which is equivalent to the formula given by Robin McLean.

Yours sincerely,
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More about Pig

DEAR SIR,

In Note 63.27 (December 1979) by S. Humphrey on the game of Pig, the author seems to have missed one important point. The object of the game is, roughly, to collect as many points as possible from successive throws of two dice, with the caveat that if a six should appear then the score drops back to zero again. It is claimed that one should go for three throws and then quit.

This would indeed be the correct strategy if one had to decide at the beginning of one's turn how many throws to attempt, but the rules suggest that one can decide after each throw whether to throw again or not. If the objective is to maximise one's expected score it is possible to use the information obtained during one's turn to improve on this strategy. What is important, after all, in deciding whether to have another throw is not how many throws one has had already, but how many points one is putting at risk.

Let us suppose, then, that our current score for that turn is r points. If we have another throw then the score will be increased by 2, 3, 4, 5, 6, 7, 8, 9, 10 points with probabilities (1,2,3,4,5,4,3,2,1)/36 respectively and will be decreased by r points with probability 11/36. The expected increase for the next throw is thus

$$\frac{1 \times 2 + 2 \times 3 + \ldots + 1 \times 10 - 11 \times r}{36} = \frac{150 - 11r}{36},$$

which is positive if and only if $r < 14$. This shows that the optimal policy is to quit when one's score reaches 14 or more. This may well be after three throws but could be after 2, 4, 5, 6 or even 7 throws.

Nor is this the full story. One is after all playing against opponents and not just going for the highest expected score by oneself. There may well be times when it is worth risking all in order to reach the target before someone else, and other times when it is better to play safe. Since one's own strategy might well affect that of one's opponents, a complete analysis of the game would be quite complex, and we have not attempted to work out the details.

Yours faithfully,
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DEAR EDITOR,

In his Note S. Humphrey suggested that the best strategy when playing Pig was to aim to throw the pair of dice three times in a turn, and then stop. More generally, if the game is played with k dice, he suggested stopping after five or six throws for $k = 1$, two throws for $k = 3$ and one throw for $k > 3$.

This strategy can be improved upon by a player using his current score in a turn to decide whether or not to throw the dice again. If his present score for the turn is x, then, using k dice, his expected score after another throw is $(x + 3k)$ with a probability of $(\frac{5}{6})^k$, and 0 with the remaining probability. Therefore his expected score is

$$v_x = (x + 3k)(\tfrac{5}{6})^k.$$

If v_x is less than the score x he already has, then it is better to stop. Therefore he should stop once his score has reached or passed x_k, given in the table:

k	1	2	3	4	5	6	7	8
x_k	16	14	13	12	11	10	9	8

For $k \geqslant 8$ this strategy is equivalent to stopping after one throw, because with k dice it is impossible to achieve a score of less than k.

Humphrey's strategy produces expected scores E_H given in the table below for different values of k. The expected scores E_S using the score stopping rule are more difficult to compute. The figures given in the table were calculated using a computer.

k	1	2	3	4	5	6	7	8
E_H	6·03	6·03	6·03	5·79	6·03	6·03	5·86	5·58
E_S	6·15	6·12	6·06	5·98	6·05	6·03	5·86	5·58

It can be seen that the score stopping rule produces the higher expected scores. It is interesting to note that whereas using Humphrey's strategy the expected score is the same whether 1, 2, 3, 5 or 6 dice are used, the score stopping rule performs best with only one die.

However, neither of these strategies is the best one for playing Pig. The objective of the game is to be the first player to reach a score of 100. This is not the same as trying to produce the highest score on each turn. For example, if your opponent has a score of 99 and you have a score of 80 and have scored 19 so far for this turn, then it is better for you to throw again, otherwise you are very likely to lose! The best strategy will take into account your current score, your opponents' scores and whether the sole objective is to win or there is also a second prize. This becomes impossibly complex and in most circumstances the score stopping rule will probably be close to optimal. In extreme circumstances common sense may perform better.

Yours sincerely,

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A fuller analysis of the game will appear in the December issue. V.W.B.

A giant leap

DEAR EDITOR,

I first saw a problem similar to that discussed by Mr Ainley (*Gazette* **63**, 272 (No. 426, December 1979)) set as a Braintwister in the *Observer* in 1962, but no doubt it has arisen on other occasions as well. Coincidentally, the problem is also mentioned in the October 1979 issue of the *Bulletin* of the IMA.

The problem set in the *Observer* asked how large a span could be obtained by building a bridge with a set of twenty-eight dominoes, each 2″ by 1″ by $\frac{1}{2}$″, the bridge to consist of two self-supporting half-spans. This is essentially a matter of extending Mr Ainley's treatment from four bricks to thirteen (since one domino on each side must be used as the base). The answer given in the newspaper the following week was the one based on the model of two self-supporting single-stepped piles, namely

$$2(1 + \tfrac{1}{2} + \tfrac{1}{3} + \ldots + \tfrac{1}{13}) = 6{\cdot}36027''.$$

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