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CONSTRUCTIONS OF BALANCED TERNARY DESIGNS

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Abstract

A construction for balanced ternary designs is given. Based on the designs so obtained, a construction of partially balanced ternary designs is given, which gives balanced ternary designs and series of symmetric balanced ternary designs in special cases.

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1. Introduction

Various authors have studied balanced n-ary designs and partially balanced n-ary designs. An excellent survey can be found in Billington [1]. We will first construct a ternary design in which equal numbers of elements occur twice in all the blocks. In other words the columns of the incidence matrix of the design contain equal numbers of entries equal to 2 (hence equal numbers of entries equal to 1 and 0). We are interested in these designs just because of their application in the main construction of the present note. Using these designs we will construct a partially balanced ternary design whose elements are the pairs of distinct elements of the original design and the blocks are formed by taking the pairs of the distinct entries of the blocks of the original design and its complement. Pairs are repeated if one of the elements has occurred twice in the original design. An example here will clarify the construction.

Construction of a balanced ternary design. Start with a finite set $\{1,2,3,4\}$. Consider all the pairs. For each pair, there are two possibilities of selecting an element. Hence for each pair we construct two blocks of size 3 as follows. If the pair is (a,b), then the blocks are $\{a,a,b\}$ and $\{a,b,b\}$ (In general we will consider all $(K-S)$-sets of a finite set, and we will talk about selecting S elements, repeating each of the S elements twice. Hence we will get $\binom{K-S}{S}$ blocks of size K for each $(K-S)$-subset of the finite set). So we get the following blocks written as columns:

$$\begin{array}{cccccccccccc} 1 & 1 & 1 & 2 & 2 & 2 & 2 & 3 & 3 & 4 & 4 & 4 \\ 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 & 3 & 4 & 4 & 4 \\ 2 & 3 & 4 & 1 & 3 & 4 & 1 & 2 & 4 & 1 & 2 & 3 \end{array}$$

The incidence matrix of the above design is

$$\begin{bmatrix} 2 & 2 & 2 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 2 & 2 & 2 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 2 & 2 & 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 2 & 2 & 2 \end{bmatrix}.$$

Observe that each column has exactly one entry equal to 2 and exactly one entry equal to 1. This is not in general true for an n-ary design. For example, see the design C^2 of Billington [1, page 52], and a ternary design of Billington [1, Example 4.4.2].

According to the notation of Billington [1] or Donovan [2], the above design is a regular ternary design with parameters

$$V=4, \quad B=12, \quad \rho_1=3, \quad \rho_2=3, \quad R=9, \quad K=3, \quad \Lambda=4.$$

Hereafter we will not write ρ_1 and ρ_2 in any design parameters.

Construction of partially balanced ternary design from the above design. First we will write the blocks of the original (above) design with their complements as follows:

$$\begin{array}{cccccccccccc} 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 & 3 & 4 & 4 & 4 \\ 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 & 3 & 4 & 4 & 4 \\ 2 & 3 & 4 & 1 & 3 & 4 & 1 & 2 & 4 & 1 & 2 & 3 \\ \hline 3 & 2 & 2 & 3 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 \\ 4 & 4 & 3 & 4 & 4 & 3 & 4 & 4 & 2 & 3 & 3 & 2 \end{array}.$$

Now form the blocks of our new design as follows: the pairs (i,j) are written as ij.

$$\begin{array}{cccccccccccc} 12 & 13 & 14 & 12 & 23 & 24 & 13 & 23 & 34 & 14 & 24 & 34 \\ 12 & 13 & 14 & 12 & 23 & 24 & 13 & 23 & 34 & 14 & 24 & 34 \\ 34 & 24 & 23 & 34 & 14 & 13 & 24 & 14 & 12 & 23 & 13 & 12 \end{array}$$

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That is, we take all the pairs (i, j) from each block and its complement to form a block of the constructed design, where $i \neq j$. Notice that if an element i occurs twice in the original block then we write the pair (i, j) twice in the corresponding block of the constructed design.

The design above has parameters

$$V = 6, \quad B = 12, \quad R = 6, \quad K = 3, \quad \Lambda_1 = 0, \quad \Lambda_2 = 8,$$

where the first associates are the pairs with one element in common and the second associates are the disjoint pairs.

Observe that the blocks are repeated twice in this particular case, and hence if we take each repeated block only once we get a design with the following parameters (see Theorem 3.5):

$$V = 6, \quad B = 6, \quad R = 3, \quad K = 3, \quad \Lambda_1 = 0, \quad \Lambda_2 = 4.$$

2. Constructions

The following theorem is a special case of Donovan [3, Theorem 6], proved independently. We include it here for the sake of completeness and its application in the present paper.

THEOREM 2.1. *Balanced ternary designs with parameters*

$$V, B = \binom{V}{K-S}\binom{K-S}{S},$$

$$R = \binom{V-1}{K-S-1}\left[2\binom{K-S-1}{S-1} + \binom{K-S-1}{S}\right],$$

$$K, \Lambda = \binom{V-2}{K-S-2}\left[4\binom{K-S-2}{S-2} + 4\binom{K-S-2}{S-1} + \binom{K-S-2}{S}\right]$$

exist, where S is any integer at most $K/2$. Each block of the design contains S elements twice and $K - 2S$ elements once.

PROOF. We first take all $(K - S)$-sets of the V-set then from each of the $(K - S)$-sets we select an S-set, whose elements occur twice in the block. Therefore the number of blocks is $B = \binom{V}{K-S}\binom{K-S}{S}$.

Checking of R. We can select an element in a $(K - S)$-set in $\binom{V-1}{K-S-1}$ ways. The element is in $\binom{K-S-1}{S-1}$ S-subsets of a $(K - S)$-set. When the element is in an S-set it counts for 2 in R. Therefore

$$R = \binom{V-1}{K-S-1}\left[2\binom{K-S-1}{S-1} + \binom{K-S-1}{S}\right].$$

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Checking of Λ. A pair of elements occurs in $\binom{V-2}{K-S-2}$ $(K-S)$-sets. Each of these sets gives $\binom{K-S-2}{S-2}$ sets where both the elements of the pair are replicated twice. These sets count for 4 in Λ.

There are $\binom{K-S-2}{S-1}$ sets where one of the elements is in the S-set. Each of these sets counts for 2 in Λ. We have to multiply this number by 2 to take care of both the elements. The number of sets where neither of the two elements is in an S-set is $\binom{K-S-2}{S}$. These sets count for 1 in the counting of Λ. Hence

$$\Lambda = \binom{V-2}{K-S-2}\left[4\binom{K-S-2}{S-2} + 4\binom{K-S-2}{S-1} + \binom{K-S-2}{S}\right].$$

Theorem 2.2. *There exists a partially balanced ternary design with parameters*

$$V' = \binom{V}{2}, \quad B' = B,$$

$$R' = \binom{V-2}{K-S-2}\left[2\binom{K-S-2}{S-2} + 4\binom{K-S-2}{S-1} + \binom{K-S-2}{S}\right] + \binom{V-2}{K-S}\binom{K-S}{S},$$

$$K' = \binom{K-S}{2} + \binom{V-K+S}{2} + \binom{S}{2} + S(K-2S),$$

$$\Lambda_1 = \binom{V-3}{K-S-3}\left[4\binom{K-S-3}{S-3} + 12\binom{K-S-3}{S-2} + \binom{V-3}{K-S}\binom{K-S}{S}, +8\binom{K-S-3}{S-1} + \binom{K-S-3}{S}\right],$$

$$\begin{aligned}\Lambda_2 = {} & \binom{V-4}{K-S-4}\left[4\binom{K-S-4}{S-4} + 16\binom{K-S-4}{S-3} + 20\binom{K-S-4}{S-2} + 8\binom{K-S-4}{S-1} + \binom{K-S-4}{S}\right] \\ & + \binom{V-4}{K-S-2}\left[4\binom{K-S-2}{S-2} + 8\binom{K-S-2}{S-1} + 2\binom{K-S-2}{S}\right] \\ & + \binom{V-4}{K-S}\binom{K-S}{S}\end{aligned}$$

where V, B, R, K *and* S *are the same as in Theorem* 2.1.

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Proof. Consider a design X obtained from Theorem 2.1. The elements of the constructed design Y are the pairs of the elements of X. The blocks of Y are obtained in the following way from the blocks of X.

Let B be a block of X. Corresponding to B, there is a block in Y, which consists of two parts. The first part contains all the pairs of the elements of B, where if an element a occurs twice then it accounts for two pairs with any other element b of B. Notice that it does not matter whether b occurs in B once or twice. That is, even if the elements a and b both have occurred twice in B, the corresponding block in Y will contain the pair (a, b) only twice, not four times. The second part consists of all the pairs of the elements in the complement of B (all elements of the set of elements of X not in B).

Checking of the parameters.

Checking of R'. A pair (a, b) occurs in those blocks of Y where the corresponding block in X contains both a and b. The pair (a, b) occurs twice when either a or b or both occur twice in the block of X, otherwise it occurs once in the block of Y. (Notice the similarity in the first part of R' and of Λ in the design X.) The second part of R' is obtained by counting those blocks of X which do not contain both a and b.

Checking of K'. This is clear from the construction of a block of Y.

Checking of Λ_1. The pairs (a, b), (a, c) are first associates. There are $\binom{V-3}{K-S-3}$ $(K-S)$-subsets of the elements of X containing a, b and c. Each of these sets forms $\binom{K-S}{S}$ blocks of X containing a, b and c. Now depending on whether a block contains all of the elements a, b and c in the S-set or two of a, b and c in the S-set of only one of a, b and c in the S-set or none of a, b and c in the S-set, each block of Y accounts for 4, 4, 2 or 1 in Λ_1. We observe that there is only one difference, namely, if the element a is in an S-set then it accounts for 4 in Λ_1.

There are $\binom{V-3}{K-S}\binom{K-S}{S}$ blocks of X containing none of a, b, or c. The corresponding blocks in Y account for one each in Λ_1. Hence the value of Λ_1 is as given in the statement of the theorem.

Checking of Λ_2. Those pairs (a, b), (c, d) are second associates where a, b, c and d are distinct elements of X. A similar counting argument gives the value of Λ_2. We have to keep in mind that those blocks of X which contain either a and b or c and d also account for Λ_2 in Y.

Example 1. Similar to the example in the introduction, starting with the design ($V = 5$, $B = 20$, $R = 12$, $K = 3$, $\Lambda = 4$), we get a partially balanced ternary design with the parameters $(10, 20, 10, 5, 2, 8)$, which is a 2-multiple of a partially balanced ternary design $(10, 10, 6, 5, 1, 4)$ constructed in Theorem 3.5

Table 1 contains a list of partially balanced ternary designs constructed from Theorem 2.2 for small values of V, K and S. for easy reference the values of V, K and S are also given. The parameters are calculated on a personal computer with maxint 32767. Table 1 does not contain the parameters for which the designs are balanced or $K' > V'$. Table 2 contains balanced and symmetric balanced ternary designs for small values of V, K and S obtained from Theorem 2.2.

Table 1

List of partially balanced ternary designs obtained by Theorem 2.2 for $V \leq 10$, $K \leq 2V$, $S \leq K/2$.

No.	V	K	S	V'	B'	R'	K'	Λ_1	Λ_2
1	4	3	0	6	4	2	3	1	0
2	4	3	1	6	12	6	3	0	8
3	4	4	1	6	12	10	5	8	0
4	4	4	2	6	6	3	3	0	4
5	4	5	2	6	12	12	6	12	0
6	4	6	3	6	4	4	6	4	0
7	5	3	0	10	10	4	4	1	2
8	5	3	1	10	20	10	5	2	8
9	5	4	0	10	5	3	6	2	1
10	5	4	1	10	30	18	6	8	10
11	5	4	2	10	10	5	5	1	4
12	5	5	1	10	20	18	9	18	8
13	6	3	0	15	20	8	6	2	4
14	6	3	1	15	30	16	8	6	10
15	6	4	1	15	60	32	8	11	20
16	6	4	2	15	15	8	8	3	5
17	6	5	0	15	6	4	10	3	2
18	6	5	1	15	60	40	10	27	20
19	6	5	2	15	60	36	9	15	24
20	6	6	1	15	30	28	14	30	18
21	6	6	2	15	90	72	12	60	42
22	6	6	3	15	20	12	9	5	8
23	6	7	3	15	60	52	13	48	32
24	6	8	4	15	15	13	13	12	8
25	7	3	0	21	35	15	9	5	7
26	7	3	1	21	42	24	12	12	14
27	7	4	0	21	35	15	9	5	7
28	7	4	1	21	105	55	11	20	33

No.	V	K	S	V'	B'	R'	K'	Λ_1	Λ_2
29	7	4	2	21	21	12	12	6	7
30	7	5	0	21	21	11	11	6	5
31	7	5	1	21	140	80	12	40	44
32	7	5	2	21	105	60	12	24	39
33	7	6	0	21	7	5	15	4	3
34	7	6	1	21	105	75	15	60	41
35	7	6	3	21	35	20	12	8	13
36	7	7	1	21	42	40	20	44	30
37	7	7	2	21	210	180	18	174	118
38	7	7	3	21	140	100	15	68	64
39	7	8	3	21	210	200	20	216	146
40	7	8	4	21	35	25	15	17	16
41	7	9	4	21	105	105	21	120	80
42	7	10	5	21	21	21	21	24	16
43	8	3	0	28	56	26	13	11	12
44	8	4	0	28	70	30	12	10	14
45	8	4	1	28	168	90	15	38	52
46	8	5	0	28	56	26	13	11	12
47	8	5	1	28	280	150	15	65	84
48	8	5	2	28	168	96	16	42	60
49	8	6	0	28	28	16	16	10	8
50	8	6	1	28	280	170	17	105	92
51	8	6	2	28	420	255	17	130	158
52	8	6	3	28	56	32	16	14	20
53	8	7	0	28	8	6	21	5	4
54	8	7	1	28	168	126	21	110	76
55	8	7	2	28	560	400	20	300	248
56	8	7	3	28	280	180	18	100	116
57	8	8	1	28	56	54	27	60	44
58	8	8	2	28	420	375	25	390	270
59	8	8	3	28	560	440	22	370	296
60	8	8	4	28	70	45	18	25	29
61	8	9	3	28	560	560	28	650	456
62	8	9	4	28	280	230	23	205	160
63	8	10	5	28	56	46	23	41	32
64	9	3	0	36	84	42	18	21	20
65	9	3	1	36	72	46	23	30	28
66	9	4	0	36	126	56	16	21	26
67	9	4	1	36	252	140	20	68	80
68	9	4	2	36	36	23	23	15	14
69	9	5	0	36	126	56	16	21	26

No.	V	K	S	V'	B'	R'	K'	Λ_1	Λ_2
70	9	5	1	36	504	266	19	114	148
71	9	5	2	36	252	147	21	72	90
72	9	6	0	36	84	42	18	21	20
73	9	6	1	36	630	350	20	180	190
74	9	6	2	36	756	441	21	210	270
75	9	6	3	36	84	49	21	24	30
76	9	7	0	36	36	22	22	15	12
77	9	7	1	36	504	322	23	226	180
78	9	7	2	36	1260	805	23	495	490
79	9	7	3	36	504	308	22	156	196
80	9	8	0	36	9	7	28	6	5
81	9	8	1	36	252	196	28	180	128
82	9	8	2	36	1260	945	27	795	610
83	9	8	3	36	1260	875	25	600	570
84	9	8	4	36	126	77	22	39	49
85	9	9	1	36	72	70	35	78	60
86	9	9	2	36	756	693	33	750	534
87	9	9	3	36	1680	1400	30	1320	1000
88	9	9	4	36	630	455	26	330	305
89	9	10	4	36	1260	1120	32	1135	850
90	9	10	5	36	126	91	26	66	61
91	9	11	5	36	504	462	33	486	360
92	9	12	6	36	84	77	33	81	60
93	10	3	0	45	120	64	24	36	32
94	10	3	1	45	90	60	30	42	38
95	10	4	0	45	210	98	21	42	46
96	10	4	1	45	360	208	26	113	120
97	10	4	2	45	45	30	30	21	19
98	10	5	0	45	252	112	20	42	52
99	10	5	1	45	840	448	24	203	248
100	10	5	2	45	360	216	27	117	132
101	10	6	0	45	210	98	21	42	46
102	10	6	1	45	1260	672	24	315	364
103	10	6	2	45	1260	728	26	350	440
104	10	6	3	45	120	72	27	39	44
105	10	7	0	45	120	64	24	36	32
106	10	7	1	45	1260	728	26	427	396
107	10	7	2	45	2520	1512	27	819	908
108	10	7	3	45	840	504	27	252	316
109	10	8	0	45	45	29	29	21	17

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No.	V	K	S	V'	B'	R'	K'	Λ_1	Λ_2
110	10	8	1	45	840	560	30	427	328
111	10	8	2	45	3150	2100	30	1470	1290
112	10	8	3	45	2520	1624	29	966	1036
113	10	8	4	45	210	126	27	63	79
114	10	9	0	45	10	8	36	7	6
115	10	9	1	45	360	288	36	273	200
116	10	9	2	45	2520	1960	35	1771	1324
117	10	9	3	45	4200	3080	33	2415	2060
118	10	9	4	45	1260	840	30	525	550
119	10	10	1	45	90	88	44	98	78
120	10	10	2	45	1260	1176	42	1302	952
121	10	10	3	45	4200	3640	39	3675	2740
122	10	10	4	45	3150	2450	35	2065	1725
123	10	10	5	45	252	168	30	105	110
124	10	11	4	45	4200	3920	42	4270	3180
125	10	11	5	45	1260	1008	36	882	726
126	10	12	5	45	2520	2464	44	2821	2092
127	10	12	6	45	210	168	36	147	121
128	10	13	6	45	840	840	45	987	728
129	10	14	7	45	120	120	45	141	104

Table 2

List of balanced ternary designs obtained by Theorem 2.2 for $V \le 10$, $K \le 2V$, $S \le K/2$.

No.	V	K	S	V'	B'	R'	K'	Λ_1	Λ_2
1	5	5	2	10	30	21	7	12	12
2	5	6	3	10	10	7	7	4	4
3	6	4	0	15	15	7	7	3	3
4	7	6	2	21	210	140	14	86	86
5	8	3	1	28	56	34	17	20	20
6	8	4	2	28	28	17	17	10	10

3. Special cases

3.1, $S = 0$. When $S = 0$, Theorem 2.2 gives us the following theorem.

THEOREM 3.1 (Saha [4]). *There exists a series of PBIBD with parameters:*

$$V' = \binom{V}{2}, \quad B' = \binom{V}{k}, \quad R' = \binom{V-2}{K-2} + \binom{V-2}{K},$$
$$K' = \binom{K}{2} + \binom{V-K}{2}, \quad \Lambda_1 = \binom{V-3}{K-3} + \binom{V-3}{K},$$
$$\Lambda_2 = \binom{V-4}{K-4} + \binom{V-4}{K} + 2\binom{V-4}{K-2}.$$

As proved in Sinha [5], we get the following series of BIBDs as a special case of the above Theorem.

COROLLARY 3.2 (Sinha [5]). *There exists a series of BIBDs* $(V', B', R', K', \Lambda = \Lambda_1)$, *where* $V = 4w^2 + 4w + 3$ *or* $4w^2 + 2$ *and* $K = 2w^2 + w + 1$, *w any non zero integer and* V', B', R', K' *and* Λ_1 *are as given in Theorem* 3.1.

3.2, $S = 1$. Equating Λ_1 and Λ_2 in Theorem 2.2, we eventually get

$$(K-1)V^3 + (-5K^2 + 5K + 4)V^2 + (8K^3 - 10K^2 - 21K + 29)V - 4K^4 + 8K^3 + 7K^2 - 17K = 0.$$

Let $V = K + t$ for some integer t. Then $\Lambda_1 = \Lambda_2$ if

$$K^3(t+2) - K^2(2t^2 + 3t + 10) + K(t^3 + 2t^2 - 13t + 12) - t^3 + 4t^2 + 29t = 0.$$

We can check that for $K = 3$ to 200 and $t = 0$ to 200 only the following values of K and t satisfy the above equation:

No.	K	t
1	3	0
2	3	5
3	4	−1
4	30	35
5	36	95
6	126	117

That is, at least for the above six cases we get balanced ternary designs.

3.3, $V = K - S$. In this case Λ_1 and Λ_2 are given by

$$\Lambda_1 = 4\binom{K-S-3}{S-3} + 12\binom{K-S-3}{S-2} + 8\binom{K-S-3}{S-1} + \binom{K-S-3}{S}$$

and

$$\Lambda_2 = 4\binom{K-S-4}{S-4} + 16\binom{K-S-4}{S-3} + 20\binom{K-S-4}{S-2} + 8\binom{K-S-4}{S-1} + \binom{K-S-4}{S}.$$

Using

$$\binom{n}{r}+\binom{n}{r-1}=\binom{n+1}{r},$$

we can prove that

$$\Lambda_2=4\binom{K-S-3}{S-3}+12\binom{K-S-3}{S-2}+8\binom{K-S-3}{S-1}+\binom{K-S-4}{S}.$$

So if $K-S-3<S$ then $\Lambda_1=\Lambda_2$ and we get a balanced ternary design.

We get a symmetric design if $V'=B'$, but as we have $B'=\binom{V}{S}$, S must be equal to 2 or $V-2$.

3.3.1, $S=2$, $V=K-S$. In this case, $K-S-3<S$ gives $V<5$. We do not get any nontrivial symmetric balanced design.

3.3.2, $S=V-2$, $V=K-S$. In this case, $K=2V-2$. Now $K-S-3=V-3<S=V-2$ for all values of V greater than or equal to 3. Hence we get a trivial series of symmetric balanced ternary designs with parameters (where $K=2V-2$ and $S=V-2$)

$$V'=\binom{V}{2}=B',\quad R'=V^2-V-1=K',\quad \Lambda=2(V^2-V-2).$$

Notice that $K'=2V'-1$ and so every block has all but one of the V' elements occurring twice, and that one element just occurs once.

There are examples where $V=K-S$ and $\Lambda_1=\Lambda_2$ but the design is not symmetric. Table 2 gives some of the designs which are balanced but not symmetric.

3.4, $K=2S$. In this case Λ_1 and Λ_2 are given by

$$\Lambda_1=4\binom{V-3}{S-3}+\binom{V-3}{S}$$

and

$$\Lambda_2=4\binom{V-4}{S-4}+4\binom{V-4}{S-2}+\binom{V-4}{S}.$$

We can prove that

$$\Lambda_2=\Lambda_1-\left[4\binom{V-4}{S-3}-4\binom{V-4}{S-2}+\binom{V-4}{S-1}\right].$$

Hence, $\Lambda_1=\Lambda_2$ if

$$4\binom{V-4}{S-3}-4\binom{V-4}{S-2}+\binom{V-4}{S-1}=0.$$

For example when $V=8$, $K=14$ and $S=6$, the design is symmetric and balanced. This is a trivial case with $K'=2V'-1$.

Now the balanced design is symmetric only if $S=2$ or $S=V-2$.

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3.4.1, $S = 2$. In this case we get

$$4\binom{V-4}{-1} - 4\binom{V-4}{0} + \binom{V-4}{1} = 0.$$

This implies $V = 8$.

3.4.2, $S = V - 2$. In this case we get $V = 5$.

For easy reference we summarise the results in this subsection

THEOREM 3.4. *When $K = 2S$ only two symmetric designs are obtained. One of them, namely* $(10, 10, 19, 19, 36, 36)$ *has $K' > V'$, and hence only the other design is listed as number* 6 *of Table* 2.

3.5, $K = 3$. When $K = 3$, $S = 0$ or 1. We are interested in the case when $S = 1$; we see that there are blocks like $a\ a\ b$, $b\ b\ a$ in X which give two identical blocks in Y. Therefore counting these blocks only once we get a smaller partially balanced design. The result can be stated as

THEOREM 3.5. *There exists a partially balanced ternary design with parameters*

$$V' = \binom{V}{2} = B', \quad R' = \frac{(V-2)(V-3)+4}{2} = K',$$
$$\Lambda_1 = \frac{(V-3)(V-4)}{2}, \quad \Lambda_2 = \frac{(V-4)(V-5)+8}{2}.$$

We can check that in this case the only symmetric design obtained is when $V = 8$ and $K = 3$. The design so obtained is balanced and symmetric, and its 2-multiple is listed in Table 2 as number 5.

It will be interesting to see when the balanced design obtained by the construction given in Theorem 2.2 gives a multiple of a symmetric design. When is it possible to get a smaller, and if possible symmetric balanced ternary design? One possible candidate is $S = \lceil K/2 \rceil - 1$. Can the construction be generalised for n-ary designs? Can the construction be generalised so that we do not have to use the designs obtained in Theorem 2.1 but smaller designs?

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