

“Ambipolar diffusion” and magnetic reconnection

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Abstract. Based on the three-fluid approximation the influence of the neutral component of hydrogen plasma on Joule dissipation of electric currents are considered. As distinguished from Mestel & Spitzer (1956) and Parker (1963) it has been shown that the magnetic flux may be not conserved in the case of the “ambipolar diffusion” due to collisions between ions and neutrals. This is explained by the ion acceleration under the action of Ampere’s force. Joule dissipation is determined by electron and ion collisions in a partially ionized plasma. Plasma evacuation from current sheets is the effective mechanism of its cooling. Thickness of a current sheet can achieve up to hundreds of kilometers in the solar chromosphere. The origin of the solar chromospheric jets observed with the Hinode satellite are discussed.

Keywords. plasmas, magnetic fields, diffusion, stars: formation, Sun: chromosphere

1. Introduction

Magnetic field diffusion and dissipation in a partially ionized plasma were considered in detail more than a half of century ago by Piddington (1954), Mestel & Spitzer (1956), and Cowling (1957). These processes play an important role in many space phenomena. Since different theoretical approaches are used up to now (e.g., Zaitsev & Stepanov 1992, Zweibel & Brandenburg 1997), this problem should be studied additionally.

Mestel & Spitzer (1956) studied star formation from the magnetic dust cloud for the first time. It was pointed out that the cloud cannot break up into fragments of mass less than $500M_{\odot}$ due to the magnetic flux conservation. However, this difficulty can be remedied if we take into account the freezing of magnetic field lines into the charged particles. According to the proposed mechanism named the “ambipolar diffusion” (e.g., Parker 1963; Spitzer 1978; Zweibel & Brandenburg 1997), the force of magnetic tension, arising due to the gravitational contraction, can drive the ionized matter and frozen-in magnetic field lines through a “gas” of neutrals thereby decoupling the field from the neutral matter. This suggests that the “ambipolar diffusion” does not alter the total magnetic flux, it simply redistributed the flux within the plasma. Consequently, the proposed mechanism cannot directly result in magnetic reconnection.

In contrast to Mestel–Spitzer’s approach, Cowling (1957) turned to the generalized Ohm’s law. He showed that the electrical conductivity (Cowling conductivity) of the non-stationary plasma can be significantly decreased owing to the ion acceleration by Ampere’s force. Collisions between ions and neutral particles become very effective in view of the high ion velocities. As a result, the magnetic flux is not conserved and the rate of magnetic reconnection might be considerably increased because of Joule (Ohmic) dissipation.

The influence of Mestel–Spitzer’s “diffusion” on magnetic reconnection was considered by many authors (e.g., Parker 1963; Zweibel 1989; Vishniac & Lazarian 1999). On the other hand, Tsap (1994) and Ni *et al.* (2007) proceeded from the generalized Ohm’s law but they did not take into account the plasma evacuation in the energy balance of a current sheet.

2. Joule dissipation in a partially ionized plasma

Using the standard notation, the simplified momentum equations for electrons (e), ions (i), and neutrals (n) of hydrogen plasma can be written as

$$n_e m \frac{d\mathbf{V}_e}{dt} = -en_e \mathbf{E} - \frac{en}{c} \mathbf{V}_e \times \mathbf{B} + n_e m \nu_{ei} (\mathbf{V}_i - \mathbf{V}_e) + n_e m \nu_{en} (\mathbf{V}_n - \mathbf{V}_e); \quad (2.1)$$

$$n_i M \frac{d\mathbf{V}_i}{dt} = en_i \mathbf{E} + \frac{en_i}{c} \mathbf{V}_i \times \mathbf{B} + n_i M \nu_{in} (\mathbf{V}_n - \mathbf{V}_i) + n_i M \nu_{ie} (\mathbf{V}_e - \mathbf{V}_i); \quad (2.2)$$

$$n_n M \frac{d\mathbf{V}_n}{dt} = n_n M \nu_{ni} (\mathbf{V}_i - \mathbf{V}_n) + n_n M \nu_{ne} (\mathbf{V}_e - \mathbf{V}_n). \quad (2.3)$$

It will be noted that adding termwise (2.1)–(2.3) and taking into account that $n_i = n_e = n$, introducing the velocity of plasma with density $\rho = M(n + n_n) = \rho_i + \rho_n$ as whole

$$\mathbf{v} = \frac{n\mathbf{V}_i + n_n\mathbf{V}_n}{n + n_n}, \quad \mathbf{v}_i = \mathbf{V}_i - \mathbf{v}, \quad \mathbf{v}_n = \mathbf{V}_n - \mathbf{v}, \quad (2.4)$$

at $|\mathbf{v}| \gg |\mathbf{v}_i|, |\mathbf{v}_n|$ we get the MHD momentum equation

$$\rho \frac{d\mathbf{v}}{dt} = \frac{\mathbf{j} \times \mathbf{B}}{c}. \quad (2.5)$$

In the case of weakly ionized plasma, when the number density of neutrals $n_n \gg n$, neglecting by collisions with electrons and inertial terms, equations (2.1) and (2.2) yield

$$\mathbf{V}_i = \mathbf{V}_n + \frac{\mathbf{j} \times \mathbf{B}}{nM\nu_{in}c}, \quad (2.6)$$

where the electric current $\mathbf{j} = en(\mathbf{V}_i - \mathbf{V}_e)$. Equation (2.6) describes the ion drift under action of Ampere’s force, i.e., the “ambipolar diffusion”. This term becomes widely used probably due to Parker (1963) (see also Nakano *et al.* 2002). However, in spite of the charge separation, the discussed process has not any relation to the classical (real) ambipolar *diffusion*, which are caused by the density gradient in the collisional inhomogeneous plasma.

Mestel & Spitzer (1956) suggested that magnetic field lines are frozen into electrons and ions and the magnetic flux is conserved. Therefore the “ambipolar diffusion” did not result in Joule dissipation (see also Parker 1963; Shu *et al.* 1987; Zweibel & Brandenburg 1997). In our opinion, such approach is not quite correct because of the following reasons.

Multiplying equations (2.1), (2.2), and (2.3) by \mathbf{V}_e , \mathbf{V}_i , and \mathbf{V}_n , respectively, using equation (2.4), after some algebra, we find that the electric energy is

$$\mathbf{j}\mathbf{E} = \rho \frac{d\mathbf{v}^2}{2dt} + nM\nu_{in} (\mathbf{V}_n - \mathbf{V}_i)^2 + nm\nu_{ei} (\mathbf{V}_i - \mathbf{V}_e)^2 + n_n m \nu_{ni} (\mathbf{V}_n - \mathbf{V}_e)^2. \quad (2.7)$$

As is easily seen from equation (2.7), the energy of magnetic field is transformed into the

kinetic and thermal ones associated with electron and ion collisions. Substituting (2.5) into (2.7), we get

$$\mathbf{jE} = (\mathbf{j} \times \mathbf{B})\mathbf{v}/c + Q,$$

where Joule dissipation is

$$Q = \mathbf{E}^*\mathbf{j} = nM\nu_{in}(\mathbf{V}_n - \mathbf{V}_i)^2 + nm\nu_{ei}(\mathbf{V}_i - \mathbf{V}_e)^2 + n_n m\nu_{ni}(\mathbf{V}_n - \mathbf{V}_e)^2, \tag{2.8}$$

and $\mathbf{E}^* = \mathbf{E} + \mathbf{v} \times \mathbf{B}/c$. As follows from (2.8), Joule dissipation is a work of the electric field on the electric current without the mechanical energy and it equals the sum of terms describing collisions between particles. Meanwhile, Mestel & Spitzer (1956) (see also Parker 1963) judged that Joule dissipation is only caused by electron collisions and did not take into consideration the ion–neutral ones.

Following Cowling (1957) (see also Zaitsev & Stepanov 1992), introducing the neutral density fraction $F = n_n/(n + n_n)$, it is easy to show from (2.1)–(2.3) that the generalized Ohm’s law at $\mathbf{j} \perp \mathbf{B}$ can be represent in terms of the Cowling (σ_C) and Spitzer (σ_S) conductivities in the form (Tsap 1994)

$$\left(\mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{B}\right)_\perp = \left(\frac{1}{\sigma_C} + \frac{1}{\sigma_S}\right)\mathbf{j}_\perp, \quad \sigma_C = \frac{c^2 n_n M\nu_{ni}}{F^2 B^2}, \quad \sigma_S = \frac{ne^2}{m(\nu_{ei} + \nu_{en})}. \tag{2.9}$$

Note that the value of F is arbitrary and the Cowling conductivity σ_C describes the energy release caused by the ion–neutral collisions.

3. Sweet–Parker model, Cowling conductivity, and solar chromospheric jets

Scaling laws, following from equations of continuity, equilibrium, and magnetic diffusion, at $\sigma_C \ll \sigma_S$ can be written as

$$\rho Lv = \rho_o l v_o; \quad \frac{1}{2}\rho_o v_o^2 = p_o - p; \quad \frac{B^2}{8\pi} + p = p_o; \quad vB = \eta_C \frac{B}{l}; \tag{3.1}$$

where the lower index (o) denotes parameters inside a current sheet, L and l are the half width and half thickness of a current sheet, respectively, v and v_o are the inflow and outflow velocities, $\eta_C = c^2/(4\pi\sigma_C)$ is the diffusion coefficient.

Combining equations (3.1), we obtain the rate of magnetic reconnection

$$v \approx \left(\frac{c^2 v_A}{4\pi\sigma_C L}\right)^{1/2},$$

which is in agreement with results of Vishniac & Lazarian (1999), Tsap (1994), and Ni *et al.* (2007).

As follows from scaling laws (3.1) and Ampere’s law the Joule dissipation rate is

$$Q = \frac{j^2}{\sigma_c} \approx \frac{B^2}{4\pi} \frac{v_A}{L} \frac{\rho_o}{\rho}. \tag{3.2}$$

Since $n_o kT_o \approx B^2/8\pi$, the characteristic Ohmic heating time

$$\tau_h \approx \frac{3n_o kT_o}{2Q} \approx \frac{3B^2}{16\pi Q} \approx \frac{3}{4} \frac{\rho}{\rho_o} \tau_A. \tag{3.3}$$

In turn, the characteristic time of the dynamical cooling caused by plasma evacuation is $\tau_e \approx L/v_A$. Joule heating will not ionize plasma within a current sheet until $\tau_e \lesssim \tau_h$ or, as is easy to see from (3.3), $\rho_o \lesssim \rho$. It should be stressed that the energy loss due to

the dynamical cooling significantly exceeds energy losses caused by the plasma radiation and thermal conductivity in the solar chromosphere.

Effective collision frequencies ν_{ei} , ν_{ea} , and ν_{ia} under the condition of the solar atmosphere are

$$\nu_{ei} \approx \frac{60n}{T^{3/2}} [s^{-1}], \quad \nu_{en} \approx 5 \cdot 10^{-10} n_n \sqrt{T} [s^{-1}], \quad \nu_{in} \approx 10^{-10} n_n \sqrt{T} [s^{-1}]. \quad (3.4)$$

Adopting $B = 30$ G, $n_n = 10^{11} \text{ cm}^{-3}$, $n = 10^{10} \text{ cm}^{-3}$, $T = 10^4$ K, equations (2.9) and (3.4) give $\sigma_C \approx 10^7 \text{ s}^{-1}$ and $\sigma_S \approx 10^{13} \text{ s}^{-1}$, i.e. the Cowling conductivity is 6 orders of magnitude less than the Spitzer one. As follows from (3.1), the characteristic half thickness of a current sheet is

$$l \sim \sqrt{\eta_C L / v_A}. \quad (3.5)$$

Taking $L = 10^8$ cm, $\sigma_C = 10^7 \text{ s}^{-1}$, $v_A = 10^7$ cm/s, from (3.5) we obtain $l \sim 100$ km ($l \sim 100$ m for the Spitzer conductivity). Consequently, the thickness of a current sheet l can achieve hundreds of kilometers in the chromosphere.

Recently thin spicules and chromospheric jets were revealed with the SOT/Hinode telescope in the line Ca II H (De Pontieu et al. 2007; Shibata *et al.* 2007). These magnetic features with the characteristic widths 100–300 km and $\lesssim 200$ km, respectively are observed outside of solar spots and active regions. To our view, the origin of these phenomena can be connected with the Sweet–Parker magnetic reconnection.

4. Conclusions

- (a) There are two types of the ambipolar diffusions: real and formal.
- (b) Joule dissipation in a partially ionized plasma is determined by collisions of neutral particles not only with electrons but with ions too.
- (c) The “ambipolar diffusion” and the Cowling conductivity describe the same phenomena in the collisional partially ionized plasma.
- (d) Plasma evacuation is an effective mechanism of its cooling in a current sheet.
- (e) Sweet–Parker reconnection can give rise to the formation of thick (~ 100 km) current sheets in the solar chromosphere.

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