

- 20) —
 21) 60
 22) × Note minutes to first decimal place

Steps 19 and 22 combine to give the ZD in degrees and minutes.

Recently Hewlett-Packard have introduced a new instrument, the HP-65, which enables the above program to be recorded on magnetic tape and then subsequently used to calculate automatically the ZD for any given values of LHA, D.R. Lat and Dec. This particular computer handles degrees, minutes and seconds directly—which can be an advantage.

The calculation of LHA is, of course, carried out in the usual manner. The intercept is also found in the normal way by comparing the calculated zenith distance with the observed zenith distance. The azimuth can easily be found from ABC³ or similar tables.

The time taken to plot a position line using this method is appreciably less than can be achieved by the use of sight reduction tables. Moreover, while it is agreed that reduction tables provide adequate accuracy, the possibility of making an error, particularly in interpolation, is much more likely than when using a computer calculator.

The main advantage, however, of the proposed method results from the fact that the position line can be plotted from the D.R. position—there is no need to work out a chosen position. Hence if a number of simultaneous sights are used they can all be plotted from the same point, i.e. the D.R. or Estimated Position of the ship. It follows that if the Estimated Position is reasonably accurate and the plot is made from this position, then the intercept will be short and the accuracy of the plot correspondingly high.⁴

REFERENCES

- ¹ Brown, H. H. *Nicholl's Concise Guide*, volume II. Glasgow.
- ² Cotter, C. H. *The Complete Nautical Astronomer*. London.
- ³ Norie, J. W. *Norie's Nautical Tables* by F. N. Hopkins, London.
- ⁴ *Admiralty Manual of Navigation*, volume II. London.

Air Navigation with a Pocket Electronic Calculator

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SOME navigators may prefer other methods and other tools, but the Hewlett Packard HP-45 has some fascinating abilities. It might be called the poor man's INS computer, though it does not of course find position. Its only disadvantages are the need for a hood in sunlight and the need for concentration and freedom from distraction. It can be used on the knee for no table is required, and it replaces conversion tables and several other mathematical tables. To a large extent it can replace plotting charts, flight planning charts, Dalton computers, protractors and dividers.

It simplifies ground speed/distance/time and fuel calculations especially, with its ability simply to convert hours, minutes and seconds or degrees, minutes and seconds to and from decimals. It calculates in the decimal mode.

Now that the United States and Canada allow straight line distances of more than 1000 miles at jet cruising level, it is useful to be able to find great circle or rhumb-line tracks between points on different charts. The needs of INS have forced chart makers to print the latitude and longitude of each airways point, so latitude and longitude can easily be read off the chart and entered into the memories of the calculator. Once the calculator is loaded with the data a standard routine program can easily be keyed in, often by memory, to produce track or distance within a minute or two. An example is the routine here set out in full for finding rhumb-line course C_r and distance D_r given L_A and L_B , the latitudes of A and B, and λ_A and λ_B , the longitudes of A and B. The formulae are:

$$\tan C_r = \frac{\text{ch}(\text{long}) \times \frac{\pi}{180}}{\text{Log}_e \tan \frac{\pi}{360} (90 + L_A) \sim \text{Log}_e \tan \frac{\pi}{360} (90 + L_B)}$$

$$D_r = \frac{\text{ch}(\text{lat.}) \times 60}{\cos C_r} \text{miles}$$

Care is needed with degrees and radians; before pressing a trigonometrical key, the appropriate mode must be selected.

Routine for rhumb-line course and distance

DEG

 L_A
 DMS →
 STO 1
 L_B
 DMS →
 STO 2
 RCL 1
 -
 if -ve, CHS
 STO 8

 λ_A
 DMS →
 STO 3
 λ_B
 DMS →
 STO 4

 RCL 3 + or -
 if -ve, CHS
 STO 5
 $\pi \times$
 180 ÷

 90 ENTER
 RCL 1 +
 $\pi \times$
 360 ÷
 RAD

TAN
 L_n
 if -ve, CHS

 90 ENTER
 RCL 2 +
 $\pi \times$
 360 ÷
 RAD
 TAN
 L_n
 if -ve, CHS

 -
 if -ve, CHS
 ÷
 DEG
 TAN -1
 STO 7

 if course in NE quadrant, read course
 if course in SE quadrant, CHS 180 +, read course
 if course in SW quadrant, 180 +, read course
 if course in NW quadrant, CHS 360 +, read course

 RCL 8
 $60 \times$
 RCL 7
 DEG
 COS ÷ STO 6
 read distance

Great circle tracks and distances can easily be calculated. The equator poses no problem if north latitude is entered positive and south latitude negative. Care is needed with $\text{ch}(\text{long.})$ when crossing longitude 180° ; if the quadrant of the track is uncertain, the cosine formula removes ambiguity. To assume the Earth is spherical seems accurate enough for all practical purposes. Intermediate points on tracks can be found by entering the appropriate formula with the longitude of the intermediate point to find its latitude. A variety of great circle formulae is available; if the approximate track is known, the sine formula for track, which is quickest but ambiguous, can be used. The formulae for solving spherical triangles can also be used for reducing astro sights.

Where airways tracks and distances are known, but clearance is received to go direct, the track and distance can be calculated by vector addition without recourse to latitude and longitude. Each track and distance is keyed in and converted to rectangular coordinates; all the coordinates are summed and the sums are converted back to polar coordinates, giving the required track and distance. Vectorial addition is much simpler than the rhumb-line or great circle routines (Fig. 1).



FIG. 1

Now that fuel is so expensive it is important to find the shortest route and the calculator makes this easy, helped by the tracks and distances marked on airways charts.

Wind can be dealt with in two ways; the triangle of velocities can generally be solved either algebraically by formula or vectorially by letting the calculator convert to and from rectangular coordinates. But if track and wind are given drift and ground speed can only be found algebraically. Note that wind is defined as the direction *from* which it is blowing, unlike heading and track.

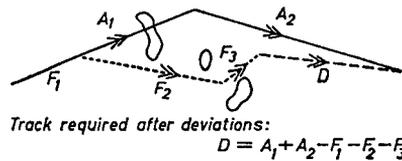


FIG. 2

Vectorial summation can also be used for deviations from track round thunderstorms (Fig. 2), for air plots or ground plots and for fixing (Fig. 3). Convenient fixes can be obtained as DME distances, due north or south of VORs whose latitude and longitude are known, or as VOR radials and DME distances from points of a network of known tracks and distances. In the former case future track and distance come from great circle or rhumb-line formulae; in the latter case from vectorial summation. Single headings through many sectors with different winds save a little time; the average wind can easily be found by

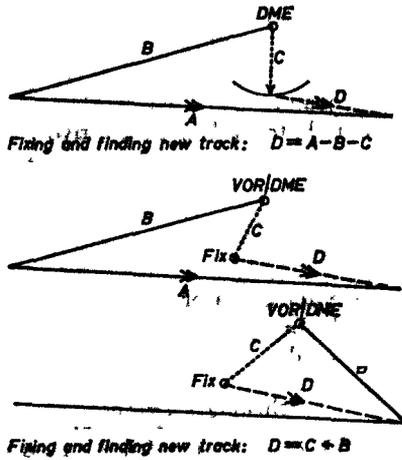


FIG. 3

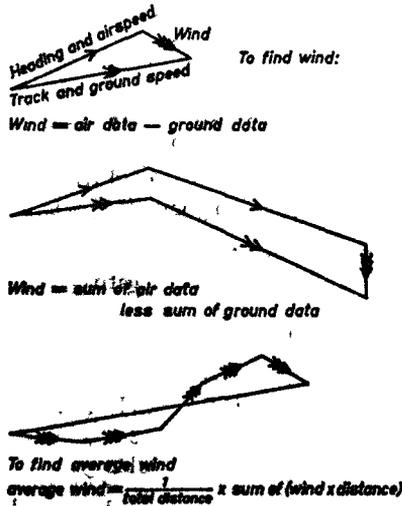


FIG. 4

vectorial summation of winds weighted by sector distance (Fig. 4). In the following formulae:

- C_r = rhumb-line track
- C_A = initial great circle track
- C_B = final great circle track
- C_x = great circle track at point X
- D_r = rhumb-line distance
- D_B = great circle distance from A to B
- D_x = great circle distance from A to X
- L_A = latitude of A, positive if north, negative if south
- L_B = latitude of B

- L_X = latitude at X
- λ_A = longitude of A
- λ_B = longitude of B
- λ_E = longitude at equator
- ch(lat.) = change of latitude from A to B, positive
- ch(long.) = change of longitude from A to B, positive
- G/S = ground speed
- TAS = true air speed
- WS = wind speed

Rhumb-line formulae: (i) enter north or south latitude positive
 (ii) before pressing trig. or DMS keys, select angular mode.

$$\tan C_r^\circ = \frac{\text{ch}(\text{long.}) \times \pi / 180}{\text{Log}_e \tan [(L_A + 90) \times \pi / 360] \sim \text{Log}_e \tan [(L_B + 90) \times \pi / 360]}$$

$$D_r = \text{ch}(\text{lat.}) \times 60 / \cos C_r \text{ nautical miles}$$

$$\cos C_r = \text{ch}(\text{lat.}) \times 60 / D$$

To convert C_r to track:

- quadrant NE read track
- SE CHS 180 + read track
- SW 180 + read track
- NW CHS 360 + read track
- $\lambda_E \approx \lambda_A \pm L_A \text{ ch}(\text{long.}) / (L_A + L_B)$

Great circle formulae: enter north latitude positive
 south latitude negative

$$\cos D_B = \sin L_A \sin L_B + \cos L_A \cos L_B \cos \text{ch}(\text{long.})$$

$$\begin{aligned} \sin C_A &= \sin \text{ch}(\text{long.}) \cos L_B / \sin D_B \\ \sin C_B &= \sin \text{ch}(\text{long.}) \cos L_A / \sin D_B \end{aligned} \left\{ \begin{array}{l} \text{NE} \text{ read track} \\ \text{SE} \text{ CHS } 180 + \\ \text{SW} \text{ } 180 + \\ \text{NW} \text{ CHS } 360 + \end{array} \right.$$

$$\cos C_A = \{ \sin L_B - \sin L_A \cos D_B \} / \cos L_A \sin D_B \left\{ \begin{array}{l} \text{E} \text{ read track} \\ \text{W} \text{ CHS } 360 + \end{array} \right.$$

$$\cot C_A = \{ \cos L_A \tan L_B - \sin L_A \cos \text{ch}(\text{long.}) \} / \sin \text{ch}(\text{long.}) \left\{ \begin{array}{l} \text{NE} \text{ read track} \\ \text{SE} \text{ } 180 + \\ \text{SW} \text{ CHS } 180 + \\ \text{NW} \text{ CHS } 360 + \end{array} \right.$$

$$\cot C_B = \{ \cos L_B \tan L_A - \sin L_B \cos \text{ch}(\text{long.}) \} / \sin \text{ch}(\text{long.}) \left\{ \begin{array}{l} \text{NE} \text{ CHS} \\ \text{SE} \text{ CHS } 180 + \\ \text{SW} \text{ } 180 + \\ \text{NW} \text{ } 360 + \end{array} \right.$$

Great circle formulae for intermediate point:

$$\tan L_X = \{ \sin L_A \cos \text{ch}(\text{long.})_{A/X} + \sin \text{ch}(\text{long.})_{A/X} / \tan C_A \} / \cos L_A$$

$$\sin C_X = \sin C_A \cos L_A / \cos L_X \begin{cases} \text{NE} & \text{read track} \\ \text{SE} & 180 + \\ \text{SW} & \text{CHS } 180 + \\ \text{NW} & \text{CHS } 360 + \end{cases}$$

$$\cos D_X = \sin L_A \sin L_X + \cos L_A \cos L_X \cos \text{ch}(\text{long.})_{A/X}$$

Wind formulae: $\begin{cases} \text{sign of drift} & \text{direction} \\ + & \text{starboard} \\ - & \text{port} \end{cases}$

$$\sin \text{drift} = \{(\text{wind speed}) \sin(\text{track} - \text{wind direction})\} / \text{TAS}$$

$$G/S = \text{TAS} \cos \text{drift} - \text{WS} \cos(\text{track} - \text{wind direction})$$

Suggested use of memories or storage registers:

1	2	3	4	5	6	7	8	9
L_A	L_B	λ_A	λ_B	$\text{ch}(\text{long.})$	D_B	C_A	$\text{ch}(\text{lat.})$ or X	nil