbachtol and others (2013) present a theoretical formulation relating conduit cross section to water pressure and discharge (Eqns (1)). Note that the following equations use the notation from Meierbachtol and others (2013, supplementary material), but that we have chosen to define $c_3 := \pi^{1/2}\rho_{\mathrm{w}} f/(2^{1/2}(\pi+2))$ differently than in the supplementary material, where the correct value was used in calculations but there is a misprint in the text. The relation in Eqns (1) is, in principle, equivalent to the formulation presented in Röthlisberger (1972), with some additional assumptions such as that the form of a subglacial conduit is semicircular and that conduit wall melt is negligible, giving a constant discharge, Q, along the conduit. The evolution of the system is described as

$$\frac{\partial S}{\partial t} = c_1 \Psi Q - c_2 N^n S, \tag{1a}$$

$$\Psi = c_3 Q^2 S^{-5/2}, \tag{1b}$$

where S, $N = p_{\mathrm{i}} - p_{\mathrm{w}}$, t and $\Psi = \frac{\partial p_{\mathrm{w}}}{\partial x} + \rho_{\mathrm{w}} g \frac{\partial z_{\mathrm{b}}}{\partial x}$ are the cross-sectional area of the conduit, the effective pressure (ice overburden pressure minus water pressure), time and hydraulic potential gradient over a varying bed respectively. Solving for $S = c_3^{2/5}\Psi^{-2/5}Q^{4/5}$ in Eqn (1b) and inserting into Eqn (1a) with $\frac{\partial S}{\partial t} = 0$, an expression for steady state can be found:

$$\Psi = c_4 Q^{-1/7} N^{5n/7}, \tag{2}$$

where $c_4 = (c_2/c_1)^{5/7} c_3{}^{2/7}$. While Eqn (2) is stated in its entirety in Meierbachtol and others (2013), a minor error, explained below, was made in rearranging the equation for numerical modeling purposes. Meierbachtol and others (2013) solve for the steady-state water pressure, which in the simplified case of a flat bottom (and $n = 3$) reduces Eqn (2) to

$$\frac{\mathrm{d}p_{\mathrm{w}}}{\mathrm{d}x} = c_4 Q^{-1/7} (p_{\mathrm{i}} - p_{\mathrm{w}})^{15/7}. \tag{3}$$

In Röthlisberger (1972), some of the numerical test scenarios concern the resulting water pressures in a conduit under a slab of ice. This leads to the ice overburden pressure term, p_{i}, in Eqn (3) being independent of the coordinate x. In this case, separation of variables can be used to simplify Eqn (3) to

$$(p_{\mathrm{i}} - p_{\mathrm{w}})^{-15/7} \mathrm{d}p_{\mathrm{w}} = c_4 Q^{-1/7} \mathrm{d}x, \tag{4}$$

after which we can integrate both sides to obtain

$$(p_{\mathrm{i}} - p_{\mathrm{w}})^{-8/7} = \frac{8}{7} c_4 Q^{-1/7} x + C. \tag{5}$$

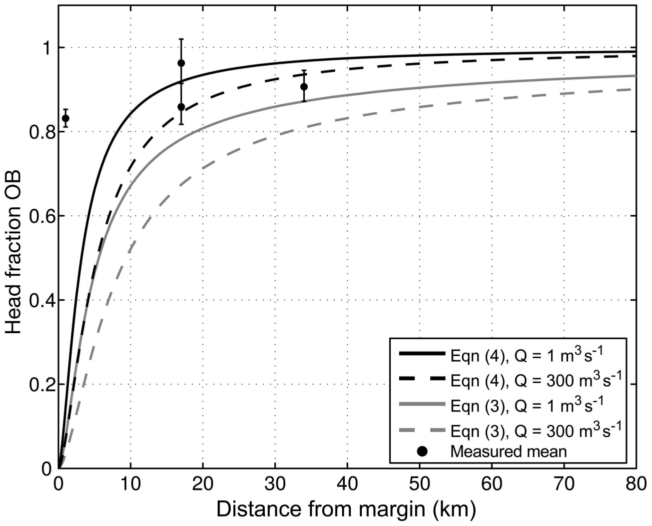

Fig. 1. Comparison of steady-state pressures as fraction of ice overburden (OB) calculated for $Q = 1\,\mathrm{m^3\,s^{-1}}$ (solid lines) and $Q = 300\ \mathrm{m^3\,s^{-1}}$ (dashed lines) for both Eqns (4) (black) and (3) (gray), with borehole pressure data shown as black dots.

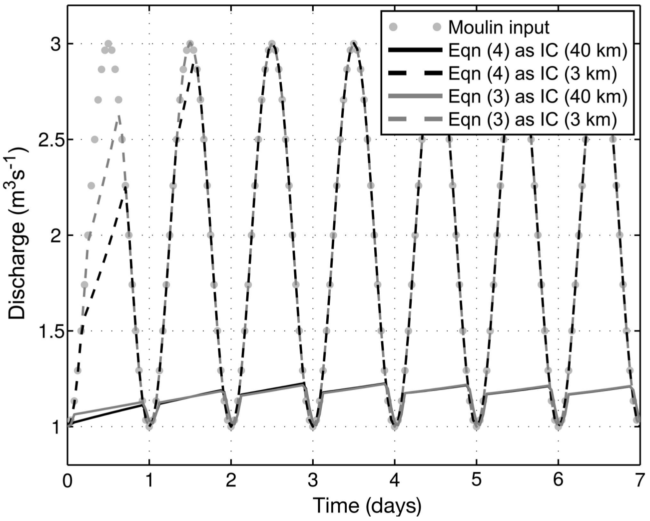

Fig. 2. Time-transient output of Eqns (1), with conduit input at moulin (gray dots), initial conditions (IC) from Eqns (4) (black) and (3) (gray). Dashed lines represent a moulin location at 3 km and solid lines at 40 km.

From this we can solve for the variable p_w and determine the constant C from the boundary conditions ($p_w = 0$ at the glacier front). Since Eqn (4) is consistent with the test case in Röthlisberger (1972), verification of the numerical output against this will produce a similar result.

In Meierbachtol and others (2013), Eqn (4) was used to model all steady-state scenarios. However, in the general case, when the ice surface depends on x, the above separation of variables is incorrect, and solving Eqn (4) will produce a different result than Eqn (3).

Figure 1 is a reproduction of figure 3A in Meierbachtol and others (2013), with the addition of the numerical solution to Eqn (3), using the same values and expression for constants and the varying ice surface as in Meierbachtol and others (2013, supplementary material). The figure shows the calculated water pressures in a channel for different discharges ($Q = 1\,\mathrm{m^3\,s^{-1}}$ and $Q = 300\,\mathrm{m^3\,s^{-1}}$), comparing the solutions of Eqn (3) (gray lines) and Eqn (4) (black lines). There is a significant difference between the two solutions, with a lower steady-state pressure for the solution of Eqn (3). Furthermore, when considering the borehole pressure data (black dots) along the length of the glacier, it becomes apparent that the observed water pressures are lower than what the (theoretical) pressure would be in a modeled steady-state subglacial conduit further from the margin (i.e. at 34 km). That is, in the steady-state case, the observations could represent water pressures from a subglacial conduit. However, Figure 1 only shows a comparison between the two solutions of Eqns (3) and (4). It is worth emphasizing, as is done in Meierbachtol and others (2013, supplementary material), that the assumptions made in the above calculations regarding conduit shape factor and (lack of) sinuosity act to lower the water pressure in the conduit, but that the system is sensitive to the choices of these factors (Ahlstrøm and others, 2005).

STEADY STATE VS TIME DEPENDENCE

The focus of the simulations in Meierbachtol and others (2013) is on the time-transient case, i.e. modeling the full set of Eqns (1), with additional assumptions. The model in this paper closely follows the model presentation in Meierbachtol and others (2013, supplementary material), in that Eqns (1) are discretized to form a set of differential algebraic equations. The conclusion made in Meierbachtol and others (2013), that a potential efficient drainage network in the ice-sheet interior does not consist of subglacial conduits (described by the model above), is supported by the transient numerical modeling and data. Even though the steady-state results are used as initial conditions to the transient cases, it turns out that in the model, basal melt rates in the subglacial conduit are limited inland. This results in a quick adjustment (due to high overburden pressure) of the transient case, making it independent, to some degree, of the different initial conditions. This means that the initial discrepancy introduced at the beginning of a time series, due to the two different steady-state solutions described above, becomes indiscernible in a matter of days. This is demonstrated in Figure 2, which is a modification of figure 3B in Meierbachtol and others (2013). In Figure 2, the two initial conditions resulting from the steady-state solutions of Eqns (3) and (4) affect the solution only over a short part of the time series.

CONCLUSIONS

Steady-state calculations of water pressures in conduits are presented as a complement to Meierbachtol and others (2013), in which these were not correctly modeled. When focusing on the evolution of conduits over time, the difference in steady-state pressure is not significant for the model, leading to the same conclusions as in Meierbachtol and others (2013). In addition, it should be noted that Eqn (2) representing conduit pressure at steady state does not in any way represent channel initiation or similar transient processes. The transient model used suggests that at inland positions small oscillations in discharge invoke pressure excursions to overburden pressure for two reasons. First, low melt energy requires long timescales for conduits to adjust to even minor perturbations in input flux. As an example, in the transient model (with the parameters used in Fig. 2) a very slow monotonic increase in discharge from 1 $m^3 s^{-1}$ to 3 $m^3 s^{-1}$ with time span in the order of 40 days is necessary for the pressure to not reach overburden at points of the ice further inland than ~50 km. Second, brief reductions in input flux may result in rapid conduit collapse in the interior setting with thick ice. Thus the steady state of this model is only a mathematically idealized indication, rather than something we would expect in nature. The sensitivity to transient processes, especially in the large-scale case of ice-sheet topography, makes the steady-state case essentially unrealistic. A scenario where this could be significant is the interpretation of conduit networks in paleo-ice-sheet settings, where the possible extent of a conduit modeled with the steady-state equation would not be restricted to the near-margin area. Therefore, caution is advisable when using the steady-state equation to represent subglacial hydrological processes, since it, at best, represents a favorable-case scenario for conduit maintenance.

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