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EPM ephemerides and relativity

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Abstract. In the seventies of the last century the EPM ephemerides (**E**phemerides of **P**lanets and the **M**oon) of IAA RAS originated and have been developed since that time. These ephemerides are based upon relativistic equations of motion of celestial bodies and light rays and upon relativistic time scales. The updated model of EPM2008 includes the new values of planet masses and other constants, the improved dynamical model with adding Trans–Neptunian Objects and the expanded database (1913–2008). More than 260 parameters have been determined while improving the planetary part of EPM2008 to 550000 observations. EPM2008 have been oriented to ICRF by including into the total solution the VLBI data of spacecraft near the planets. The real uncertainty of EPM ephemerides has been checked by comparison with the JPL's DE ephemerides. Some estimates of the post–model parameters have been obtained: $|1-\beta| < 0.0002$, $|1-\gamma| < 0.0002$, $\dot{G}/G = (-5.9 \pm 4.4) \cdot 10^{-14}$ per year, the statistic zero corrections to the planet perihelion advances.

Keywords. Relativity, celestial mechanics, ephemerides, radar astronomy

1. Historical introduction: general relativity in EPM ephemerides

In the seventies of the last century to support space flights the EPM ephemerides (**E**phemerides of **P**lanets and the **M**oon) of IAA RAS originated at about the same time as DE ephemerides and have been developed since that time.

After the brilliant explanation by Einstein the strange ($43''$/cy) discrepancy between theoretical predictions and observations of the secular motion of Mercury perihelion, the planet ephemerides are to be constructed on the basis of General Relativity. The relativistic basis for constructing ephemerides was provided many years ago in the papers by Estabrook (1971), Will (1974), and it has been used for JPL (Standish, 1976), IAA RAS (Krasinsky *et al.*, 1978), and MIT (Ash *et al.*, 1967) ephemerides for more than 30 years. However, for the workers in Russia the main guide was the book of Brumberg (1972). Moreover, the relativistic equations of the planet motion may be given in different coordinate systems of the Schwarzschild metric (parameter α), namely, standard, harmonic, izotropic, etc. However, planet coordinates turned out to be essentially different for the standard and harmonic systems. Brumberg (1979) proved that ephemeris construction and processing of observations should be done in the same coordinate system, in which case the dependence on the coordinate system (parameter α) vanishes. Later on, the resolutions of IAU (1991, 2000) recommended to use harmonic coordinates for BCRS. Actually, harmonic coordinates have been used for all modern ephemerides since long ago.

Our first ephemerides of the inner planets which were analytical (Krasinsky *et al.*, 1978) in contrast to more perfect analytical ephemerides of the Moon and planets by Chapront and Bretagnon were compared with optical and radar observations. Simultaneously we also computed numerical planet ephemerides. Our comparison revealed that numerical ephemerides were able to present accurate observations much better than any analytical theories did.

In the eighties of the previous century (for example, Krasinsky *et al.*, 1986) we tested relativistic effects processing the observations available at that time. A purely Newtonian theory was developed and results were tested by both the relativistic and the Newtonian theories. It was proved that the relativistic ephemeris for any observed planets provides considerably better fit of the observations (by 10%) than the Newtonian theory even if latter incorporates the observed perihelion secular motions. Moreover, at that time attempts to estimate PPN parameters β, γ and the rate of changing of gravitation constant $\dot{G}/G$ were also made.

All the modern ephemerides: DE – JPL (Folkner *et al.*, 2008), EPM – IAA RAS (Pitjeva, 2009), INPOP – IMCCE (Fienga *et al.*, 2008) are based upon relativistic equations of motion for celestial bodies and light rays as well as relativistic time scales. The numerical integration of the equations of celestial bodies motion has been performed in the Parameterized Post–Newtonian metric for General Relativity in the TDB time scale; the relativistic effects of the signal delay (the Shapiro effect), and path-bending of the radio-signal propagation in the gravitation field of the Sun, Jupiter, Saturn and the reduction of observations from the proper time of the observer to the coordinate time of the ephemerides are taken into account while processing observations.

2. Present EPM2008 ephemerides

EPM ephemerides are computed by numerical integration of the equations of celestial bodies motion in the barycentric coordinate frame of J2000.0 by Everhart (1974) method over the 400 years interval (1800–2200) using the program package ERA-7 (ERA: **E**phemeris **R**esearch in **A**stronomy) developed to support scientific research in dynamical and ephemeris astronomy (Krasinsky & Vasilyev, 1997). This paper concerns a planet part of the EPM ephemerides; the group of George Krasinsky is now developing a lunar part of the EPM ephemerides and fitting it to the LLR data (Yagudina, 2009).

The mass values of the planets have been taken from the recent best determinations by different authors obtained from the data of spacecraft orbiting and passing near planets or from the observations of satellites of these planets (http://maia.usno.navy.mil/NSFA/CBE.html). All other constants have been obtained inside the EPM2008 ephemeris fitting process.

The updated model of EPM2008 includes Eris (which surpasses Pluto in the mass) and the other 20 largest Trans–Neptunian Objects (TNO) into the process of the simultaneous numerical integration in addition to nine planets, the Sun, 301 biggest asteroids, the Moon as well as the lunar physical libration, and takes into account perturbations due to the solar oblateness and perturbation from the massive ring of small asteroids.

Moreover, some tests have been made for estimating the effect of other TNO on the motion of planets. Their perturbations have been modeled by the perturbation from a circular ring having a radius of 43 AU and the five versions of different masses. The minimum mass of this ring is equal to the mass of 100000 bodies with 100 km in diameter and density is equal to 2 g/cm^3, it amounts to 110 masses of Ceres. The maximum mass of the ring is expected to be 100 times the minimum mass. Other test versions of the TNO ring surpass the minimum mass by 25, 50, and 75 times. The effect of the ring is only noticeable for more accurate observations – the spacecraft data, especially for ones from spacecraft near Jupiter and Saturn. The rms residuals and the weight unit errors for the data after fitting the standard and test EPM ephemerides have shown that all the test masses of the TNO ring except the minimum mass are too large and make the

Table 1. Mean values and rms residuals for radiometric observations.

Planet	Type of data	Time interval	N	$< O - C >$	σ
MERCURY	τ [m]	1964–1997	746	0	575
VENUS	τ [m]	1961–1995	1354	-2	584
	Magellan dr [mm/s]	1992–1994	195	0	0.007
	MGN,VEX VLBI [mas]	1990–2007	22	1.6	3.0
	Cassini τ [m]	1998–1999	2	4.0	2.4
	VEX τ [m]	2006–2007	547	0.0	2.6
MARS	τ [m]	1965–1995	403	0	719
	Viking τ [m]	1976–1982	1258	0	8.8
	Viking $d\tau$ [mm/s]	1976–1978	14978	-0.02	0.89
	Pathfinder τ [m]	1997	90	0	2.8
	Pathfinder $d\tau$ [mm/s]	1997	7569	0	0.09
	MGS τ [m]	1998–2006	7342	0	1.4
	Odyssey τ [m]	2002–2008	5257	0	1.2
	MRO τ [m]	2006–2007	380	0	2.5
	spacecraft VLBI [mas]	1984–2007	96	0.0	0.7
JUPITER	spacecraft τ [m]	1973–2000	7	0.0	11.8
	spacecraft VLBI [mas]	1996–1997	24	-1.8	9.5
SATURN	spacecraft τ [m]	1979–2006	33	1.0	20.2
URANUS	Voyager-2 τ [m]	1986	1	1.9	105
NEPTUNE	Voyager-2 τ [m]	1989	1	0.0	14

Notes: VEX, MGS, Odyssey, MRO data are normal points representing about 400000 original observations.

data residuals worse. Thus, the upper limit of the mass of the TNO ring ($5.26{\cdot}10^{-8} M_{\odot}$) has been obtained.

Database, to which EPM2008 have been adjusted includes (in addition to previous observations since 1913) the recent spacecraft measurements, namely, ranging to Venus Express (VEX), Odyssey, Mars Reconnaissance Orbiter (MRO) and VLBI data of Odyssey and MRO (2006–2008), three-dimensional normal point observations of Cassini (2004–2006), along with CCD Flagstaff and TMO data of the outer planets and their satellites (2006–2008). These measurements have resulted in a significant improvement of planet orbits, especially for Venus and Saturn and the orientation of the EPM2008 ephemerides to ICRF. The most part of observations has been taken from the database of the IAU Commission 4 created by Myles Standish and continuing by William Folkner.

About 260 parameters have been determined while improving the planetary part of EPM2008 to more than 550000 data:

- the orbital elements of all the planets and 18 satellites of the outer planets observations those have been used to improve the orbits of these planets;
- the value of the Astronomical Unit in m;
- three orientation angles of the ephemerides relative to the International Celestial Reference Frame (ICRF) and their velocities;
- 13 rotation parameters of Mars and the coordinates of the three landers on the martian surface;
- masses of the ten asteroids that perturb Mars most strongly, mean densities for three taxonomic classes of asteroids (C, S, M), the mass and the radius of the asteroid ring, the ratio masses of the Earth and the Moon;
- the solar quadrupole moment (J_2) and 21 parameters of the solar corona for different conjunctions with the Sun;
- eight coefficients of Mercury's topography and the corrections to the surface levels of Venus and Mars;
- five coefficients of the phase effect correction for the outer planets;
- constant bias for spacecraft and some radar planet observations, that were interpreted as calibration errors of the instruments or as systematic errors of unknown origin;

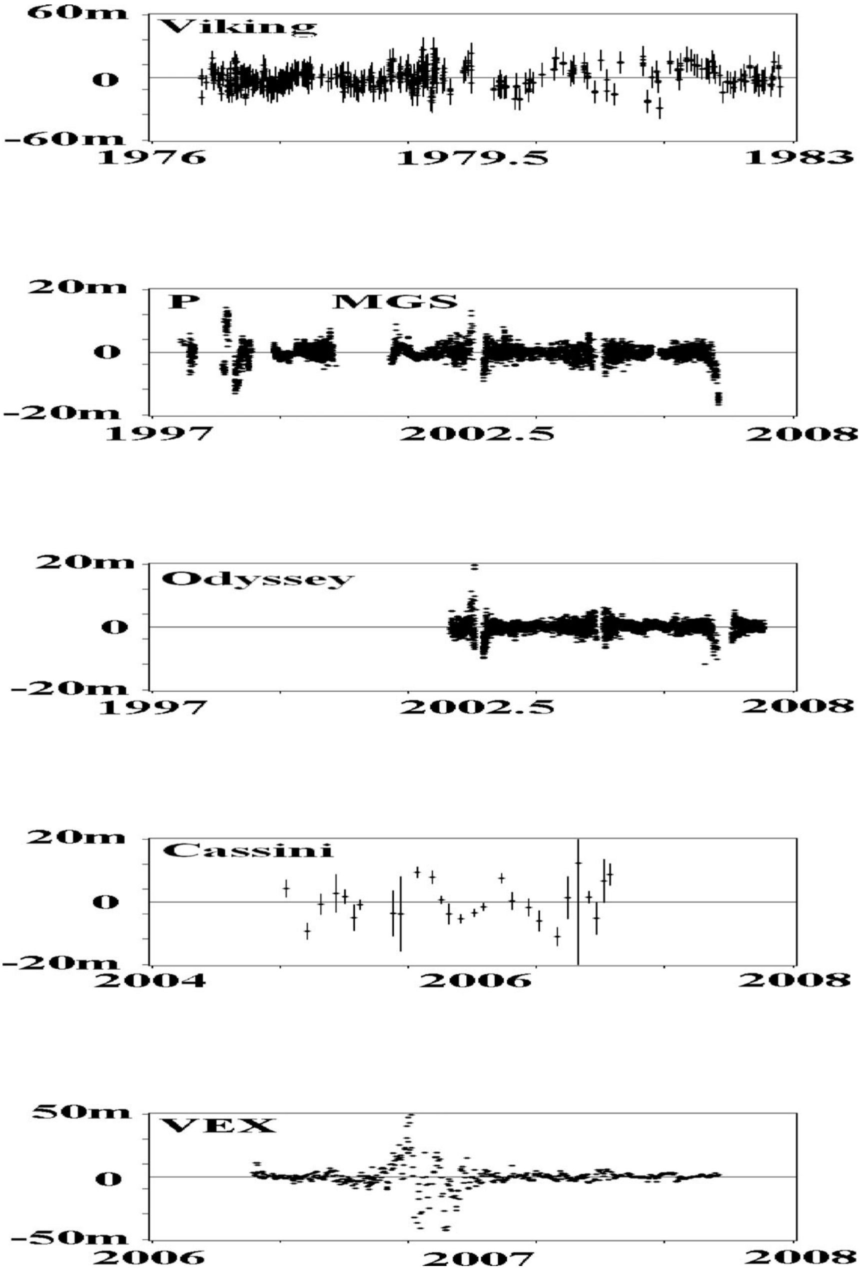

Figure 1. Viking, Pathfinder (P), MGS, Odyssey, Cassini, VEX range residuals

- the post-model parameters (β, γ, $\dot{G}/G$, secular trends of the planet perihelia and semi-major axes).

Mean values and rms residuals of observations are presented in Tables 1, 2 and on Fig. 1, 2. The data residuals don't exceed their *a priori* accuracies. The rms residuals of ranging for Viking are 8.8 m, for Pathfinder 2.8 m, for MGS and Odyssey 1.2–1.4 m, for Cassini (Saturn) 3.0 m, for VEX 2.6 m.

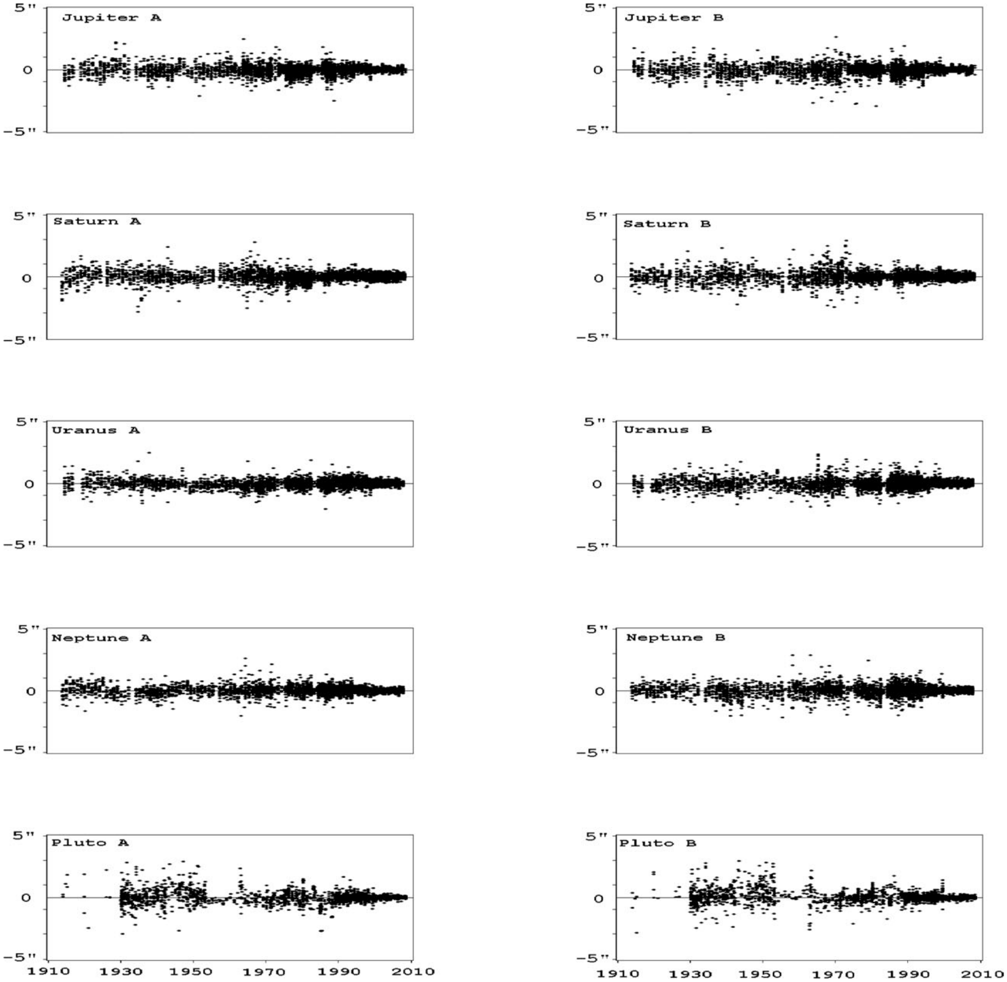

Figure 2. Residuals of the outer planets 1913–2008 in $\alpha\cos\delta$ (A) and in δ (B), the scale $\pm 5''$.

EPM2008 have been oriented to ICRF by including into the total solution the 118 ICRF-base VLBI measurements of spacecraft (Magellan, Phobos, MGS, Odyssey, VEX, and MRO) 1989–2007 near Venus and Mars. Several solutions for recent and previous data are given in Table 3.

The obtained values of the Astronomical Unit, the Moon-Earth mass ratio

$$\mathrm{AU} = (149597870697 \pm 3)\ \mathrm{m}, \quad M_{\mathrm{Earth}}/M_{\mathrm{Moon}} = 81.3005676 \pm 0.0000030$$

and masses of several asteroids (Table 4) are presented with their real uncertainties estimated by comparing the values obtained in dozens of different test LS solutions that differed by the sets of observations, their weights, and the sets of parameters included in the solution, as well as by comparing parameter values produced by independent groups. The discussion of these values and their comparison with values obtained by other authors are given in the paper by Pitjeva & Standish (2009).

3. Comparison of DE and EPM ephemerides

The differences between various ephemerides are useful to know since they are indicative of the realistic accuracies of the ephemerides. The comparison of our recent EPM2008

Table 2. Mean values and rms residuals for optical observations and spacecraft encounters* α and δ in mas, 1913–2008.

Planet	N	$< O - C >_\alpha$	σ_α	$< O - C >_\delta$	σ_δ
VENUS*	4	1.5	2.0	1	6.5
JUPITER	12518	15	187	-30	199
JUPITER*	16	0.1	1.9	-4.1	6.1
SATURN	14296	-1	167	-3	160
SATURN*	68	2.2	2.9	4.2	5.9
URANUS	11446	6	178	2	208
URANUS*	2	-45	9	-25	12
NEPTUNE	10982	7	160	9	205
NEPTUNE*	2	-11	3.5	-14	4.0
PLUTO	5134	1	191	6	197

Table 3. The rotation angles for the orientation of EPM onto ICRF.

Time interval	Number of obs.	ε_x mas	ε_y mas	ε_z mas
1989–1994	20	4.5±0.8	−0.8±0.6	−0.6±0.4
1989–2003	62	1.9±0.1	−0.5±0.2	−1.5±0.1
1989–2007	118	-1.53±0.06	1.02±0.06	1.27±0.05

Table 4. Masses of Ceres, Pallas, Juno, Vesta, Iris, Bamberga in $(GM_i/GM_\odot)\cdot 10^{-10}$.

(1) Ceres	(2) Pallas	(3) Juno	(4) Vesta	(7) Iris	(324) Bamberga
4.71 ±0.03	1.06 ±0.03	0.129 ±0.008	1.32 ±0.03	0.040 ±0.008	0.046 ±0.008

ephemeris with the standard DE405 and the latest DE421 ephemerides has been made (Fig. 3, Table 5). The differences of heliocentric distances for the inner planets between EPM2008 and DE405 or DE421 are small. It is necessary to say about the real accuracy of DE405 ephemerides. Right now, 12 years after the DE405 construction and 27 years after observations of Viking-1 (with more accurate data included in this ephemeris) the residuals for modern data for Odyssey don't surpass 200 m (Konopliv *et al.*, 2006), and as Fig. 3, Table 5 demonstrate it. It is evident that modeling the Mars motion is more difficult than other planets because of a large number of asteroids perturbing its orbit. The availability of a number of spacecraft near Jupiter and Saturn (besides optical observations) allows their ephemerides to be known better than those of other outer planets. The Fig. 3, Table 5 show the significant progress in agreement (and in reduction of uncertainties) of the orbits of all the planets especially owing to the VEX data (they were kindly given to us by Dr. Fienga) and the Cassini data.

4. Relativistic tests and estimation of post–model parameters

At present, the relativistic terms from the Sun and all the planets are included into the motion equations for integration. However, it is interest to estimate how relativistic terms from different planets influence modern planet observations. In addition to the basic EPM2008 (case 1) with the total account of all the relativistic terms in motion equations, three test ephemerides have been constructed:

a) without Saturn relativistic terms (case 2);
b) without Jupiter relativistic terms (case 3);
c) taking into account only solar relativistic terms (case 4).

Then all these ephemerides were improved to all 550000 observations of planets of different types (1913–2008). All the angular observations (both classical and modern CCD

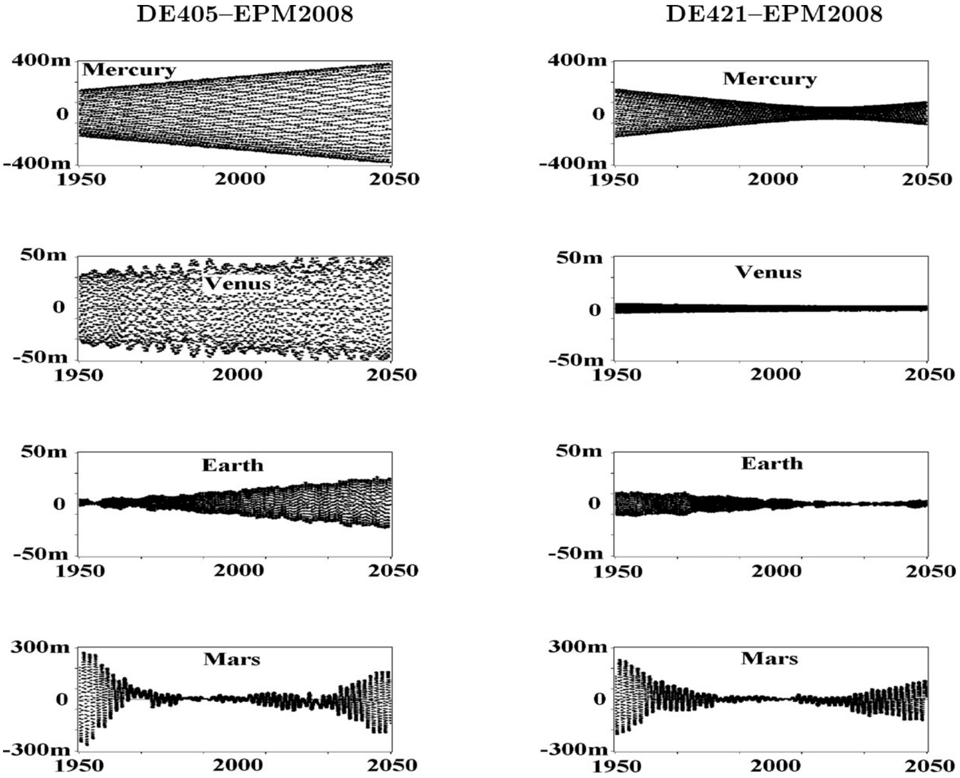

Figure 3. Differences in the heliocentric distances of inner planets for DE and EPM ephemerides, 1950–2050.

Table 5. Maximum differences in the heliocentric distances of planets for DE and EPM ephemerides, 1950–2050.

Planet	DE405–EPM2008	DE421–EPM2008
Mercury	384 m	185 m
Venus	53.7 m	4.6 m
Earth	26.8 m	11.9 m
Mars	272 m	233 m
Jupiter	19.7 km	4.8 km
Saturn	29.3 km	0.4 km
Uranus	864 km	310 km
Neptune	6100 km	848 km
Pluto	29000 km	1800 km

ones) and even VLBI data from spacecraft near planets don't show any differences in all cases. It is only the high-precision ranging that shows small differences. All the results are in given in Table 6. The results demonstrate that Saturn terms (from the comparison of the cases 1 and 2) don't affect the residuals of the observations. Comparison of the 3 and 4 cases show that all the other planet terms (except Jupiter) don't have effects also. Actually, for modern planet observations it is necessary to take into account only relativistic terms from the Sun and Jupiter; moreover, before the appearance of the recent spacecraft ranging data with 1–2 m accuracy it had been possible to take into account only the solar relativistic terms without introducting any errors.

The high-accuracy modern observations not only made it possible to improve the orbital elements of planets and values related to the ephemerides but enable to determine

Table 6. The rms residuals in m and the weight unit errors σ_0 for EPM ephemerides, accounting for different relativistic members.

Observations	planet ranging	Martian landers	Martian spacecraft	Venus Express	Cassini at Saturn	σ_0
Interval	1961–1997	1976–1997	1998–2008	2006–2007	2004–2006	1913–2008
Numbers n.p.	2504	1348	13903	547	31	97101
All relativity	612.20	11.75	2.03	2.59	3.29	0.874
Without Saturn	612.37	11.76	2.03	2.62	3.29	0.875
Without Jupiter	613.14	11.95	2.36	3.74	5.24	0.916
Only Sun	613.25	12.04	2.37	3.85	5.63	0.926

Table 7. Variations of $\dot{G}/G$ and $\dot{a}_i/a_i = \mathrm{R}_i$ per year with 3σ uncertainties.

$\dot{G}/G \cdot 10^{-14}$	$\mathrm{R}_{Ve} \cdot 10^{-14}$	$\mathrm{R}_{Ea} \cdot 10^{-16}$	$\mathrm{R}_{Ma} \cdot 10^{-16}$	$\mathrm{R}_{Ju} \cdot 10^{-12}$	$\mathrm{R}_{Sa} \cdot 10^{-13}$
-5.87 ±4.44	8.99 ±8.73	1.36 ±0.99	2.36 ±1.65	9.14 ± 69.48	6.74 ±50.73

some small physical parameters characterizing the fundamental properties of our physical space. The EPM2008 ephemerides have been used to analyse these data. Unfortunately, the real accuracy of the parameters is reduced by order of magnitude or more because of systematic errors of observations of an unknown origin, impossibility to completely allow for the delay in the solar corona, and large correlations between parameters. However some estimations may be obtained, their real uncertainties were obtained from the comparison of many different versions of the solution. The PPN parameters and the quadrupole moment of the Sun (J_2) producing various secular and periodic effects in orbital elements of planets have been estimated from the simultaneous solution:

$$J_2 = (2.0 \pm 0.5) \cdot 10^{-7}, \ |\beta - 1| < 0.0002, \ |\gamma - 1| < 0.0002.$$

The variability of $G\dot{M}_\odot/GM_\odot$ should cause the corresponding variation of the semi-major axes of the planetary orbits. In this case the angular momentum integral holds: $GM_\odot(t) \cdot a(t) = \mathrm{const}$, then $G\dot{M}_\odot/GM_\odot = -\dot{a}_\mathrm{i}/a_\mathrm{i}$.

As Dr. Nikolay Pitjev has proposed an attempt to estimate these values. The values of variation of the semi-major axes of the planetary orbits are found stable and have a quite good accuracy for planets covered by the high-accurate data from spasecraft. The results obtained simultaneously for $\dot{G}/G$ and $\dot{a}_\mathrm{i}/a_\mathrm{i}$ per year of the semi-major axes with their 3σ uncertainties are given in Table 7. It is to be noted that all the semi-major axes of the planets are increasing while $\mathrm{GM}_\odot$ decreases (the Sun is losing its mass), as it should be. The average weighted value obtained from $\dot{a}_\mathrm{i}/a_\mathrm{i}$ is

$$G\dot{M}_\odot/GM_\odot = (-1.63 \pm 1.50) \cdot 10^{-16} \text{per year}.$$

This result is preliminary, it demands further improvement and discussion. The obtained value is significantly less than the adjusted $\dot{G}/G$ value (Table 7) and the supposed mass reduction of the Sun owing to the solar radiation and wind (of the order $-8 \cdot 10^{-14}$/year). This discrepancy may to be part of the reason for comet falling on the Sun. The main result (Table 7) is

$$\dot{G}/G = (-5.9 \pm 4.4) \cdot 10^{-14} \text{per year} (3\sigma).$$

The corrections to the perihelion advances for the planets show to what extent the constructed model of the planet motion corresponds to the observations. In particular, the corrections for the inner planets demonstrate correspondence to General Relativity and to the value of the solar oblateness included into ephemerides. The corrections for the outer planets show agreement or non-agreement to the Newtonian theory of gravitation.

Table 8. Corrections to the perihelion advances of planets ($''$/ cy) and their real uncertainties.

Mercury	Venus	Earth	Mars	Author
42.98	8.62	3.84	1.35	Brumberg, 1972
0.11±0.22	−3.03±0.71	−0.12±0.16	−0.35±0.24	Pitjeva, 1986
−0.017±0.052	—	—	—	Pitjeva, 1993
−0.0040±0.0050	0.024±0.033	0.006±0.007	−0.007±0.007	Pitjeva, 2009
Jupiter	Saturn	Uranus	Neptune	Pluto
0.067±0.093	−0.010±0.015	− 3.89±3.90	-4.44±5.40	2.84±4.51

The obtained values (Table 8) are within the limits of their real uncertainties, in other words, the corrections to the planet perihelion advances are statistic zero.

5. Conclusion

Further improvement of the planet ephemerides and their parameters depends on the accuracy of modeling which results from the better knowledge of masses of celestial bodies including asteroids and TNO as well as decreasing errors of radiometrical data which originate from the ageing delay due to the solar corona and the spacecraft transponder.

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